SIDDHĀNTA-DARPAṆA

(English Translation with Mathematical Explainations & Notes)

vol. II

ARUN KUMAR UPADHYAYA, IPS
Msc. AIFC

NAG PUBLISHERS

Present second volume translates all the verses in English. Translation is not literal but in mathematical terms, but preserving the technical terms in Sanskrit. Verses in praise of god have been left out, not because of disrespect. With all devotion inspired by Samanta Chandrashekhar, this is not the purpose of the second volume. In addition to translation, each formula has been explained or derived according to modern mathematics and astronomy. The methods have been compared with other Indian astronomers and some times with other countries and with modern astronomy. This was the method and purpose of Samanta himself.

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१. **नाम** - अरुण कुमार उपाध्याय, पिता श्री चन्द्रशेखर उपाध्याय तथा माता श्रीमती जगतारिणी देवी (दोनों का जन्म १९०५ में) की कनिष्ठ सन्तान।

२. **जन्म** - ३०/३१-८-१९५२ ईस्वीय सन् ठीक अर्धरात्रि अर्थात विक्रम सम्वत् २००९ भाद्रशुक्ल एकादशी, जन्म स्थान - आरा, जिला भोजपुर (बिहार) अक्षांश २५° ३६' उत्तर, देशान्तर- ८४°४२' पूर्व।

३. **शिक्षा** - १९६१ में रेलवे स्कूल जमालपुर (मुंगेर) बिहार में कक्षा ६ से विद्यालय शिक्षा आरम्भ।

१९६३ से १९६६ तक उच्च विद्यालय शिक्षा तेनु अज (विक्रम गज) रोहतास जिला में। इस अवधि में स्वाध्यायी छात्र के रूप में कामेश्वर सिंह संस्कृत विश्वविद्यालय से प्रथमा और मध्यमा परीक्षा। पिता द्वारा संस्कृत और ज्योतिष का शिक्षण।

पटना विश्वविद्यालय की जातिवादी राजनीति के कारण भौतिक विज्ञान अध्ययन में बाधा, अत: १९७४ में भारतीय वन सेवा, पंजाब संवर्ग में योगदान, पर अध्ययन का निश्चय और दृढ़।

१९७६ में भारतीय पुलिस सेवा, उड़ीसा संवर्ग में प्रवेश। १९८१ में भुवनेश्वर निवास में चार मास तक गणित का स्वाध्याय एवं उत्कल विश्वविद्यालय से गणित स्नातकोत्तर परीक्षा उत्तीर्ण। अभी कटक में पदस्थापित।

४. **ग्रन्थ की प्रेरणा तथा समर्पण** - गणित के अध्ययन क्रम में ब्रह्म स्फुट सिद्धान्त का अध्ययन। १९९१ के बाद अकर्मक पदों पर रहते समय सिद्धान्त दर्पण का अनुवाद एवं वैज्ञानिक व्याख्या। परिवार परम्परा में संस्कृत एवं संस्कृति की ज्योति प्रज्वलित रखने की प्रेरणा पिता और गुरु श्री चन्द्रशेखर उपाध्याय से मिली, जिनकी अपूर्ण इच्छा की पूर्ति के रुप में यह व्याख्या लिखी गयी। अत: उन्हीं को

SIDDHĀNTA-DARPANA

(1899 A.D.)

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SUBJECT INDEX

Units of length and time in 7 pages

B. SPHUṬĀDHIKĀRA

C. TRIPRAŚNĀDHIKĀRA

D. GOLĀDHIKĀRA

INTRODUCTION

(1) Arrangement of the book -

Scope - Original book was written in 2,500 Sanskrit verses in Oriya script on palm leaves. It was published with introduction in English by Prof. Jogesh Chandra Roy of Ravenshaw College, Cuttack by Calcutta University in 1899. Subsequently same edition was reproduced with approximate Oriya translation by Paṇḍit Vīr Hanumāna Shāstrī, by Utkal University, Bhubaneswar (then at Cuttack). This was reprinted by Dharmagrantha Stores, Cuttack. Some parts have not been translated and explained. First volume of this book renders the Sanskrit verses in devanāgari script with literal Hindi translation. It also contains the original introduction.

Present second volume translates all the verses in English. Translation is not literal but in mathematical terms, but preserving the technical terms in Sanskṛit. Verses in praise of god have been left out, not because of disrespect. With all devotion inspired by Samanta Chandrashekhar, this is not the purpose of the second volume. In addition to translation, each formula has been explained or derived according to modern mathematics and astronomy. The methods have been compared with other Indian astronomers and some times with other countries and with modern astronomy. This was the method and purpose of Samanta himself.

Technical terms and their calculations cannot be explained in words alone. So a general mathematical and technical introduction is given at beginning of each chapter with bibliography or source reference for further study. In that light only, the methods proved in the chapter can be understood. Where-ever considered useful, methods have also been explained with examples, based on text as well as modern astronomy.

In Sanskrit verse, some number or statement has been continued in many verses due to poetic and literal explainations. They have been clubbed together for translation. For brevity and simplicity, many parts have been given in chart form. Chapter 23 contains only verses in praise of god. Most of these verses have two or more meanings. It cannot be expressed in other language, nor it is related to the main topic. It is, therefore, omitted.

2. Numeration

Decimal system of writing numbers originated in India. Arabs called them Hindu numerals. Europeans learnt from Arabs and termed them Arabic numerals. This system uses 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, each increasing by one. For writing greater numbers, successive positions towards left are used, each place having ten times the value of position on its right side. Similarly, fractions are written towards right from the unit place after giving a point, called decimal. Each place has value of 1/10th of the value of its predecessor towards left.

Modern computers use binary system with two symbols 0 and 1 only, each place value

increasing two times towards left. In angular and time measurements of Indian astronomy, continued till today, multiples or divisions by 60 at each step is used. This was used in Sumerian mathematics for all numbers and is called sexa-gesimal (60) system.

Āryabhaṭa, I, has given the following order of place values, each ten times the preceding -

Eka (units place), Daśa (ten place), Śata (hundred), Sahasra (thousand), Ayuta (ten thousand) Niyuta (hundred thousand or lakh), Prayuta (ten lakhs or a million), Koṭi (ten millions or 1 crore) Arbuda (10 crores), Vṛnda (100 crores) etc.

Śankara Varman in his Ṣaḍratnamālā (1,5-6) has given the following sequence in multiples of 10 -

Eka (1), Daśa (10), Śata (100), Sahasra (1,000), Ayuta (10,000), Niyuta (or lakh, 10^5), Prayuta (10^6), Koṭi (10^7), Arbuda (10^8), Vṛnda (10^9), Kharva (10^{10}), Nikharva (10^{11}), Mahāpadma (10^{12}), Śanku (10^{13}), Vāridhi (10^{14}), Antya (10^{16}) and Parārdha (10^{17})

Lalita-vistara, a Buddhist text gives powers of 10 beyond 100 koṭi (i.e. 10^9), each increasing 100 times the previous -

Koṭi (10^7), ayuta (10^9), niyuta (10^{11}), kañkara (10^{13}), vivara (10^{15}), akśobhya (10^{17}), vivāha (10^{19}), utsanga (10^{21}), bahula (10^{23}), nāgabala (10^{25}), titilambha (10^{27}), vyavasthānaprajñapti (10^{29}), hetuhila (10^{31}), karāhu (10^{33}), hetvindriya (10^{35}), samāptalambha (10^{37}), gaṇanāgati (10^{39}), niravadya (10^{41}), mudrābala (10^{43}), sarvabala (10^{45}), visajñāgati (10^{47}), Sarvasajñā (10^{49}), vibhūtañgamā (10^{51}), tallakśaṇa (10^{53}).

Āryabhaṭa notation - Varga letters (k to m) should be written in varga places (unit place and hundred times at each step) and avarga letters (y to h) in the avarga places. Varga letters take the numerical values (1,2,3 .....25) from k onwards. Numerical value of the initial avarga letter y is ñ plus m (i.e. 5+25), next letters are 40 to 90. In nine places of double zeros, nine vowels should be written (one vowel for each pair of varga and avarga letters).

Kaṭapayādi notation was before Āryabhaṭa and is believed to have been used in vedas in portions related to astronomy or mathematics. It was very popular in Kerala. Each digit is represented by a consonant letter. Vowels and half letters have no meaning. Digits are written from right to left to form a number. Numbers 1 to nine and the 0 are indicated by letters starting from k, ṭ, p, or y, hence the system is called kaṭapayādi.

1	2	3	4	5	6	7	8	9	0
k	kh	g	gh	ñ	c	ch	ja	jh	ñ
ṭ	ṭh	ḍ	ḍh	ṇ	t	th	d	dh	n
p	ph	b	bh	m					
y	r	l	v	ś	ṣ	s	h	l	

Sūryasiddhānta and other works including the present book have used words to indicate each digit again written from right to left. These have already been indicated in the 1st volume for purpose of literal hindi translation, and need not be repeated here.

3. Transliteration of Sanskrit letters

	vowels						
Short	अ	इ	उ	ऋ	लृ		
	a	i	u	ṛ	l		
Long	आ	ई	ऊ	ए	ओ	ऐ	औ
	ā	ī	ō	e	o	ai	au

Anusvāra ं = ṃ, ṁ

Visarga : = ḥ

	Consonants							
	क्	ख्	ग्	घ्	ङ			
	k	kh	g	·gh	ñ			
	च्	छ्	ज	झ्	ञ			
	c	ch	j	jh	ñ			
	ट्	ठ्	ड्	.ढ	ण			
	ṭ	ṭh	ḍ	ḍh	ṇ			
	त्	थ्	द्	ध्	न्			
	t	th	d	dh	n			
	प्	फ्	ब्	भ्	म्			
	p	ph	b	bh	m			
य्	र्	ल्	व्	श्	ष्	स्	ह् ळ्	
y	r	l	v	ś	ṁ	ṣ	h l	

4. A brief survey of Indian astronomy

Astronomy has come from old French word 'astronomie' which in turn was derived from Lātin 'astronomia' and Greek 'astronomos' - meaning star law.

'Jyotiṣa' in Sanskrit means the same - 'Jyoti' means source of light i.e.a star in a sky; study of star groups and motion of planets observed through them is jyotiṣa.

Greek astronomy had its origin in Nile river and Sumerian civilisation. Western astronomers try to establish that vedic jyotiṣ is originated from Sumer and later Indian astronomy is influenced by Greek. But internal astronomical evidence suggests that text of vedāṅga jyotiṣa was written in 2976 B.C. when summer solstice started (verse 6 tells that Maghā month began when Sun was in mid Aśleṣā. Full moon point was 1°13′ east of mid maghā i.e. 8° east of Regulas at present at 150°56′ - difference of 68°56′). However, system is much older, and many changes have been made from Taiṭṭirīya Saṁhitā.

Vedāṅga jyotiṣa is found in two texts - Ṛkveda has 36 verses on the topic and yajurveda has 43 verses. Many are common, but the system is entirely different. Yajur jyotiṣa was written 624 years after Ṛk jyotiṣ according to internal evidence. Compiler of Ṛk jyotiṣ is 'Lagagha.' According to difference in day lengths, mentioned in verse 7 and 22, they refer to a place of 34°50′ North latitude. In northern borders of India, this is near Almā - Atā of Kyrgiz. Since it was first seat of learning, first school is called alma-meter is Greek.

So far, authors have assumed that both versions of vedāṅga jyotiśa denote 5 years yuga (or cyclic period). Accordingly text of Ṛk jyotiśa had to be modified and twisted. But now Sri P.V. Holay of Nagpur in his Vedic Astronomy (1988) has proved that the original text of Ṛk jyotiśa indicates a 19 year yuga - after which solar and lunar years start together. There are 7 extra months in a yuga, their adjustment is such that 5 solar years start within 6 days of new moon. Such

approximately concurrent years are called Saṃvatsara. Other types of years are Anuvatsara, Parivatsara, Idvatsara and Idāvatsara. Thus the statement that a yuga has 5 samvatsaras doesn't mean that 5 years make a yuga as assumed so far. It means only that out of 19 years in a yuga, 5 are saṃvatsaras. 19 years cycle was later on discovered by a Greek astronomer Meton in 432 B.C. and is called Metonic cycle. However, this cycle was used by Sumerians and Chinese also in their calender much before the Greeks. It is certain that astronomy in the whole world had single system. Irrespective of origins, there was exchange and compilation of ideas, and same standard was adopted as in the modern sciences. Thus houses of zodiac and constellations have the same names in all the languages. There is similar correspondance in medical names of Greek origin and their Sanskrit names in yoga or āyurveda.

Vedāṅga Jyotiṣa was followed by Garga Saṃhitā and Paitāmaha Siddhānta and Jain works Sūrya-pannati and Jyotiśkaraṇḍaka with minor changes. This period was followed by so called Siddhānta period. According to traditional Indian belief, there were 18 such siddhāntas - (1) Sūrya (2) Paitāmaha (3) Vyāsa (4) Vāśiṣṭha (5) Atri (6) Parāśara (7) Kāśyapa (8) Nārada (9) Gārgya (10) Marīci (11) Manu (12) Aṅgirā (13) Lomaśa (or Romaka) (14) Pauliśa (15) Cyavana (16) Yavana (17) Bhṛgu and (18) Śaunaka, Five of these siddhāntas - Saura, Paitāmahā, Vāśiṣṭha, Romaka and Paulisa - were codified by Varāhamihira in his Pañcasiddhāntikā (184 B.C.) who has emphasised that the Saura was most accurate of them.

Saura or Surya Siddhānta has no human authorship. Second verse of the text states that when short time (or 121 years in Kaṭapayādi) was remaining in end of Satyuga, Sun god taught this to Maya asura. Yuga system of this originates from Viṣṇudharmottara purāṇa according to Brahmagupta which is modification of old Brahma (or Paitāmaha) siddhānta.

In Varāhamihira's Saura, a period of 180,000 years has been stated which contains 66,389 inter calary months and 10,45,095 ommitted lunar days (tithis). Modern Sūrya Siddhānta tells about a mahāyuga (or yuga) of 43,20,000 years divided into Kṛta, Tretā, Dvāpara and Kali ages in ratio of 4:3:2:1 (12,000 divine years) with 1/12th period each in beginning and end as sandhyā (twilight period). 360 solar years are called a divine year. Paitāmaha siddhānta is crudest and has 5 years yuga like yajuṣ jyotiṣa. Vāśiṣṭha has improvement and deals with true motion of 5 planets. Sidereal year has been stated of 365 1/4 days.

Pauliśa siddhānta is more accurate and gives days counts (ahargaṇa) and sine tables. It gives solar year of 365.2583 days. Al-Baruni has regarded Pauliśa as a Greek from Alexandriā (Sachau I, p/153).

Romaka gives a luni-solar cycle of 2850 years with 1,050 intercalary months and 16,547 omitted lunar days. Length of year is 365 days 5h 55'12" and synodic month is 29 days 12h 44'2.2". It deals with equations of centre for Sun and moon.

Among present compiled texts, Āryabhaṭīya of Āryabhaṭa I (476 A.D.) of 121 verses is the first.

It is a brief codification of existing knowledge after observatory (khagol village) near Kusumpur (modern Patnā, capital of Bihar) was destroyed in Hūṇa attack. It is more an attempt to preserve the science in verse form, than to write a text book. For brevity, he has devised his own number system, as explained before. Subsequent astronomers made appropriate corrections and devised simpler methods of calculations in their texts.

Jyotiṣa has three parts - (1) Gaṇita - corresponding to modern astronomy and mathematical methods (2) Phaliṭa - Astrology (3) Horā or Saṁhitā - auspicious times, natural phenomena, signs in human beings and animals etc. Gaṇita is written in three styles - (1) Siddhānta is a text for calculation from beginning of yuga. (2) Tantra starts the calculation from beginning of Kaliyuga (17/18-2-3102 B.C. Ujjain miḍ-night) (3) Karaṇa uses short methods for current years ephemeris with reference to a recent base year. Its literal meaning and use is same as that of a handbook or a manual.

A brief list of astronomers and their works is indicated below -

5th-6th Century - Āryabhaṭa I (Āryabhaṭiya and Āryabhaṭasiddhānta, the later available only in quotations).

6th Century - Prabhākara, pupil of Āryabhaṭa, Varāha-mihira (Pañcasiddhāntikā and Bṛhatsaṃhitā).

6-7th Century - Bhāskara I (Mahābhāskarīya, Laghubhāskarīya and Āryabhaṭīya - bhāṣya); Brahmagupta (Brahma-sphuṭa-siddhānta and Khaṇḍa-

khādyaka), Haridatta (Grahcāra-nibandhana) Devācārya (Karaṇa-ratna).

8-9th Century - Lalla (Śiṣya-dhī-vṛddhida-Tantra) Govinda - Svāmin (Mahābhāskarīya-bhāṣya) Śaṅkaranārāyaṇa (Laghu-bhāskariya vivaraṇa) Pṛthūdaka svāmin (Brahma-siddhānta vāsanā bhāṣya) and Khaṇḍa-Khādyaka vivaraṇa.

10th Century - Vaṭeśvara (Vaṭeśvara-siddhānta) Muñjāla (Laghumānasa), Śrīpati (Siddhānta-śekhara) Āryabhaṭa II (Mahāsiddhānta), Bhaṭṭotpala (Khaṇḍa-Khādyaka vyākhyā and Vṛhatsaṃhitā - vyākhyā) Vijayanandin (Karaṇa Tilaka).

11th Century - Someśvara (Āryabhaṭīya Vyākhyā) Śatānanda (Bhāsvatī)

12th Century - Bhāskara II (Siddhānta Śiromaṇi with Vasanā bhāṣya, Karaṇa Kutūhala), Mallikārjuna Sūri (Sūrya siddhānta Vyākhyā) Sūryadevayajvan (Āryabhaṭīya Prakāśikā and Laghumānasa Vyākhyā) Caṇḍeśvara (Sūrya-Siddhānta Bhāṣya).

13th Century - Āmrāja (Khaṇḍa-Khādyaka-Vāsanā bhāṣya)

14th Century - Makkibhaṭṭa (Gaṇita Bhūṣaṇa), Mādhava of Saṅgamagrāma (Sphuṭa candrāpti, Agaṇitagrahacāra, Venvaroha), Madanapāla (Vāsanārṇava on the Sūrya siddhānta), Viddaṇa (Vārṣika Tantra).

15th Century - Parameśvara (Dṛggaṇita, Goladīpikā) Grahaṇamaṇḍana, Grahaṇa-nyāya-dīpikā, Āryabhaṭīya vyākhyā, Bhaṭdīpikā, Mahābhāskarīya vyākhyā, Laghubhāskarīya vyākhyā, Sūrya siddhānta-vyākhyā and

Mahābhāskarīya bhāṣya vyākhyā) Yallaya (Āryabhaṭīya vyākhyā, Jyotiṣa darpaṇa, Laghumānasa-kalpataru and Kalpavallī on the Sūrya-siddhānta), Rāma Kṛṣṇa Ārādhya (Sūrya siddhānta Subodhini) Cakradhara (Yantra Cintāmaṇi) Nīlakaṛṭha Somayāji (Jyotirmīmānsā, Golasāra, Candracchāyā gaṇita, Siddhānta Darpaṇa, Tantra Saṃgraha and Āryabhaṭīya bhāṣya).

16th Century - Jyeṣṭhadeva (Yuktibhāṣya, Dṛkkaraṇa) Śaṅkara Vāriyar (Karaṇa sāra, Tantra-saṅgraha, Yukti dīpikā), Bhūdhara (Sūrya siddhānta vivaraṇa) Tamma yajvan (Grahaṇādhikāra, Sūrya siddhānta - Kāmadogdhri), Gaṇeśa Daivajña (Grahalāghava, Tithi-Cintāmaṇī, Pratodayantra and Siddhānta Śiromaṇi-Vyākhyā), Acyuta Piśāraṭi (Karaṇottama, Sphuṭanirṇaya with vivaraṇa, Uparāgakriyā - Krama, Rāśigola sphuṭānīti); Rāma (Rāma vinoda)

17th Century - Viśvanātha (Grahaṇārtha Prakāśikā, Grahalāghavatīka, Karaṇakutūhala Udaharaṇa) Caṇḍīdāṣa (Karaṇa Kutūhala Tīkā), Putumana Somayāji (Karaṇa paddhāti, Pañcabodha, Nyāyaratna) Nityānanda (Siddhāntarāja and Siddhānta sindhu)

18th Century - Mahārāja Sawāī Jayasiṃha (Yantrarāja racanā, Jayavinodasāraṇī), Jagannātha Samrāṭa (Samrāṭa Siddhānta)

19th Century - Śaṅkaravarmana (Ṣaḍratnamālā)

For easy calculation of pancāṅga, many astronomical tables have been prepared. These are called Koṣṭhaka or Sāraṇī. Early examples are Grahajñāna by Āśādhara (epoch-20-3-1132), Laghukhecara siddhi by Śrīdhara (20-3-1316),

Makaranda by Makaranda (epoch 27-3-1478) Kheṭa-muktāvalī by Nṛsimha (31-3-1566)

(5) Astronomers of Orissa

Orissa was part of the Indian tradition of Jyotiṣa from vaidic and siddhānta period. Astronomy and mathematics were related to Yajñas whose time was found with astronomy and construction was as per geometric diagrams. In Orissa, Brāhmanic titles related to yajñas still exist like - Hotā; Udgāta, Brahmā, Pāṭhi, Pati, Vāgmī etc. It is quite probable that Taittirīya Samhita and Āraṇyaka, Aitareya and Gopaṭha Brāhmaṇa etc. - the āraṇyaka granthas forming origin of astronomy flourished in places like western Orissa which were famous as araṇya or mahākāntara. Another indication of rise of astronomy is the sea trade from Orissa coast to East Asia and upto Roman Empire. Due to popularity of Bāli yatrā, it is thought that sea trade of Orissa was only with Bāli - a small island in Indonesia. However, the relations must have developed with other areas of South East Asia and Chinese coast and intermediate islands of Andamāna group must have formed base for supply of food etc. Late Dr. N.K. Sahu in his history of Orissa states that silk of Sambalpur was known in Roman empire also. This confirms that ships from Orissa and other parts of India were going to different parts of the world. Technically, visit to America was also possible and traditional jyotiṣa texts mention a town 90° east of Ujjain (yamakoṭi) which should be in New Zealand (southern hemisphere). Hence yama is lord of south direction. 180° east of Ujjain in Siddhapura in North hemisphere. At this longitude there is a

town near Mexico where greatest Pyramid was built - Vālmīki Rāmāyaṇa calls it a gate built by Brahmā to indicate end of east direction i.e. 180° East of Ujjain at prime meridian in Indian Astronomy (Kishkindhā kāṇḍa). To a layman this discussion appears irrelevant to astronomy. However, sea journey (and plane journey in modern times) is not possible without knowledge of astronomy. There are no landmarks in sea or sky for finding the way. Hence navigation requires accurate determination of longitude, latitude and direction. These three are discussed in an important chapter Tripraśnādhikāra' of Sūrya siddhānta. It is noteworthy that Columbus could undertake his journey in open sea only because method of finding longitude was discovered in western astronomy ten years before that. That was from Turkish ships who had learnt astronomy from India. Vice versa, longitude determination in remote past indicates that India was well versed in navigation round the globe.

Transport of rice from Orissa was marked by Śālivāhana Śaka in 78 A.D. - it means transport of rice (Śāli = rice, Vāhana - carriage). As a product of Auḍra (Orissa), rice was called Auḍrīya i.e. Oryza in greak. This has become rice in English (omitting '0') and Orissa as name of the state. Navigation history indicates traditional study of astronomy in Orissa.

Sūrya Siddhānta has been given by sun god, whose worship is most common in coastal areas and river ports in India and elsewhere (Japan, Egypt, Mexico, Peru etc.). Jyotiṣa study might have suffered during Buddhist era in Orissa. It again

picked up after Varāhamihira in orissa like other parts of India. Gañga period (650 to 900 A.D.) records of Orissa indicate that Brāhmans were well versed in Vedāñga of which jyotiśa is a part. One person has specifically been mentioned as siddhānti. Śatānanda was most famous of old astronomers of Orissa.

Śatānanda - He was son-of Śañkara and Sarasvatī of Purushottampura (Purī) who completed his famous work Bhāsvatī' in 4200 yugābda (1099 A.D.). He has made calculations with reference to Purī. Full name of Bhāsvati was Pañca siddhānta sāra or Pañcasiddhānta - Bhāsvati on pattern of Pañcasiddhantikā of Varāhamihira. However, he has followed Sūrya siddhānta only which is considered most accurate. It is a Karaṇa grantha following solar year starting from Sāyana meṣa saṃkrānti. It was popular for its accurate calculation of eclipse - ग्रहणे भास्वती धन्या Commentaries on Bhāsvati-

(1) Sañsāraprakāśikā by Kāśiśekhara

(2) Bālabodhinī Tīkā of Bhāsvatī in 1543 A.D. by Balabhadra son of Vasanta of Kauśika gotra in Umā town of Jumila state.

(3) Oriya translation by Trilocana Mohānti in Yugābda 4747. Other books of 'Śatānanda are - (1) Śatānanda Ratnamāla - a saṃhitā book like Ratnamālā of "Śrīpati, his elder contemporary. (Palm leaf manuscript No 268, Orissa Museum). (2) Śatānanda Saṃgraha - work on smṛti. No manuscript is available.

Only Bhāsvāti is available with Hindi commentary by Mātṛ Prasāda Pandey by Chaukhambhā, Vārāṇasī.

Other astronomers of Orissa -

(1) Jayadhara Śarmā of Kotarahanga near Sākhīgopāla (Purī) received grants from Bhanja kings in 1231-12[illegible] A.D. for his mastery on Jyotiṣa. Though he was famous, no book by him or his forefathers is available.

(2) Gajapati Kapileśvara Deva (1435-1466 A.D.) of Cuttack who started Kapila era got another book written after his name called Kapila Bhāsvatī. But no manuscript is available.

(3) Govinda Dāsa of Nāgeśa gotra son of Hīrā Devī was a great astrologer. He constructed a dola-maṛḍapa in sacred town of "Śrī Kūrma". No work by him is available.

(4) Trilocana Mahānti - He translated Bhāsvatī in Oriyā verses in 4747 yugābda (1646 A D).

(5) Gajapati Nārāyaṇa Deva of Parla Khemundi wrote Āyurdāya Kaumudī in 26 chapters around 1650 A.D.

(6) Vipra Nāmadeva - He wrote a saṃskṛta commentary Sarvabodhinī on Sūryasiddhānta in 1721 AD.

(7) Dhanañjaya Ācārya - wrote a Pālaka Pañjikā for 1665 Śaka (1733 A.D.). 18 chapters of his Jyotiṣa candrodaya are available in Orissa museum. He wrote another work Jātaka Candrodaya.

(8) Māgunī Pāṭhī, son of Mārkaṇḍeya Pāṭhī wrote an Oriya commentary Mandārtha bodhinī on Siddhānta Śiromaṇi in 1741 A.D. In 1744 A.D. he wrote another commentary in Oriya on Grahacakra of Kocanācārya.

There is an incomplete work Jyotiṣ Śastra by Mārkaṇḍa who may be his father.

(9) Mahāmahopādhyāya Dayānidhi Nanda wrote Śiśubodhinī in 1707.

(10) Mahāmahopādhyāya Chapadī Nanda wrote Bālabodharatna Kaumudi in 1763.

(11) Son of Śrīnivāsa Miśra wrote Jyotiṣ tattva Kaumudī in 18th century. First 12 chapters are available.

(12) Gadādhara Pattanaik S/o Padmanābha in 18th century wrote Ravīndu grahaṇam on basis of Kocanā-cārya in 18th century.

(13) Gopīnātha Dāsa (Patnaik) wrote Āyurdāya Śiromaṇi and Śuddhāhnika Paddhati.

(14) Caitanya Rāja Guru - wrote Laghusiddhānta on pattern of Sūrya Siddhānta and wrote one Oriyā commentary on it.

(15) Yajña Mishra S/o Viśvaṃbhara wrote Jyotiṣa Cintāmaṇi or Ratnapañcaka whose incomplete manuscripts are available.

(16) Mahīdhara Mishra wrote Mahīdhara Samhitā in 18th century and a commentary on Amarakoṣa.

(17) Prajāpati Dāsa - (Unknown time) - Grantha Saṃgraha, pañcasvara and Saptāṅga.

(18) Bhānuśekhara Dāsa (18th century) Taraṇī Prakāśika , a commentary on Jātaka Ratnākara.

(19) Dāśarathi Mishra (18th century) - Jyotiṣa Saṁgraha.

(20) Kṛṣṇa Miśra (18th century) - Nakśatra Cūḍāmaṇi, Kāla Sarvasva.

(21) Tripurāri Dāsa - Oriya poet of 17th century - He wrote the following books on Kerala astronomy - Kerala Sūtra, Keralīya daśā and Prakṛta Kerala.

(22) Nīlakanṭha Praharāja and his son Yogī Praharāj - Their books Smṛti Darpaṇa and Vaidyahṛdayānanda have been published by Madras Govt.

(6) Sāmanta Candraśekhara and his role

Brief Biodata - He was born on 11.1.1936 (Tuesday) i.e. Pauṣa Kṛṣṇa 7/8th 1892 Vikramābda (1757 śaka) For an astronomer it is proper to give his birth time by planetary positions which is free of a calendar system.

Birth time - 09-04 IST based on Kumbha lagna and daśā calculation

Birth place - Khaṇḍaparā (Purī)

Latitude 20°15′ North, longitude 85°6′East Lagna 310°40′ (Prāṇapada in 5th house, Navāmśa is Makara.

Ayanāmśa 21°34′

Sun 268°28′	Moon 172°35′
Mars 263°26′	Mercury 271°45′
Jupiter 77°40′	Venus 292°25′
Saturn 192°46′	Rāhu 34°56′
Uranus 306°59′	Neptune 281°31′

Balance of Moon daśā of birth - 6 months 23 days.

Important events of his life - He did not have formal university education. Even though he was born in a royal family, he sufferred poverty

and unhappy family throughout his life. He sufferred from chromic dyspepsia and stomach inflammations frequently. At the age of 22, he married princes Sītā Devī of Aṅgula Rāj family. Due to his ugly looks his father-in-law showed reluctance to give his daughter in marriage in lagna maṇḍapa. When he showed his deep knowledge of Śāstras and mastery over Saṃskṛta verses, his marriage was solemnised (possibly on 28-2-1858). He had 5 sons and 6 daughters, out of which two sons had expired. He was banished from Khaṇḍapādā by his ruler, being his own cousin. But due to his knowledge, he gained wide fame and his rights were restored by the then commissioner of Cuttack. He was also given a 'Sanad' in honour of his achievements. His work "Siddhānta Darpaṇa" cannot be fully understood by a person unless he is well versed in Indian astronomy as well as modern mathematics. Whatever is known to common public about the book or its author is based on the English introduction by Prof. Jogesha Chandra Roy. This is based on personal interviews and not on a study of the book. So, many vital points have been left out. Sāmanta expired on 11.6.1904. On the basis of his horoscope he had foreseen his death; which is expressed by his son Gadādhara in an Oriya verse-meaning - "father called me near and told that moon had entered his māraka nakśatra, and there was no escape from death". In last but one verse of Siddhānta Darpana, he has expressed desire that his body should fall at the feet of Lord Jagannātha. On his last day, he went for darshana of lord. At the time of bowing before Jagannātha, he expired. At every place in

the book, he has shown his deep faith in lord and the scriptures. He has accepted his experimental observations only when they found support in some scripture.

His works - Siddhānta Darpaṇa is work of his whole life. At the end of every chapter two fold purpose of this book is explained - (1) Bālabodha - i.e. a text book and (2) Gaṇita-Akśi Siddhi - i.e. tally of calculation and observation.

For text book purpose, this is a treatise on Indian astronomy containing relevant positions of all text books from Śākalya Saṃhitā to siddhānta books starting from Āryabhaṭa. Quotation from Atharvaveda is unique in Indian astronomy; as it is only correct figure for sun's diamater, in Indian astronomy or in western astronomy before advent of telescope about 300 years ago. It is most voluminous book on astronomy with 2500 verses. Next largest are Vateśwara Siddhānta with 1100 verses or Siddhānta Śiromani with 900 verses.

Gaṇita-Akśi Siddhi has three fold significance. As every other science, purpose of astronomy is to tally mathematical calculations with observations.

Books starting with Āryabhata have only formulated or coded the existing knowledge, they have not indicated source of such figures. Methods are often in-completely explained and only refer to Vedic origin which is not clear. Thus purpose of math is only to find calculation methods for finding the observed position. Siddhānta books are not concerned about mathematical models, theories of gravitation or theories of motion. We are satisfied

when calculations give correct result and not bothered whether Sun or earth is centre of motion.

Third aspect is that there is slight change in planetary motion over long periods of time as stated in Sūrya siddhānta. This happens due to tidal friction. But siddhānta texts after Āryabhaṭa have assumed constant motion throughout yuga or a kalpa of 1000 yugas. Due to approximation of constants or errors in calculation methods there is some deviation in observed results. In every period astronomers have corrected the constants given in Sūrya Siddhānta. according to need. These are called Bīja corrections.

Researches of Candrashekhara :

(1) Moon's Motion (a) Traditionally moon's equation was of the form -

$$300'49.5'' \text{ Sin } (nt-\alpha) + 2'23.25'' \text{ Sin } 2 (nt-\alpha)$$

2nd term is equation of apsis introduced by Brahmagupta. This form is correct, but constant is slightly wrong.

(b) Śrīpati had found effect of Sun's attraction on moon motion (called evection). This has been introduced as Tungāntara correction by Candraśekhara given as

$$-160' \cos(\theta - \alpha) \text{ Sin } (D - \theta) \times \frac{\text{Moon's apparent daily motion}}{\text{daily mean motion}}$$

Error is about 4′ only.

(c) Bhāskara II had observed a fortnightly variation in moon's motion giving an error of maximum of 6 daṇḍas in middle of pakśa.

Comparing with his own observation, Sāmanta gave the Pākśika equation as

38′12″ Sin 2 (D-θ)

where D′ is moon corrected by Ist and 2nd equation

(d) Due to Sun's annual motion, a digaṃśa correction also was introduced.

±11′27.6″ Sin (Sun's distance from apogee)

These equations almost give the modern value and are to be further checked after 1000 years.

(2) Ayanāṃśa - According to modern theory, earth's axis is rotating in a conical motion completing almost uniform circular motion in 25,726 years. Sāmanta has assumed libration theory that the motion is not circular with 360° rotation but pendulum like oscillations within values of 27°, but with uniform motion. Present value of Ayanāmśa tallies with both theories. Only after 300 years or so error may be noticed. He has corrected the value of Sūrya siddhānta slightly (6,40, 170 revolutions in a kalpa, instead of 6 lakh revolutions according to Sūrya siddhānta). Liberation theory is not supported by modern astronomy but it may be correct according to methods of projective geometry used in Jain Astronomy (Thesis by Sri S.S. Lishk).

(3) Mandocca gati - According to classical mechanics, planets move in elliptical orbit whose major axis is fixed in space. Partly due to acti of other planets (mainly jupiter) and partly due to general theory of Relativity (1917 - Einstein), force of attraction reaches at speed of light, not

instantaneously - mandocca (apogee) is moving slowly. For mercury, it was calculated to be 1° in 11,000 years which was tested in 1919. In 300 years since Tycho Brahe, it is only 1/36° of an angle. For other planets it is so slow that it cannot be measured even by modern instruments. Indian astronomy gives 1° in 12,000 years for mercury and 39 revolutions of Saturn in 1 Kalpa or 1° in 3 lakh years. 'Cosmology' by Nārlīkara gives its rate of movement as

$$n = \frac{6\pi\ G\ M}{L\ Tc^2}$$

where M = mass of Sun, T = period of planet Sāmanta has introduced a new concept of Parocca for Mars and Saturn which moves with constant circular motion around which mandocca oscillates. This is not supported by relativistic equation. But it may be probable due to effect of Jupiter between Mars and Saturn, which can be tested only by a computer calculation. Another doubt is that such a motion cannot be observed in one life time. Even Moon's equation of motion is based on 1000 years of observation and needs same time more to test it. Sāmanta has not mentioned the basis of his correction.

(4) Discussion of other theories - Prof. J.C. Ray had not read Siddhānta Darpaṇa and wrote introduction on basis of personal discussions. But Sāmanta has treated him as student and has criticised his opinious about modern physics in his chapters on discussions (Vāsanā-rahasya).

(a) Jain theories have been criticised because they were based on projective geometry and become absurd according to spherical geometry. As a single sphere of earth is drawn as two circular maps in projective geometry, two Suns and two Moons were assumed in Jain theories. But dimensions of imaginary mountain 'Meru' have been quoted on the basis of Jain theories only.

(b) In Indian astronomy, for calculation purpose, it is immaterial whether earth in fixed or it moves. In both views, relative motion will be same giving the same result. Sāmanta has used modern physics to refute the theory that Sun is not the centre of motion. It is mass centre of solar system, which is away from Sun's surface at a distance of 1-1/2times its radius in direction of Jupiter (effect of other planets can be neglected). It appears that, Sāmanta was too skeptic of European theories whom he has called 'golden theory' as they were supported with hope of getting gold medal (17-160)

(c) His other objection was that if earth moves on its axis; why Jupiter moves faster being the heaviest. This has been explained later on by presuming that Jupiter and Sun were twin stars. Due to loss of matter, Jupiter gained in angular speed, to preserve the momentum. The other objection as to why we observe the same side of moon - has not been explained so far.

(d) If stars are all like Sun and are equally spaced in all directions, there should be no day and night - every time equal light should come from all directions. This was called Olber's paradox in modern astronomy and was explained only in 1930 when expansion of universe was observed (It is mentioned in Indian scriptures also) Due to expansion, the farther stars have lesser effect and

only the Sun causes day and night. Sāmanta has correctly refuted the argument of absorption of light of stars by gases etc.

(5) Diameter of Sun : Diameter of Sun had been heavily under estimated to be about 10 to 14 times the diameter of moon by all astronomers in India and outside. After talescope it was known to be 400 times. Distance of Sun being 400 times that of Moon, it will cause much greater difference in amount of solar eclipse at two places. This might have prompted Sāmanta to correct it. But he has referred to Brahmavidyā upaniṣad and Atharva veda (describing expansion of ऊ) to get the value of 72,000 yojans (19-40,50) and (8-12). This become 162 times diameter of Moon.

Siddhānta Darpaṇa has taken value of 1 yojans as 4.9 miles. Had he taken it to be 11 miles (Aryabhata 7.5 miles, Jain theories 9.2 miles) as it was in vedic times, he would have got correct value of Sun's diameter. This shows the absolute faith of Candraśekhara in ancient scriptures without which he never confirmed any result. To some extent it was justified, as seen from correct assessment of Sun's size in vedas compared to all ancient measures.

Some of his observations may appear biased or excessive, but they show a marvelous grasp of modern physics. Some of the points were not properly understood by top astronomers of his time

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14.4.1997 Arun Kumar Upadhyay

Rāmanavamī Cuttack

MADHYAMĀDHIKĀRA

Madhyama = mean. This portion deals with average or mean motion of planets. Calculation of mean position is done from beginning of Kalpa or yuga for siddhānta, from Kali beginning in a tantra and from epoch of this book (12-4-1869 Monday - Ist saura Caitra 1791 Śaka) as a Karaṇa book. Siddhānta Darpaṇa explains all the three methods and in addition last chapter gives easy method for calculations of pañcāñga. There are 33 mathematical tables in the end for ease in calculations..

The book is in two halves. First half deals with the (gaṇita) methods of astronomy, 2nd half deals with explainations and discussions and special topics (Gola)

First half of the book contains three parts called Adhikāras. First part is madhyamādhikāra, with 4 chapters (called Prakāśa), Part 2 is Spaśṭādhikāra with 2 chapters. Part 3 is Tripraśnādhikāra with 9 chapters.

Second half is called gola and has two parts. Part 4 is Golādhikāra containing 6 chapters. Part 5 is Kālādhīkāra containing 3 chapters.

First chapter of part 1 is called kāla varṇana explained below.

Chapter - 1

MEASUREMENT OF TIME

1. An Introduction to the Units of measurements

Any natural science involves theories and experiments which verify each other. We test the theory by measuring certain quantities and see whether they are according to the theory. The deviations or errors cause refinement in the theory.

As in physical sciences (particularly mechanics), the units of measurement in astronomy are of length, degree and time. Basic units in physics are of length, time and mass. Degree is a dimensionless quantity because it is ratio of length of arc to length of radius.

Practical units of quantities are based on human experience. Length is similar to hand or feet length, mass is mass of rice measured by spread of palms, time units is based on breathing time of human beings.

However, standardisation of length units is based on dimension of earth or comparison of some light wavelength. Similarly time units are fixed according to rotation periods of sun and moon or more accurately time taken by light to travel a particular distance. We can see that in modern physics as well as in ancient India standardsation method was exactly the same. Units of angle also

are based on the number of days (about 360) in a year, and hence sexāgesimal (divisions by 60) system was more convenient.

2. Units of length

In British system of units, foot was the basic unit equal to average length of human feet. In old Greece and Rome; cubit (18″= one hand) and stadia were also based on human measurements. For smaller units, aṅgula (finger width) was the basic unit in India (0.75″ or 1.88 cm).

In Tiloya Pannati (Jain Text), 1 aṅgula = 8^9 Trasareṇu In Anuyogadvāra Sūtra (″), 1 aṅgula = 8^{10} ″

In Siddhānta Jyotiṣa (Śrīpati), 1 aṅgula = 8^6 trasareṇu

Successively smaller units of Siddhānta are

Aṅgula - yava - yūka - līkshā - Bālāgra - Reṇu - Trasareṇu. Bālāgra (hair end) is Angul $\div 8^4 = 1/4 \times 10^4$ cm (micron) Thus the dimensions are really correct has hair is 3-4 micron wide.

According to Tiloyapannati lowest division is 1 paramāṇu = 1 aṅgula (1.88 cm) x 8^{-13} cm. = 3.5 x 10^{-12} cms.

This is of the order of nuclear diameter.

In Lalita vistara (Buddhist text), units are divided by 7 at each stage. According to it, 1 paramāṇu = 1 aṅgula (1.9 cm) x 7^{-10}cm = 0.66 x 10^{-8} cm.

This is equal to the Bohr rodius of Hydrogen atom.

Larger units are multiples of aṅgula or a 'puruṣa' or person (about 6 ft height). It is same as 'fathom' used to measure depth of sea or river.

Bigger units in Tiloyapannati are -

6 Aṅgula = 1 pāda (foot)

2 pāda = 1 vitasti (span)

2 vitasti = 1 hasta (forearm or cubit)

2 hasta = 1 rikku or kisku

2 Kisku = 1 daṇḍa (staff) or dhanuṣa

2,000 daṇḍa= 1 Krośa

4 Krośa = 1 yojana

Same units have been used by Paulish Siddhānta, Śrīpati and subsequent siddhānta texts. Lalita vistara, however makes 1 kosa = 1000 dhanuśa only equal to 1/2 Jain or Siddhānta yojana.

In the time of Napolean, attempt was made to link length unit 'metre' with dimensions of earth. So 1 metre was proposed to be 10^{-7} of distance between equator and north pole. Subsequently, it was learnt that it was 1 crore and 486 parts of this distance. Still, the standard length of platinum bar kept at Paris is used as metre. Nautical mile is also based on earth's dimension but it is not a decimal fraction. It is length of 1 minute of arc at equator (about 6080 ft. or 2 kms)

In same way yojana has been defined to be an exact fraction of earth's diameter or circumference in polar circle.

Varāhamihira - Circumference 3200 yojana

Āryabhaṭa - Diameter = 1050 yojana

Sūrya Siddhānta - Diameter = 1600 yojana

Siddhānta Śiromaṇi - Circumference = 4,800 yojana

(This is followed by Siddhānta Darpaṇa also)

Thus, yojana is 5 miles according to Siddhānta Ṣhiromani and 7.52 miles according to Āryabhaṭa.

Anuyogadvāra Sūtra (Jain) gives

1 Ātmā yojana = 7,68,000 aṅgula = 9-1/11 miles estimated according to current measurements of earth. Dr. L.C. Jain opines that 1 Pramāṇa yojana is 500 Ātmā yojana = 4,5 45.45 miles. M.B. Panta opines that 5 yojana (40 or 45.5 miles) was called Mahāyojana used for measuring distances of stars.

For example, 'Triśaṅku' star is named on basis of its distance from earth.

Tri-Śaṅku = 3 x 10^{13} Mahāyojana = 207 light years. This is actually the distance of that star now known as Beta-crucis in Southern cross constellation.

Similarly, it is said that Agastya had crossed Vāridhi (10^{14}) or drunk ocean and had gone south. It is now known as 'Argo-Navis' star at 80°5' south latitude, indicating naval journey. This star is 652 light years away ; 10^{14} mahāyojan is about 690 light years.

Prof. S.S. Dey of Calcutta has observed that Egyptian names of planets mercury, venus, mars, jupiter and saturn give their distances from Sun in yojana if names are interprated in Kaṭapayādi system.

At present metre is defined as 16,50,763.73 times the wave length of radiation of Krypton-86 isotope for transfer of electron between 2p and 4 d states. With accurate measurement of velocity

of light, it is proposed to link time and length units. In fact truṭi (a unit of time) was also defined as time taken by light to travel 1 yojana.

3. Measurement of Time

Principle of time measurement is to choose a unit which is equal to the time of a periodic event (which repeats itself after fixed intervals). Examples of such events are - vibration of quartz crystal or metal spring, pendulum (all used in clocks), rotation of earth (1 day = 24 hours), synodic rotation of Moon (1 month) or apparent rotation of Sun around earth (1 year).

Basic unit of time in jyotiṣa is 'asu' (meaning mouth) or 'prāṇa' (breathing) as it is approximately time (4 seconds) taken by a man in breathing in and out. Since our mental feeling of time is based on breathing only, units bigger than asu can be felt and are called 'Mūṛtta' (tangible). Smaller units are called Amūrtta (imaginary). Astronomically, it is time taken by earth in its daily motion (360° in 24 hours) to move by 1' (=1°/60).

'Amūrtta' or small units - In gaṇita-Sāra-Saṃgraha (Jaina) 1 prāṇa has been divided into 44466-2458/3773 Āvalikās. Possible reason for such peculiar ratio is that a muhūrtta (1/30 of a day of 24 hours) was equal to 3773 prāṇas in one system and 1,67,77,216 āvalikās in another system.

A solar day is divided into 60 daṇḍa or ghaṭikā (like 24 hours). Each ghaṭī is divided into 60 pala (each 24 seconds) which is again divided into 60 vipalas.

Thus 1 asu or prāṇa is equal to 1/6 pala or 10 vipalas

1 asu (respiration) = 5/2 kāṣṭhā

1 kāṣṭhā = 4 long syllable (gurvakśara)

1 gurvakśara or vipala = 9/2 nimeṣa (twinkling of eye)

1 nimeṣa = 100 lava

1 lava = 100 truti (1 Truṭi is time taken by a sharp needle to pierce a soft lotus petal)

1 Truṭi = 3 Trasareṇu

1 Trasareṇu = 3 aṇu

1 Aṇu = 2 paramānu

Thus 1 Truṭi $= \frac{\text{1 asu (4 second)}}{10 \times 9 \times 50 \times 100}$

In this time, hight will travel about 2.68 Kms (1/3 or 1/4 yojans or 1 krośa approximately).

1 paramāṛu Kāla = 1/8 Truṭi = 5×10^{-7} seconds approx.

Larger Units -

10 gurvakśara or Vipala = 1 prāṇa

6 prāṇa = 1 pala or vighaṭī

60 vighatī = 1 ghaṭikā

60 ghaṭikā or daṇḍa = 1 day (24 hours)

30 days = 1 month (approximate time from one full moon to the next)

12 months = 1 year (approximate time of apparent rotation of Sun)

360 years = 1 divya varṣa (divine year)

43,20,000 years = 1 yuga

72 yugas = 1 manu

14 Manu = 1 Kalpa (day of Brahmā) Āryabhaṭa

Thus in this system 1008 yugas make a kalpa.

Sūryasiddhānta gives 1000 yugas in a kalpa with 14 manus of 71 yugas each with 15 sandhis of 1 satyuga (4/10 yuga) each.

2 kalpa = 1 ahorātra (day-night) of Brahmā

30 days of Brahmā = 1 month of Brahmā

12 months of Brahmā = 1 year of Brahmā

= 7,25,760 yugas (Āryabhaṭa)

or 7,20,000 yugas (Sūrya Siddhānta)

100 years of Brahmā = Life of Brahmā (Mahākalpa or Parā)

50 years is called Parārddha = 1.5×10^{17} years

In one mahāyuga there are 0.4×10^{17} asus

Hence 10^{17} is called parā or parārdha.

Concept of yuga - 'Yuga' of Ṛkveda was of 19 years after when mutual motion of moon and sun repeats itself. Later on, this period was called metonic cycle in Greece. Yajur jyotiṣa gave a yuga of 5 years which is a simpler system of tallying lunar and solar years. In vedāṅga jyotiṣa 19x8+8 = 160 years was next bigger yuga after which lunisolar calender tallies more accurately. Viśvāmitra had smaller yuga of 3339 tithis = 111 synodic months + 9 tithis. This was half of Saros cycle of Chaldea (223 synodic months or 18 tropical years and 10.5 days) after which ellipses are repeated. His greater yuga was of 3339 synodic years or 3240 sidereal years. One third of this period 1080 sidereal years was used in determining Indian Erās. This gave rise to small chaturyuga of 4x1080 = 4320 years. One Mahāyuga is 1000 times this unit and 1 Kalpa is 1000 mahāyuga. This is

based on astronomical hymn of Viśvāmitra (RV III 9-9) - 3339 dyus (days/ tithis/parts of sky) worshipped Agni (Sun) by revolutions in the sky. This concept has been used for divisions of constellation in vedānga jyotiṣa.

Siddhānta texts have formed a Mahāyuga in which all the seven planets Sun and Moon and 5 faint (Tārā) planets make complete revolutions. After a yuga they come to the same position. Thus a position of these planets will occur only once in a yuga and is most accurate method of indicating a time in a yuga. This is one of the purposes of preparing a horoscope.

Rotation of mandocca (apogee) of planets is still slower and their full rotations are completed only after 1000 yugas or a Kalpa. Slowest is śani whose mandocca makes only 39 rotations in a kalpa.

It may be mentioned that a period of kalpa of 4 bilion years is approximately same as life of earth or the solar system. 2 kalpa or 1 day/night of Brahmā is considered to be the time from when universe is expanding and will contract again. Life of Brahmā $3x10^{17}$ years is approximately half life period of proton decay after which basic elements of the universe will dissolve themselves.

4. Other examples of accurate measurements -

Verses of Veda composed by known astronomers Viśvāmitra, Atri, Śunahśepa, Hiraṇyastūpa, Kutsa, Utathya, his son Dīrghatamas, his son Kākṣīvat and daughter Ghoṣā - should be read according to Kaṭapayādi system for their mathematical meaning.

Nāsadīya and other verses of these sages indicate theories of creation of universe which are similar to modern cosmology.

Ṛkveda (I-164-2) tells that the seven join the body in constant circular motion of earth (rātham). Orbit round Sun is elliptical (called Trinābhicakram) because elipse has 3 nābhis (1 centre and 2 focus)

Cakram = 2 x π= 6.283, ratha = 72

Hence in Krośa units (= 2.5 miles)

7 x Cakra x ratha = 6.283x7x72x2.5 = 7915 miles which is diameter of earth.

Second line indicates Sapta (7) nāma (50) vahati (moves in orbit). If movement is taken in 1 lava

$$= \frac{\text{muhurta}}{60} = 48 \text{ seconds.}$$

then orbital velocity of earth is

$$= \frac{7 \times 50 \text{ Krosa}}{1 \text{ lava}} = \frac{7 \times 50\ 2.5 \text{ miles}}{48 \text{ seconds}} = 18.5$$

miles/sec.

Tri (3) nābhī (40) Cakra (2π = 6.283) gives acceleration due to gravity if length unit is taken as hasta 19.8″ and time as lava. For small units both are divided by 60.

Vilava = 4/5 sec., 1/60 hasta = 0.33/12 ft.

$$g = 3 \times 40\ 6.283 \times \frac{0.33}{12} \times \left(\frac{5}{4}\right)^2 = 32 \text{ ft/sec}^2$$

Fourth line gives arc of an imaginary sphere on which moon moves.

Yātrā (21), viśva (44), Bhuvana (44) gives 21x44x44 = 40, 656 when unit is moon's distance ÷ radius of earth. Dīrghatamas gives a theory of star formation in RV (I-164-8) - Māta (steller cloud formed by hydrogen atoms) absorbs light (garbharasā) and is further excited by gravilational contraction (pitaraṃ). Dhiti (69) manasā (708) gives diameter of hydrogen atom if we take unit of length 60x60 times smaller than 1/60 hasta (0.33 inches)

Dhiti × mānasa

$$= \frac{0.33}{60 \times 60} \times \frac{1}{60 \times 708} \times 2.54 \text{ cm.}$$

= 0.478×10^{-9} cm = radius of hydrogen atom when it is divided by mātā pitaram (65x261), this gives 2.8×10^{-13} cm, the distance at which nuclear interaction works. Atomic radius divided by Sā (7) garbharasā (7243) gives 10^{-13} cm which is diameter of proton or electron.

Velocity of light - Smṛtiśastra tells - we salute with our respect to sun who traverses 2202 yojans in 1/2 nimiṣa.

In purāṇa - 1 nimiṣa = 16/75 seconds

In Līlāvatī, 1 yojana = 4x8000 cubits = 9.09 miles

Hence velocity of light is

$$\frac{9.09 \times 2202}{8/75} = 1.86 \times 10^5 \text{ miles/sec.}$$

Bhāskara nimiṣa is 8/90 seconds, Manu's yojana is 4 Krośa of 4000 cubits each. Then velocity is 3×10^5 km/sec.

5. Measurement of angles - Since apparent revolution of Sun around earth in a year is in about 360 days, a circle is divided into 360° degrees

(aṃśa) so that motion in one day is about 1°. Its average motion in 1 month (lunar node to node) is about 30° hence 30° is 1 rāśi. Further sub divisions are always by 60 at each step because it is a simple factor of 360 and there were 6 days weak (ṣaḍaha) in vedas. 1 extra day was added to some weeks making it 7 days, this day was not regular weak day hence tradition of weekly holiday arose. Since moon's node makes 12 rounds when Sun makes 1 round, the clock also copies that motion. Minute hand makes 12 rounds when hour hand makes 1 round.

Angular and time measurement both are divided into 60 units so that they tally with sun's motion.

One rotation = 12 rāśi = 360 Aṁśa (degree)

1 Aṁśa = 60 Kalā (minute)

1 Kalā = 60 vikalā (second)

Tatpara and parātpara are further divisions.

Thus angular motion of sun corresponds to time units

1 rāśi = 1 month, 1° = 1 day

1′ = 1 daṇḍa, 1″ = 1 pala

1 tatpara = 1 vipala etc.

This division of angle continues throughout the world till day. Time units are slightly different, but still in divisions of 60. Actually hour is derived from 'Horā' (Ahorātra) i.e. two divisions of a rāśi (like day-night divisions of a day). Earth rotates 1 circle i.e. 24 hours in 1 day, hence 1 hora in 1 hour.

6. References to introduction

(1) For units and dimensions any standard text book of physics for +2 or graduate standard may be referred. First chapter is on units and dimensions.

(2) For vedic astronomy see P.V. Holay's book.

(3) For other interpratation of vedas see - Issues in vedic Astronomy and Astrology published in 1992 by Rashtriya Veda Vidyā Pratishthan, New Delhi-2.

(4) For details of siddhānta texts individual texts may be referred. Some information is compiled in chapter 7 of Indian Astronomy - A source book.

Translation of the text (Chapter I)

Verses 1-9 - Maṅgalācaraṇa - Prayer to Lord Jagannātha and other gods.

Verses 10-11 - Pratijñā - Scope and purpose of the book - It deals with only gaṇita jyotiṣa. Purpose is to explain difficult methods of mathematics in simple language to a common man.

Verse 12 - Mathematical methods (Pāṭigaṇita) has already been perfectly explained by Śrī Bhāskarācārya in his text Līlāvatī . Without repeating the same, motion of planets is discussed straight away.

Verse 13 - Comparison with earlier Siddhāntaṣ - Some special subjects have been dealt with in this siddhānta, not found in earlier texts, for satisfaction of the learned.

Verses 14-15-Importance of jyotiṣa - Veda guides everyone in yajña. Muhūrta (auspicious

time), for that is known through jyotiṣa. If veda is taken in human form, Jyotiṣa is eyes, Vyākaraṇa is mouth (grammer), Nirukta (dictionary) is ears, kalpa (Purāṇa) is hands, Śikśā Śastra (reading) is nose, chanda (prosody) is feet. Thus jyotiṣ is the part of veda through which all other parts can be understood.

Comments - Many portions of vedas, Samhitāṣ and all the text books explain jyotiṣa in similar words which need not be repeated.

Verses 16-19 - About Siddhānta - Without mathematical astronomy, the whole jyotiṣa is usṣless. Siddhānta deals with time scale from Truṭi (smallest unit) to Kalpa (biggest unit used in jyotiṣa), arithmatics (including indeterminate equations of first and second degree), algebra, evolution and creation of world, orbits of earth, planets and stars, eclipse and conjunction of planets and description of various instruments like jyā, dhanu etc. and elements of mensuration. Among all śāstras jyotiṣa is highest; in jyotiṣa itself, siddhānta is best; and in siddhānta also gola (=sphere including Bhūgola =geography and khagola = astronomy) is most important. A country prospers due to presence of men well versed in gola. Otherwise, animal behaviour spreads. Siddhānta gives all the four results (Dharma, Artha, Kāma and Mokśa). So Sūrya (creator of Sūrya siddhānta) has kept it a secret to be given only to good and pious prson.

Verse 20 - First half of this text deals with time units, ahargaṇa (count of days), bhagaṇa (revolutions in sky), graha ānayana (calculating planetary positions), jyā (sine) koṭijyā (cosine) etc.

spaṣṭa śara (actual position of planets as seen in the sky), tripraśna (three problems about daily motion).

Second half deals with different theories, creation and dissolution (sṛṣṭi and laya), earth (geography), kakśā (orbit), yantra (instruments), description of countries, prayers to lord Jagannātha and Kautuka Pañjikā (easy preparation of almanc).

Verse 21 - Parabrahma was in the beginning. It created puruṣa and prakṛti (2), mahattatva (intellect-1), ahaṅkāra (ego-1), tanmātrā (elements 5), mahābhūta (5 types of creations or beings), 5 organs of sense, 5 organs of action, and one mind - a total of 25 elements. It supervises all.

Verse 22 - Jyotiṣa Cakra - After creation, Brahmā caused the sphere of ākāśa to rotate from east to west in a daily motion (seems as a result of rotation of earth on its axis from west to east) with respect to two dhruvas (north and south poles on the axis). With a slower motion, the planets move west to east relative to stars in nīca and ucca circles (earth is not the centre of their or bits).

Verses 23-30 - Kāla has two meanings - one is destroyer of world and the other is reckoning of time. Time units are of two types - Sūkśma or amūrtta is very small unit which cannot be felt by senses, but calculated or measured by instruments.

Prāṇa is of 4 seconds, in which a person breathes in and out. This is the smallest sthūla or mūrta time unit to be felt by senses.

Divisions of time are - 1 lava = 100 truṭi

30 lava =1 nimeṣa = 1/135 seconds

18 nimeṣa = 1 kāṣṭhā

27 nimeṣa	=	Time to pronounce long vowel =0.2 seconds
20 long vowels		=1 prāṇa (4 seconds)
2 Prāṇas		=1 Kalā
3 Kalā		1 vighaṭikā or pala (24 seconds)
10 vighaṭī	=	1 kśaṇa
6 kśana	=	1 ghaṭī or daṇḍa (24 minutes)
2 daṇḍa		1 muhūrta
30 muhūrta	=	1 nākśatra dina

Note - Nākśatra dina is the time of rotation of earth with respect of stars (23 hours 56 minutes) called sidereal day and is slightly smaller than civil day or solar day (between sunrise to sunrise) of average 24 hours.

30 nākśatra day = 1 nākśatra māsa

From sunrise to next sunrise is called sāvana dina (or civil day)

30 Sāvana dina = 1 sāvana māsa

30 tithis = 1 cāndramāsa

NB-1 Cāndramāsa is period from moon's node (amāvasyā - in same direction as sun, or Pūrṇimā, at 180° from Sun) to the same node next time. Tithi is 1/30th part of that period equal to the time in which moon gains 12^0 difference over sun.

Verse 31 - Detailed description of these nākṣatra and civil days, months etc. will be done in second part of the book.

12 solar months = 1 solar year

Solar (Saura) year is one day of deva or asura (divya or Asura varṣa)

Note - 1. Sauradina is the interval of time during which sun moves 1^0 of ecliptic. In saura

māsa it moves 30° or 1 rāśi and in 1 year it makes a complete round of 360°.

2. Krāntivṛtta or ecliptic is the apparent path of stars from east to west in plane of equator. One complete round is called bhagaṇa which is divided into 360° amśa (degrees). Each subdivision is in 60's as follows -

60 viliptās, vikalās, or seconds
= 1 liptā, kalā or minutes

60 liptās = 1 aṃśa, Bhāga or degree

30 aṁśa = 1 rāśi or sign

12 rāśis = 1 Bhagaṇa or revolution

Verse 32 - When sun is moving north of ecliptic (for six months), it is day for devas and night for asuras (day in north pole and night in south pole). When sun is moving south of ecliptic, it is night for devas and day for asuras.

360 divya or asura days = 1 divya or asuravarṣa

Verse 33-39 - Time scales greater than a year

Period	Divya years	Solar Years
Satya yuga	4,800	17,28,000
Tretā yuga	3,600	12,96,000
Dvāpara yuga	2,400	8,64,000
Kali yuga	1,200	4,32,000
Total - yuga or Mahāyuga	12000	43,20,000

There is a sandhyā, 1/12th of the yuga, included in each yuga at its beginning and the end. Total sandhyā is 1/6 of the yuga.

Sandhyā at beginning or end	Divya years	Solar years
Satya yuga	400	1,44,000
Tretā yuga	300	1,08,000
Dvāpara yuga	200	72,000
Kali yuga	100	36,000

1 day of Brahmā is called kalpa and it consists of 1,000 yuga. Kalpa is divided into 14 manvantaras of 71 yuga each. They are separated by 15 sandhyā periods in between and at the end, in addition to manu period. Each sandhyā is equal to one satya yuga i.e. 4/10 of a yuga.

Thus, 1 kalpa = 14 manu + 15 sandhyā

= 14 x 71 yuga + 15 x 4/10 yuga

= 994 yuga + 6 yuga = 1000 yuga

Note - According to Āryabhaṭa, a kalpa has 1008 yuga divided into 14 manus of 72 yugas each.

Verse 40-46 - Current time - At present 50 years of Brahmā have passed. In the 51st year (of 2nd parārdha or half life of Brahmā), this is the first day, called Śvetavārāha kalpa. In this kalpa, six manvantaras have passed, namely - (1) Svāyambhuva (2) Svārociṣa (3) Auttami (4) Tāmasa (5) Raivata and (6) Cākśuṣa

Current manvantara is Vaivasvata in which 27 yugas have passed. In 28th yuga, Satya yuga, Tretā and Dvāpara have gone. First fourth part of kali era is continuing.

Time from beginning of creation till beginning of kaliyuga in this kalpa -

Beginning sandhyā		= 17,28,000 years
6 Manvantaras	= 6x71x43,20,000	= 1,84,03,20,000 years

6 sandhyās of 6 manus 6x17,28,000 = 1,03,68,000 years
27 yugas in 7th manu 27x43,20,000 = 11,66,40,000 Years
Satya, Tretā, Dvāpara = 38,88,000 years
Total = 1,97,29,44,000 years

In kaliyuga, as in March 1996, 5098 years have passed. These are to be added to find the years passed since beginning of kalpa.

Verse 47-52 - Motion of planets started at mid night at Laṅkā (a point of equator through which prime meridian through Ujjain passes). The day was named as Ravivāra, Caitra Śukla pratipadā. All the planets reach the same position at midnight of Brahmā (after interval of 1 kalpa)

At the end of Brahmā's day (kalpa) all planets vanish. Author doesn't agree with Bhāskara that earth remains.

Verse 53 - Bhagaṇa is one revolution of a planet starting from Aśvinī to Revatī end as seen in the sky.

Explaination - Path of revolution along Zodiac (apparent path of planetary movement, more correctly of sun - rāśivṛtta) covers 360° divided in 12 rāśis of 30° each. Almost the same circle is path of moon (inclined at 5° angle) There are 27 nakśatras of 13°20′ each in which moon stays for about 1 day each. There was system of unequal division of nakśatras also which will be discussed later on. Aśvinī nakśatra is Ist and meṣa rāśi also start from 0° of zodiac. Last nakstra is Revatī

Verse 54 - Division of angular measurements according to previous ācāryas -

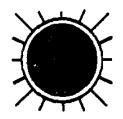
1 Bhagaṇa 12 rāśi, 1 rāśi = 30 Aṃśa

1 Aṃśa = 60 kalā, 1 kalā = 60 vikalā

1 vikalā = 60 parā 1 parā = 60 viparā

Note - Parā and viparā are not used by other texts nor in modern mathematics.

Verse 55 - Prayer of Lord Jagannātha

Verse 56 - Śri Candraśekhara has written this dṛksiddha gaṇita (as observed in sky) in simple language, so that even children can follow it.

Units of Time and Length –written and added later

Units of measurement-A physical quantity is measured by 2 components-

A basic unit of quantity used as standard for comparison of other quantities.

A number which is ratio of measured quantity to standard .Basic unit should be available, reproducible, convenient to handle and easy to compare through experiments.

How many units are sufficient-For mechanics, 3 units are sufficient-length, mass, time. All other units of measurement can be derived from them. In 1901, Giorgi proved that that by adding a quantity related to electric properties, all physical quantities can be measured. To show the inter-relation between electricity and magnetism, or to explain the property of medium (vacuum or material), another quantity is required. Thus 5 quantities are sufficient to explain all physical quantities. These are called 5 *Tanmātrās* in *Sānkhya*' philosophy. For this 5 dimensional view of world, there are 5 x 5 = 25 elements in *Sānkhya*.

5 fold division of units is described by 5 *Mā* (=to measure) *chhandas*-

Mā, pra-mā, prati-mā, upa-mā, sa-mā

For same type of quantity e.g. length-

Mā = basic unit (e.g. meter), *Pra-mā* = multiples (kilometer etc.), *Prati-mā* = sub-multiples, *Upa-mā* = Related length units (Foot, Nautical mile, light-year etc.), *Sa-mā* = Link with other units (with time through velocity of light, or with area, volume etc.)

Inter-relation of units of same or different kinds is called *asrī-vaya* (*upamā + samā*)

Various types of inter-relations are called *Vaya-chhandas*. In context of length, they are classified as -.*Mā* =*Pṛthvī* (earth) -Standard rod, earth or earth-like compact body-sun, solar system, galaxy.

Pramā- Antarikśa = Intermediate. Regions between and beyond earth(s).

Pratimā -Space,volume.

Asrīvi-Directions.

Modern units of length-(1) Foot- Based on human foot-

(2) Meter-It was defined in 4 ways-(i) Length of pendulum with half time-period of 1 second, (ii) 10^7 part of arc length from equator to north pole, (iii) 16,50,763.73 times wavelength of Kr_{86} radiation between energy states of 2_p10 and 5_d5, (iv) Distance traveled by light in 29,97,92,458 part of 1 second.

(3) Nautical mile-1 minute arc of equator.

(4) Astronomical unit (AU)-Semi-major axis of earth orbit around sun.

(5) Persec -Distance at which AU subtends an angle of 1 second.

(6) Light year-Distance traveled by light in 1 year=1016 meters approx.

1 Persec=3.26 light years, 1 AU=1.5x1011 meters.

Seven *yojanas*-(1) *Nara yojana* =32,000 hands-used for human possessions of land.

(2) *Bhū-yojana*—1000 or 1600 parts of earth diameter (sūrya-siddhānta -8km-*yojana*)

(3) *Bha-Yojana*—27 *bhū-yojanas* = 216 kms, used for sun distance and size of galaxy

(4) *Prakāśa-yojana*—Distance traveled by light in 1 *truṭi* = 1/33,750 seconds

(5) *Dhāma-yojana-Kśara dhāma* =720 parts of equator circumference = 55.5 kms.

Akśara dhāma-Measure of space with earth as standard in exponential scale. Number of powers of 2 is equal to *akshara* in *chhanda*. Distance d = r x $2^{(n-3)}$, r = radius of earth, n = unit of distance

(6) Sun diameter as *yojana* for solar system in *purāṇas*.

(7) *Pramāṇa-yojana*—Starting from solar system, scale for each successive *loka* in 500 times longer units at each step.

Micro-units-Smaller worlds are successively 1 *lakh* times smaller-man (meter size), *kalila* (cell, 1 *lakh* part of meter), *Jīva* (atom of $10^{(-10)}$ meter size), *Kuṇḍalinī* (nucleus of 10^{-15} meter size), *Jagat*-particles of 3 types (10^{-20} meter size-not defined-*Chara* (lepton), *sthāṇu* (Baryon), *Anupūrvaśah* (link -particles or meson)

Deva-dānava (10^{-25} meters size)-Creation from 33 types of *devas* only, not from 99 types of *asuras* (*dānava*), Created world is 1/4th part of *puruṣa*. 3/4th is field or dark matter.

Pitar (proto-type, Parents)-10^{-30} meter size.

Ṛṣi (string-*rassi* in *hindi*) 10^{-35} meters size.

Micro or smallest unit called *paramāṇu* by *Varāhamihira* = 8^4 parts of *angula* = 4.5 micron. *Śrīpati* calls it *trasareṇu* equal to 60 atoms. So, atom =1.2 x 10^{-7} cm.

Lalita-vistara-Paramāṇu-raja = *Angula* x 7^{-10} = 0.6 x 10^{-8} cm

Tiloya pannati—Trasareṇu = Angula x 8^{-9} = 1.4 x10^{-8} cm.

Single object or *Brahma* is indicated by *Anguṣṭha* (Thumb).In *Puruṣa-sūkta, angula* means 96 parts of human length, or earth, solar system earth, galaxy as per context.

Measures of Solar system-Modern estimate (NASA 2002) is that Woort cloud is boundary of solar-system at distance of 50,000-1,00,000 AU from sun. Indian measures-

Samvatsara is *āditya* (energy-field) of sun. Fields of galaxy and universe are called *Varuṇa* and *Aryamā*. This is sphere of 1 light year radius with center at sun.

1,575 crores diameter in unit of sun-diameter = 2.310 light year.(called *ratha* of sun). Outer wheel diameter is 6,000 i.e. outer boundary of Kuiper-belt. Modern estimate is about 70,000 plutonic bodies of above 100 km size. *Purāṇas* tell 60,000 *bālakhilyas* of *Anguṣṭha* size i.e. 96 parts of earth diameter =135 km approx. *Sūrya-siddhānta* calls it *Nakśatra-kakśā* of sun at 60 AU. This has been called *Aloka* (dark) earth of 100 *crore yojanas* (8km). *Loka* (lighted) earth is of 50 crore *yojana* (8 km) diameter. This has 7 *dvīpa* /oceans of *Priyavrata* formed by motion of planets. Inner wheel of sun is of 3000 sun diameter i.e. up to Uranus orbit. Practical or *Indra* zone is of 1000 sun diameter (*sahasrākśa, akśa* = sun or eye) up to Saturn orbit.

Saturn being at the end of solar effect is called son of sun.

Planetary distances in *Bhuvar-loka* are in terms of earth diameter. Size of next *Svar-loka* will be in 500 times bigger unit. Distance from sun to pole (*Dhruva*) is 14 lakh x 500 earth diameter which is distance of Woort cloud.

Earth diameter x $2^{(30)}$ in *akśara-dhāma* units. With three spherical zones inside earth as image of 2 bigger earths, there are 33 zones. Energy (*prāṇa*) of each is a *deva*. Signs of 33 *devas* are letters from k to h – its arrangement (*nagara*) is Indian scripts called *Devanāgarī*.

Earth/man = Solar earth/Earth.=10^7,called *koṭi* (limit).This is called *Maitreya-maṇḍala* or *Sāvitrī* (2^{24} of earth size) in which creation occurs. *Dyu* (sky) of solar system is 10^7 times sun size. Earth is *crore* times or 24 *dhamas* bigger than man and is called *Gāyatrī*. *Sāvitrī* x 2^{24} = *Sarasvatī* (creative field of galaxy). *Sarasvatī* x 2^{24} = *Veda* (creative field of universe or *Veda-puruṣa*,10 times bigger than universe).Creative aspect is *Niyati*.

Measures of Galaxy-(1) *Sūrya-siddhānta* gives 1.87 x 10^{16} *Bha-yojanas* (216 km) = 1,23,000 light years (modern estimate 1 lakh LY)

(2) Earth size x 2^{46} is galaxy Its creative field is *Kūrma* (*Goloka* of *Brahmavaivarta purāṇa*) of 52 *dhāma* units i.e.2^{49} earth size. For 49 *ahargaṇa* (*dhāma* units) there are 49 letters in *Devanāgarī* script from a to h. 3 extra units of *Kūrma* is creator, conscious being is called *kśetrajña* in *Gītā* chapter 13, so 3 letters are added at end-*kśa, tra, jña*.

(3) Circumference is 0.5 *Parā* (10^{17}) *dhāma yojanas* (55.5 kms). Diameter comes to about 1 lakh LY.

(4) Size of *Kūrma* in *Narapati-Jayacharyā* is hundred thousand (10^5) *Śanku* (10^{13}) = 10^{18} *yojanas*. As a *puruṣa* of Galaxy, it is 10 times bigger, so galaxy is about 10^{17} *yojanas*. In space, earth is lotus of 1000 petals, 1 petal = 1 *yojana* (*Āryabhaṭa*)

(5) Galaxy is 1 crore times solar earth-*Sāvitrī*.

(6) This is *Janah loka* of 2 crore *yojana* radius (1*yojana* = 500 x 500 sun diameter)

Bhuvar-loka is sphere of 15 *ahargaṇa* (*dhāma*) around earth i.e. 2^{12} of earth size which is also called *Varāha*. Viewed from sun, it is 100 *yojana* (= sun diameter) high and 10 *yojana* high, up to lunar orbit. This covers up to 60% distance of Venus orbit. Earth's exclusive zone extends up to 9 *ahargaṇa* i.e. 64 times earth radius. Moon is within it at 61 radius. Spiral arm of galaxy is called *Śeṣa-nāga*. Near sun, it has 1000 sun like stars called 1000 heads of *Śeṣa*. Region of this is called *Maharloka*, whose size is given as-

1000 times size of solar system (*sahasra-śīrṣa puruṣa*,1000 heads of *Śeṣa*)

43 *ahargaṇa* =earth x 2^{40} (*triṣṭup chhanda* has 44 ± 2 or 43 letters in *Māheśvara sūtra*)

1 crore *yojanas* (1 *yojana* =500 sun diameters)

Middle *loka* between earth (*bh*ū) and *satya loka* 10^{20} times bigger, so it is 10^{10} times earth size.

Tapah loka is given in 4 ways-

2^{64} times earth size for 64 letters in *Brāhmī* script.

8 crore *yojana* (1 *yojana* =sun diameter x 500 x 500)

(3) 864 crore light year radius –equal to day-night of *Brahmā*.

(4) Earth orbit/earth =*Tapah loka*/galaxy

Satya loka (1) Galaxy x 10^7 (or 2^{24})

(2) *Maharloka* /earth = *Satya loka / Mahar*

(3) 12 crore *yojana* (1 *yojana* = sun diameter x 500^3)

Time units

Modern unit-1 second = 86,400 parts of mean solar day.

Due to fluctuations and slowing down of earth rotation by tidal friction, new definition was adopted in1967-it is 9,19,26,31,770 times the period of light radiated by transition between two ground states of Cesium-133 atom.

Nine Indian Time-units-

(1)Brahma-Time period of creation from formless (*avyakta*) to forms is called a day of *Brahmā* or *Kalpa*. In same period of time, creation dissolves into *avyakta* (*Gītā* 8/17,18). Day of *Brahmā* has been defined as of 1000 *yugas* in *Gītā, purāṇas*, *Sūrya-siddhānta* etc. Each *yuga* is of 12,000 *divya*-years, 1 *divya*-year =360 solar years. Thus, 1 day of *Brahmā* = 432 crore years.

(2) Prājāptya-Prajāpati started *yajña* (*Gītā* 3/10), so *prājāpatya* period is period of galaxy from where creation started. Rotation period of galaxy is called *Manvantara* of about 31.68 crore years. Present stage is 7th *manvantara*. This has been called 7th day in Bible. After 4, 5th days sun, moon, earth were created. So, day cannot mean here rotation of earth or even of sun. This is rotation of first creation galaxy. Modern estimate of period of sun revolution around center of galaxy is 20-25 crore years.

(3)Divya year is of 360 solar years, arrived in 3 ways. This is approximate period of revolution of imaginary planet at 60 AU (or average rotation of about 60,000 *Bālakhilyas* at same distance), called *Aloka* (dark) boundary of 100 crore *yojana* diameter, This has been called *parivarta yuga* in *Vāyu, Matsya purāṇas*, which is cycle of historic changes. Third view is that north-south motion of sun is like day-night cycle. This cycle of 1 year is 1day (*divya*). Taking round number 360 for days in a year, *divya* year is of 360 years.

(4) *Guru* scale-In period of 60 years Saturn and Jupiter complete integral revolutions-2 and 5. Alternatively, *Angirā*-effect (upward convection due to radiation pressure takes 60 years to complete (*Aitareya Brāhmaṇa* 18/3/17,*Taittirīya Brāhmaṇa*, 2/2/3/5-6)

(5) *Pitar māna*- Synodic revolution of moon in 29.5 days is called 1day of *pitars*. *Varāha* is 15 *ahargaṇa* or 4,096 times earth size-that is *parjanya*. Intermediate level is *pitar*, 64 times earth size called *pitar*. Moon orbit is 61 times earth size. So *pitars* of human beings also reside on outer region of moon. Our bright half of month is night of *pitars* and dark half is their day.

(6) *Sāvana māna*-Sunrise to next sunrise is *sāvana* or civil (practical) day.1 month = 30 days, 1 year = 12 months.

(7) Solar-Apparent revolution of sun around earth is yea r(sidereal). 1/12th part is 1 month (1 *rāśi* = 30^{0}. Movement). 1 day is 1^{0} movement.

(8) Lunar-Synodic revolution of moon around earth is a lunar month.5 *angas* of *pañchānga* are defined from moon and sun—*Tithi* = (M-S) /12^{0}, 1*tithi* = 2 *karaṇa*, *yoga* = (M+S) /13.3. *Nakśatra* is average daily motion of moon. *Vāra* (day) is cyclic naming of days.

(9) *Nākśtra* (sidereal)-Axial rotation of earth with respect to fixed stars is sidereal day of 23 hours 56 minutes. Month, year have 30 and 360 days.

Micro units of time-*Śatapatha Brāhmaṇa* (12/3/2/1,5) divides mean day length of 12 hours successively by 15 parts into units named as *muhūrtta* (48 minutes), *kśipra, etarhi, idānī, prāṇa, aktana* (or, *ana*), *nimeṣa, lomagartta, svedāyana*. It is stated that stars (*nakśatra*) in galaxy (*Brahmāṇḍa*) are like its *loma-gartta* (roots of skin hairs). Number of *lomagartta* in a year (*Samvatsara*) is equal to the number of stars in a galaxy, so this unit of time is called *lomagartta* which is equal to *muhūrtta* x 15^{-7} = about 80.000 parts of a second. Similarly its 15th part is *svedāyana* equal to about 11, 20,000 parts of a second. In this time, light travels about 270 meters. Rain drops (*sveda*) move (*ayana*) about same distance without breaking or joining, so this time unit is called *svedāyana*. Estimate of number of stars in galaxy was done after 1985 which is correctly estimated in *Śatapatha-Brāhmaṇa* to be 10 [11]. Logic of division by 15 is given that ratio of earth orbit to earth size is same as ratio of *Tapah-loka* (visible universe) and galaxy, both equal to 2 [15].

Seven *Yugas*-By joining two cycles of time, a *yuga* is formed. *Munīśvara* in his astronomy text *Siddhānta Sārvabhauma* has stated 5 *yugas*-

5 years, 5 x 12 = 60, 12 x 60 = 720, 600 x 720 =*kaliyuga, kali* x 10 =1 *yuga*.

Like 7 *yojanas*, there are 7 *yugas*, depending on completion of various *yajñas*—

(1) *Sanskāra yuga*—Education and other reforms projects are completed in 4 to 19 years which is a *sanskāra-yuga*. (a) *Gopada-yuga*-is of 4 years like modern leap year system. Its year starts in *godhūli-velā* (literally cow-dust-time, when cows return home at sunset, dust is raised) like Hebrew or Islamic calendars. West Asia was place of *Asuras*, called *Niśācharas* because their day started with sunset.This is described in *Aitareya Brāhmaṇa* (7/13). Suppose,1st year starts at 6 PM on 4-1-2001.This year *Kali* will end at 12 PM on 4-1-2002 when people will be sleeping, So, *kali* is called sleeping.2nd year *dvāpara* will end on 5-1-2003 at 6AM, so *dvāpara* is called twilight. 3rd year *tretā* will end at 12 AM on 5-1-2004, when people will be standing, so *tretā* is called standing.4th year will end on 4-1-2005 at 6PM as there is leap year in 2004. Here year is of 365 ¼ days. (b) 5 year *yuga* - *Yājuṣa jyotiṣa* has described 5 years *yuga* in which lunar years match with solar year by adding 2 extra (*adhika*) months. Years are named by adding prefixes *sam-, pari-, id-, idā-, anu-*, to the word *vatsara*. (c) 12 years *yuga* is revolution period of Jupiter around sun. These are named like months of lunar year-*chaitra, valśākha*, etc. (d) *Rāhu yuga*-It is called Saros cycle in Babylonian astronomy. This is relative motion of Sun and *Rāhu* (node of

Moon) in 18 years 10,5 days in which eclipse cycle repeats. Its half period of 3339 *tithis* is also approximately eclipse cycle stated by *Viśvāmitra* (*Rk* 3/9/9). (e) 19 year *yuga*—This is followed in *Ṛk-jyotiṣa*, as explained by *Sri Prabhakara Holay*, *Nagpur*. In this period, lunar years match with solar year more accurately (less than 2 hours error) by adding 7 *adhika* months. Years are classified into 5 types according to 5 blocks of 6 *tithis* in which a solar year starts. In 1 *yuga,* there are 5 years of *samvatsara* type.

(2) Manuṣya (human) *yuga* -- 60 years active life of man is called *Angirā* period in which 6o year cycle of *guru*-years occurs. In 100 years, *Saptarṣi* (Ursa major, ursa =*ṛṣi*) moves *1 nakśatra*, i.e.27th part of zodiac circle. In *Ṛk-jyotiṣa* calculation, moon moves 1 *nakśatra* ahead in 100 years. The line joining two eastern stars moves 1 *nak-śatra* back in 100 years. This year count has been called *Laukika* in *Rājatarangiṇī*. 1/3rd of *divya-dina* or *pari-varta yuga* of 360 years is 120 years which is human life for astrological timing of events.

(3) Parivarta yuga-This is *divya-dina* of 360 years in which historical changes (*parivartana*) occurs. 71 such *yugas* make *manu-yuga* of 26,000 years (precession period of earth's axis) in *Brahmāṇḍa Purāṇa* (1/2/29/19)

(4) Sahasra yuga-Bhāgavata purāṇa (1/1/4) states 1000 year *satra* of *Śaunaka* in *Naimiṣāraṇya*. Compilation of *Pu-rāśas* took about 200 years, but its effect on social norms lasted for thousand years. It could be revised only in time of *Vikramāditya* of Ujjain (82BC-19AD, era started in 57 BC) as per *Bhaviṣya purāṇa* (3/3/1/2-4). During 3000 years, seasons shifted back by 1 ½ months.720 years of *Munīśvara* is of 2 *parivarta yugas. Sahasra* (1080) years is 3 *parivarta*. Even *Gautama Buddha* planned his religion for 1000 years. Prophet *Mohammed* predicted Islam to last for 1400 years. *Saptarṣi yuga* is of 2700 years. It is described in two ways in *Brahmāṇḍa, Vāyu purāṇas*. 2700 solar years are called *divya* years, *Mānuṣa* year is 12 revolutions of moon around earth in 327.5364 days. *Saptarṣi* era is also stated to be of 3030 *mānuṣa* years =2717 solar years. *Romaka yuga* of *Pañchasiddhāntikā* (*Varāhamihira*) is of 2850 years =19 year *Ṛk-yuga* x 150,

(5) Dhruva or *Krauñcha yuga*-This is of 9090 *mānuṣa* years or 8100 solar years. This is exactly 3 times *saptarṣi yuga* and about 1/3rd of *Ayana* or *Manu-yuga*. Position of north pole of earth makes a circle in 26000 years and is close to 3 stars so the period is divided into 3.On earth regions around north pole are called *Krauñcha-dvīpa*, so it is called *Krauñcha yuga* also. In north India, *Guru* years are calculated as per actual mean motion of *guru* in 361.14 days (*Sūrya-siddhānta*). In 85 solar years there are 86 guru years. In south India, solar years are named as guru years (*Paitāmaha-siddhānta*). In 85 x 60 = 5100 years, both cycles are completed. On 11-2-4433 BC when *Rama* was born, the year was start of *guru* cycle in both systems (1st *Prabhava* year) as per *Viṣṇudharmottara purāṇa* (82/7,8). *Matsya* incarnation had occurred 5100 years before that in 9533 BC. Herodotus gives date of sinking of last island of Atlantis in 9564 BC. This is approximately period of last glacial flooding.

(6) Ayana (Precession) *yuga*-Earth's axis rotates around pole of ecliptic (earth orbit) in 26, 000 years. This has been called *Manu yuga* in *Brahmāṇḍa purāṇa* (1/2/29/19).Glacial age on earth is due to two cycles-Precession in 26000 years in reverse direction and advance of earth aphelion in 1 *lakh* years. Glacial region is around north pole. When it is inclined away from sun or earth is at aphelion it gets less heat. When both combine, it is Glacial ice age. Its cycle is in 21600 years—1/21600 = 1/26000 +1/100000

Civil cycle is taken as middle of the two. By taking 24000 years cycle, there is positive error in12000 years and negative in other half (*Brāhma-sphuṭa-siddhānta, madhyamādhikāra, 60, Siddhānta-śiromaṇi of Bhāskara-II, Bhū-paridhi,* 7). Each half of 12000 years is taken as *yuga* of 12,000 *divya* years in every *purāṇa*, *Mahābhārata*, etc. In *Mahābhārata* period *Avasarpiṇī* (descending) period was running in which part *yugas –Satya, tretā, dvā-para, kali*, of 4,3,2,1,parts come in that order. Of this order, *Kali* started on17/18-2-3102 BC Ujjain mid-night. The other half is called *Utsarpiṇī* (ascending) in which *kali* to *satya yugas* come.

(7) Astronomical *yuga*—This is 360 times *ayana yuga* of 43,20,000 years in which all planets up to Saturn complete integral number of revolutions (*Bhagaṇopapatti* in *Siddhānta-Śiromaṇi of Bhāskara*-II). Two other cycles depend on it which is not verified so far-(i) Movement of magnetic pole and magnetic reversal, (ii) Movement of geographical pole in north south direction (*Indra-Vijaya* of *Madhusudan Ojha)* or equivalent continental-shift.

Time has been equated to full-pot (*Pūrṇa-kumbha*) or volume .and parallel with 7 *chhandas* is shown in *Kāla-sūkta* of *Atharva-veda* (19/7), *Bṛhatī-sahasra* (36000) days is life period of man (*Aitareya Brāhmaṇa*). Understanding of these *yugas* explains *purāṇic* chronology since 62,000 BC.

(Detail article with references is in Hindi, titled—"*Chhanda-Adhārita Māpa-Vijñāna"*

Chapter - 2

REVOLUTION OF PLANETS

Subject - This chapter deals with total revolutions of planets in a kalpa, adhika māsa (gain of lunar months above 12 in a solar year), kśaya tithi (shortage of lunar dates from months of 30 civil days), rotation of orbits. From that; the average daily motion of planets have been calculated.

1. Explanations - All the results given in this chapter are assumed and no hint is given as to how these numbers have been found. Obviously they are highly accurate and have been followed since time immemorial. All texts from Sūryasiddhānta to Siddhānta Darpaṇa have followed the same practice.

It is possible that the saṃhitā and Brāhmaṇa texts gave the methods and observations of planetary motions. The teaching of science and mathematics was like present day text books of college, and not in verse form which is useful for memorising only. This became necessary when educational institutes and their books were destroyed due to foreign invasions. Mathematics in modern text book form has been found in Bākhsalī manuscripts of mediaval preiod (edited in 3 vols by G.R. Kay - New Delhi-7)

All ancient authorities have admitted that these results are not based on observations. Sūrya siddhānta has stated that these were given by Sūrya to Mayāsura in Romaka town in 21, 63, 223 B.C. (121 years before the end of Satya yuga). To some extent, it is correct. Even with most modern

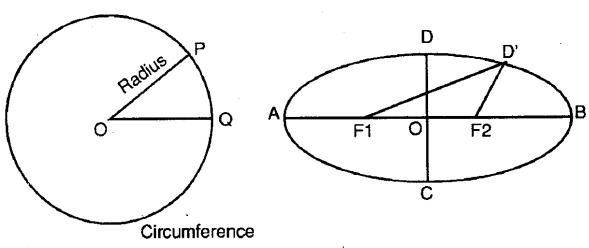
equipments, calculation of motion for billions of years cannot be made on observations during a life time only. It needs systematic observations for at least 500 years for studying motion in orbits, and at least 10,000 years and much more, if rotation of orbits, or change of earth's axis is to be calculated. Thus the result could have been otained only from observations through the ages, preserved by generations (like Vedas, it has to be 'Apauruṣeya' i.e. god given or beyond a human being).

2. Origin of complete revolution numbers in a kalpa - There are two assumptions by ancient authorities - In general, it is assumed that the figures have been obtained on the basis of observations through ages. Total motion in a kalpa has been calculated on basis of observed rates.

Siddhānta Darpaṇa has followed pattern of Siddhānta Śiromaṇi of Bhāskara II except for some new improvements. Bhāskara has assumed that concept of yuga and kalpa has been derived from the observed motion of planets. The planets repeat their positions after every yuga (the grand year), as in civil year earth comes back to the same position round the sun. However, if we consider rotation of orbits; its cycle is repeated only after 1000 yugas or a kalpa. For example, orbit of Saturn rotates only 39 times in a kalpa, so its motion cannot be perceived within a yuga. Text books of Tantra and karaṇa are not concerned with such slow motion.

Another presumption is that theories of planetary motion and constants of orbit have been given in vedas. We do not know the technical terms and method of presentation of astronomy as explained in saṃhitā and Brāhmaṇa texts. Varāhamihira was probably last who understood

contents of all 3 parts of jyotiṣa from vedas. It is presumed that 10,000 verses of Ṛkveda contain records of astronomical observations for 10,000 years or yugas of 5 or 19 years. Though only Āryabhaṭa I has specifically mentioned two motions of earth, it appears that many others knew about movement of earth. It is clear from names Jagat (moving), saṃsāra etc. Methods of calculation followed by other astronomers also indicate that they were following some theories known in vedas or other texts but not specified in astronomy works. Whatever may be the nature of planetary motion, it contines to be observed against the background of same Zodiac of 12 rāśis or 27 nakśatras, and from earth only. From scattered observation charts through the ages, theories of circular or elliptical orbits have originated. They cannot be observed directly.

3. Circle and Ellipse

O Centre, radius OP or OQ
Fig. 1A-Circle

AB = Major axis, CD = Minor axis
O Centre, Focus F1 and F2
Fig. 1B-Ellipse

Definition - Circle is path (locus) of a point (circumference) which remains at fixed distance (radius) from a fixed point called centre. (Figure 1 A)

Circle is a round figure on a paper looking same from all directions. Ellipse is elongated form of circle - stretched in two opposite directions (called major axis)

Definition - Ellipse is locus of a point whose sum of distences from two points F_1 and F_2 (called focus) is constant. Thus $DF_1+DF_2 = D'F_1+D'F_2 =$ AB (major axis). Smallest width is minor axis CD. AB and CD are perpendicular at their middle point O, called centre (Figure 1B)

Kaplar's laws of planetary motion indicate that planets move in an ellipse round the Sun which remains at one of the focus (not at the centre). Newton's law of gravitation were derived from Keplar's laws and vice versa.

In circular orbit (special type of ellipse where both focus are at same point), speed of planet will remain constant. In elliptical orbit, it will be fastest when the planet is closest to Sun (at A if sun is at F_1). It will be slowest when farthest from sun (at B). B is called Aphelion (Apex=top, helios = Sun) or mandocca (slow+top) or ucca in short in jyotiṣa.

One feature of circular orbits remain the same in elliptical orbit alṣo. Though the speeds vary at different position, the area covered by line from sun to the planet in unit time remains the same. Thus POQ area in circle or DF_1D' area in ellipse, covered in unit time remain constant.

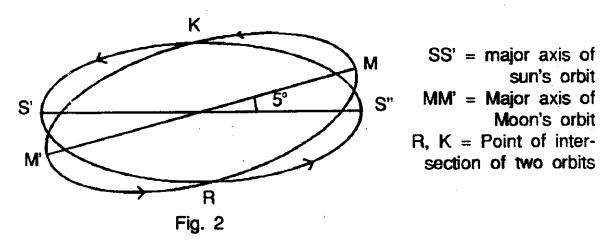
4. Śīghrocca and Mandocca - The relative motion of sun and earth remains the same, whether it is observed from sun or earth. In either case, it will be elliptical motion with same speed. Similarly,

moon also moves in elliptical orbit round the earth. The position of sun or moon where its speed is lowest (at highest point in orbit), is called man-docca.

Orbits of other planets around sun are also elliptical. However, when we observe from earth, it is a composition of two elliptical motions - one is relative motion of sun around earth and the second is motion of planet round the sun. The planet in smaller orbit is called śīghrocca, as the average motion is faster in smaller orbit. The highest point in slower and bigger elliptical orbit is called mandocca.

In bigger orbit, a planet's motion will appear slow due to two reasons. At larger distance, gravitational attraction of Sun is small and planet moves at small speed to counter the attraction by its centrifugal force. If speed is more, it will go still farther and loose speed, till it settles into a stable orbit. Due to larger distance the angular speed appears still slower.

5. **Pāta**- Orbit of sun around earth and orbit

Fig. 2

of moon around the earth are not in the same plane. They are inclined at angle of about 5° (Figure 2).

If sun orbit is taken as reference level, at point R, moon appears moving towards north or upwards. Moon itself is not at R, it may be at any point on its orbit MKM'R. R is merely an imaginary point of intersection and is called uttara-pāta (ascending mode) or Rāhu.

At point K motion of the moon appears south wards, hence it is called dakśiṇa-pāta (descending node) or ketu.

When moon is at one of the pātas on pūrṇimā (180° away from sun) or amāvasyā (same direction as sun), eclipse occurs. Thus Rāhu and Ketu are said to cause eclipse. Rāhu and Ketu are called chāyā graha as they are only imaginary points. They have nothing to do with shadow of earth or moon. R and K are always in opposite direction from earth as seen from the diagram. They are moving in reverse direction to the direction of planetary motion. Their revolution is called bhagaṇa of pāta (in about 19 years).

Similarly, pāta of other planets also move. But their motion is so slow that it is noticed only in a kalpa.

Motion of Ucca - Motion of ucca of moon is visible in a yuga (one revolution in about 9 years). Motion of other planets is very slow and can be noticed only in a kalpa.

6. Change in values of bhagaṇa - Sūrya Siddhānta, first chapter states that motion of planets vary with time and hence its observation needs to be corrected after long lapse of time.

It is known in modern astronomy that earth's rotation on its axis is slowing down at the rate of

14 seconds per century due to tidal function. Due to decrease in angular momentum of earth, moon is moving away at the rate of 8mm every year to conserve the angular momentum of earth-moon system. Due to tidal forces of galaxy and sun and friction of solar atmosphere, motion of planets also will slow down. But its values are not known either in siddhānta texts or in modern astronomy. It can be inferred to some extent by comparison with old records of solar eclipse in vedas, or comparing old values of bhagaṇas with present values.

Thus, if we calculate the average motion of the planets on the basis of their total motion, their values will differ from the real observation. Another reason of error will be inaccuracy and approximation of mathematical methods and calculations. To correct these, various astronomers have introduced correction terms for their era. Candraśekhara was last among them. In addition, he introduced 3 correction terms for moon's motion, whose error was noticed due to its faster motion.

REFERENCES

(1) For knowledge of circle and ellipse any college text book on plane coordinate geometry can be referred. For example Loney's coordinate geometry.

(2) For concept of intersection of two orbital planes any book on solid geometry can be referred. Fuller discussion will be in books of spherical Trigonometry by Gorakha Prasāda or by Todhunter.

(3) Development of planetary theories of motion have been excellantly explained in 'The structure of the Universe' by Sri J.V. Nārlīkara.

(4) Historical discussion of zodiac and frictional slowing of planetary orbits is given in 'An intelligent Man's guide to Science' by Isaac Asimov.

(5) For comparison of values of bhagaṇas given in different texts any good commentary of standard texts may be referred. One may read the histories of astronomy, referred to earlier in introduction.

Translation of the text

Verses 1-2 - Bhagaṇas of planets in a kalpa (West to east)

Ravi, Budha, Śukra	4,32,00,00,000
Candra	57,75,33,36,000
Mañgala	2,29,68,71,112
Bṛhaspati	36,41,55,205
Śani	14,66,49,716
Budha Śīghrocca	17,93,69,67,141
Śukra Śīghrocca	7,02,22,57,860
Mañgala, Bṛhaspati, Śani Śīghrocca	4,32,00,00,000

Note : 1 - Sun - Ravi, Sūrya, Arka etc. in sañskṛta

Mercury - Budha; Venus - Śukra, Mars - Maṅgala, Kuja, Bhauma; Jupiter - Guru, Bṛhaspati; Saturn - Śani

2. Budha, Śukra are in inner orbits around sun, so their revolutions are same as sun as they appear tied with it as seen from earth. Their revolution is equal to number of solar years in a kalpa by definition (1 year corresponds to 1 revolution of sun)

3. Śīghrocca of Bṛhaspati, Śani and Maṅgala is due to earth's orbit round the sun. Hence it is equal to apparent revolution of sun round the earth.

Verse 3 - Mandocca Bhagaṇa in a kalpa from west to east.

	Siddhānta Darpaṇa	**Surya Siddhānta**
Ravi	334	387
Candra	48,81,17,940	48,82,03,000
Maṅgala	310	204
Budha	410	368
Guru	805	900
Śukra	557	535
Śani	70	39

Note - Only source of these figures is Sūrya siddhānta. Author has not indicated source of his corrections.

Verse 4 - Bhagaṇa of pāta (East to West)

Note - Pāta is calculated according to inclination of orbit with Ecliptic. Since it is path of sun, there is no pāta for sun.

Planets	Bhagaṇa in a kalpa	Sūrya-siddhānta
Candra	23,22,98,033	23,22,38,000
Maṅgala	298	214
Budha	552	488
Guru	945	174

Śukra	110	903
Śani	545	662

Note - Source of different figures and large variations in figures for guru and śukra is not explained.

Verses 5-6 - Nākśatra dina is the time between rising of any nakśatra to its next rising (equal to time of revolution of earth on its own axis)

The time between rising of a planet to its next rising is called sāvana dina for that planet. (For example sunrise to next sunrise is sāvana sūrya dina). This corresponds to rotation of earth with respect to that planet.

(i) Total number of nākśatra dina in a kalpa 15,82,23,78,28,000

(ii) Sāvana dina of a planet = Nākśatra dina - graha bhagaṇa

(iii) Cāndra māsa = Candra bhagaṇa - Sūrya bhagaṇa

Verse 7 : In a kalpa (or a Mahāyuga)

No. of adhimāsa = No. of cāndramāsa - No. of Sauramāsa

No. of Kśhaya dina (Lost days) = 30x No. of Cāndramāsa - No. of sāvana days

Verses 8-11 :

No. of solar months in a kalpa	51,84,00,00,000
No. of Cāndra months "	53,43,33,36,000
No. of adhimāsa "	1,59,33,36,000
No. of Sauradina "	15,55,20,00,00,000
No. of Cāndra dina "	16,03,00,00,80,000

No. of Sūrya (sāvana) dina " 15,77,91,78,28,000

No. of Kśaya tithi 25,08,22,52,000

Note - No. of sāvana dina and kśaya tithi here is same as that of Sūrya siddhānta, where the figures given are for a mahāyuga. However, sāvana days in a mahāyuga are different according to other texts -

Sūrya siddhānta of Pañca siddhāntikā - 1,57,70,17,800

Āryabhaṭa - 1,57,79,17,500

Brahma sphuṭa siddhānta, siddhānta Śiromaṇi 1,57,79,16,450

Mahāsiddhānta - 1,57,79,17,542

Verse 12 - Definition - At any given time kendra of a graha (angle) = position of a planet-position of its ucca.

Compared to Śīghra ucca it is called Śīghra kendra, compared to manda ucca, it is called manda kendra.

Ucca and pātā bhagaṇas are not completed in a yuga except for moon, so their bhagaṇas are stated for a kalpa (1,000 yugas)

Verse 13 : Śīghrocca = drāk, cala, āśu, capala etc.

Mandocca = Mṛdu, ucca, manda etc. (synonyms)

Verse 14-15 - No. of asu (prāṇa = 4 seconds) in a day-

1 average (madhyama) nākśatradina = 21,600 asu

1 madhyam saura dina = 21,976 asu

1 madhyam Cāndra dina = 21,320 asu

1 madhyam sāvana dina = 21,659 asu

Sāvana dina is commonly used by people which is divided into 60 ghaṭika or daṇḍa.

Verse 16 - Bhagaṇa = 1 complete revolution = 360° aṃśa. Bhagaṇa kalā = bhagaṇa x 360 x 60

Dainika kalā of a graha = graha bhagaṇa kalā in kalpa/sāvana dina in kalpa

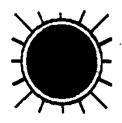
Time for 1 bhagaṇa of a graha = sāvana days in kalpa/graha bhagaṇa in kalpa

Verses 17-18 : Like division of full circle rotation in 360° (aṃśa) and then further sub-divisions by 60 in each step, learned men have divided a sāvana dina also by 60 at each step to daṇḍa and pala etc. One complete revolution (bhagaṇa) of sun takes days 365/15/31/31/24 daṇḍa, pala etc.

Verse 19 - Madhyama guru takes days 361/5/27/27/13 in one rāśi at average speed.

Verses 20-24 - Daily motion of planets is described in liptās (1/60 aṃśa) and 10 furhter sub-divisions in steps of 60. By multiplying this daily motion with no. of days (passed from beginning of kalpa to desired day), madhyama graha (position with average speed) is obtained.

Sun (Ravi)	59-8-10-10-24-12-30-4-10-4
Candra	790-34-52-3-49-8-2-16-10-11
Mañgala	31-26-30-6-47-44-32-49-3-4
Budha Śīghra	243-32-16-7-17-17-59-43-42-44
Guru	4-59-5-37-0-36-41-17-1-51
Śukra Śīghra	96-7-37-47-57-50-39-32-31-35
Śani	2-0-26-55-2-53-21-2-4-54
Candra ucca	6-40-54-31-0-44-5-52-45-39
Candra pāta	3-10-47-40-40-26-11-25-13-30

Verse 25-26 - Krānti vṛtta in sky is the sudarśana cakra of Jagannātha with which he removes fear, produces light and destroys all in the end. With this prayer Śri Candraśekhara Siṁha completes second chapter of Siddhānta Darpaṇa describing bhagaṇa of grahas.

Chapter - 3

MEAN PLANETS

Scope - This chapter describes methods for calculating value of madhya graha (position calculated from average motion). This coincides with sphuṭagraha (true position) twice in every bhagaṇa (revolution). Since the planetary motions started from mandocca position; at mandocca, sphuṭa and madhyama positions should be same (for sun and moon).

Verse 1 - Ahargaṇa (count of days) - Ahargaṇa for iṣṭa dina (desired day) is counting of days from beginning of kalpa (in siddhānta text). This is needed to know the graha on iṣṭa dina of any varṣa, māsa or tithi.

Note - In tantra, ahargaṇa is counted from beginning of mahāyuga (or sometimes, from the beginning of kaliyuga). In karaṇa text, ahargaṇa is counted from any reference year or beginning of current year itself for preparation of pañjikā.

Verses 2-8 - Steps in calculation of ahargaṇa -

1. Add the saura varṣas for 6 manu, 7 manu sandhi (each equal to satya yuga), 27 mahāyuga, 3 pādayuga and years passed in current kaliyuga.

Note : In the present Śvetavārāha kalpa, 6 manus out of 14 have passed. In the current 7th vaivasvata manu, 27 yugas have passed. At beginning of kalpa and after each manu, a sandhi equal to one satya yuga exists.* In current

mahāyuga, satya, Tretā and Dwāpara have passed. Kaliyuga started on 17/18-2-3102 B.C. Ujjain midnight.

2. Deduct 1, 70, 60, 400 years

Note - According to verse 24 of madhyamādhikāra in Sūrya Siddhānta, Brahmā took this time of 47,400 divya varṣa to create stars, planets and living beings. The present stable motion of planets started after that.

3. Multiply by 12 to make it months and add the number of months (māsa) elapsed from Caitra (Cāndra months in current year are almost equal to saura māsa).

4. Keep the result (no. of completed saura months) at two places.

5. At first place, multiply it by no. of adhika māsa (1,59,33,36,000) in a kalpa and then divide it by sauramāsa in a kalpa. Result will be adhimāsa related to the saura varṣa.

6. Add this to the no. of māsa from kalpa beginning obtained at step 3.

7. Multiply Cāndramāsa by 30 and add the days completed in the present month (Cāndra māsa)

8. Keep the result at two places.

9. At one place, multiply it by kalpa tithi kśaya (25, 08, 22, 52, 000) and divide by number of kalpa tithi. Substract the result from kalpa tithi at the second place. Difference is number of sāvana tithis from kalpa beginning. Divide it by 7. Remainder will give the week day counted from ravivāra (sunday) as 1.

Mathematical comments - 1. The methods are based on rule of 3 (Trairāśika) or ratio and proportion.

(a) $\frac{\text{Adhīmāsa till iṣṭa dina}}{\text{Sauramāsa till iṣṭa dina}} = \frac{\text{Adhimāsa in a kalpa}}{\text{Sauramāsa in a kalpa}}$

(b) $\frac{\text{kśaya tithi till iṣṭa tithi}}{\text{gata tithi (elapsed tithi)}} = \frac{\text{kśaya tithi in a kalpa}}{\text{Total tithi in a kalpa}}$

Tithi is a cāndra dina.

2. Ratio between Cāndra and saura māsa, tithis; Saura māsa + adhimāsa = Cāndra māsa

Within current year, they are almost equal.

Cāndra māsa x 30 = Cāndra tithi

Cāndra tithi is almost equal to ṣāvana dina in a current month

Cāndra tithi - kśaya tithi = sāvana dina. Sāvana dina is time from sunrise till next sun rise.

3. Kalpa had started on ravivāra at midnight at Laṅkā which is at equator on 0° longitude of India (passing through Ujjain).

Verses 9-13 : Errors in approximation of sauramāsa and Cāndra tithis (as explained in mathematical notes above sl 2)

Adhimāsa - While calculating adhimāsa only the quotient (result) is taken and remainder is left out. If remainder is almost equal to divisor, or if an adhimāsa has passed recently (in past 1 year), then 1 is added to know correct adhimāsa. However, if the remainder is almost zero or an adhimāsa is to come soon, then 1 is to be substracted.

Kśaya tithi - Similarly, if in calculation of kśaya tithi, remainder is almost equal to divisor

and within a week kśaya tithi has passed, then 1 is added to the result. (If pañcamī comes after trītīyā, then caturthī is kśaya tithi upto daśamī, if remainder is more than half the divisor, 1 is to be added to kśaya tithi. Thus 1 will be substracted from ahargaṇa. If remainder is almost zero, 1 is added to ahargaṇa. Correctness of ahargaṇa can be checked with week day.

Verse 14 - Māsādhipati - Divide ahargaṇa by 30, multiply the result by 2, add 1 and divide by 7. Remainder will indicate week days counted from ravivāra as 1. (soma 2, mañgala 3, budha 4, guru 5, Śukra 6, Śani 0) Ruler of this day will be māsādhipati.

Derivation - Each civil month is of 30 days (civil). Ruler of Ist day is māsādhipati. Ahargaṇa divided by 30 gives the number of civil months. In each month of 30 days; 4 weeks are completed (4x7=28 days) and 2 days remain. Hence for each month; 2 remainder days are taken. 1 is added because the first day of kalpa was ravivāra, Ist day.

Verse 15 - Divide ahargaṇa by 30. Remainder is the days gone (gata dina) in current month. Gata dina substracted from 30 gives bhogya (remaining days) dina of the month.

Derivation is obvious from earlier verse.

Verse 16 - Divide ahargaṇa by 360, Multiply result by 3 and add 1. Divide the result by 7. Remainder indicates week days starting from ravi as 1, which is the varṣapati. The remainder left after division of ahargaṇa is bhukta dina (past days) of current year.

Comments : (1) Māsāsidhipati and varṣādhipati are used only for calculating kāla bala in horoscopes, or in mundane astrology for forcasting events of the year. It has no importance in gaṇita jyotiṣa.

(2) Each civil year is of 360 civil days. Hence the quotient after division by 360 into ahargaṇa, is number of completed civil years. Remainder will be past days of the current year.

(3) In 1 year of 360 days, 360 ÷ 7 = 51 weeks and 3 extra days remain. Hence each completed year gives 3 days for count of week days, Next day will be first day of current year, hence 1 is added to find varṣādhipati.

Verse 17-20 - Lord of first day of māsa (month) is māsādhipati, and lord of first day of varṣa is varṣādhipati.

Śatānanda (author of Bhāsvati karaṇa) and his followers have different opinion. Lord of the day on which meṣa saṃkrānti falls is the lord of the year (varṣādhipati). To calculate the number of days in that year, the following rule has been given.

Calculate the daṇḍa, pala etc. from time of entry of ravi in meṣa saṃkrānti to the time of beginning of next day. Multiply it by 4 and keep it in 3 places. Divide the number at third place by 37 and add the result at second place. Divide at second place by 8 and add this result at first place. The result in daṇḍa etc. will indicate the number of days for which the varṣapāti will rule. For remaining days of the year, (360 - days of rule of varṣapati) lord of day next to saṅkrānti will rule.

According to this rule, no graha can rule for more than 271 days.

Mathematical symbol : let T = time in daṇḍa etc. from entry of ravi in meṣa to next sunrise.

$$\frac{4T}{37} = T' + R \text{ (remainder smaller than 37)}$$

$$\frac{4T + T'}{8} = T'' + R' \text{ (remainder smaller than 8)}$$

$$4T + T'' = D \text{ daṇḍa} + p \text{ pala etc.}$$

D is the number of days for which varṣapati will rule. Lord of the day after meṣa Saṅkrānti will rule for 360 - D days.

2. This appears to be a convention by Śatānanda, hence no derivation of the rule is given.

3. Maximum days of rule of varṣapati -

$$T < 60 \text{ daṇḍa}$$

$$T' < \frac{4T}{37} = \frac{4 \times 60}{37} = 6.5$$

$$T'' < \frac{4T + T'}{8} < \frac{4 \times 60 + 6.5}{8} < 31$$

$$D = 4T + T'' < 4 \times 60 + 30 = 271$$

Hence maximum days of rule from saṅkrānti day is 271 days.

Verses 21-22 - Formula for calculating graha for indicated day - Multiply ahargaṇa by kalpa bhagaṇa and divide by kalpa sāvana dina. Result will be lapsed bhagaṇa. Multiply remainder by 12 and again divide by kalpa sāvana dina. Again multiply by 30, 60 and 60 and divide by kalpa

sāvana dina to obtain aṁśa (degree) kalā (minutes), vikalā (seconds)

Explaination (1) By ratio and proportion

$$\frac{\text{Bhagaṇa till iṣṭa dina}}{\text{Bhagaṇa in a kalpa}} = \frac{\text{Ahargaṇa}}{\text{Sāvana dina in a kalpa}}$$

(2) Fraction of bhagaṇa are converted to rāśi etc. according to the scale -

1 Bhagaṇa = 12 rāśi, 1 rāśi = 30 aṁśa

1 aṃśa = 60 kalā, 1 Kalā = 60 Vikalā

(3) Ist rāśi is meṣa starting from 0° to 30° in krānti vṛtta (ecliptic). 0° starts from a fixed point marked by star groups in Indian astronomy. In western system, 0° is marked by point of intersection of equator with ecliptic plane, where motion of sun appears northwards. Difference between the two initial points is called ayanamśa. Axis of earth rotates one round in 25,762 years. In Indian system also calculation of day length, lagna etc. are done from this ayanamśa sāyana point.

(4) 12 rāsiś are 1. meṣa 2. vṛṣa, 3. mithuna, 4. karka, 5. siṃha, 6 kanyā, 7. tulā, 8. vṛścika, 9. dhanu, 10. makara 11. kumbha, and 14. mīna.

Verse 23 - (Quoted from Sūrya Siddhānta) - Same method is used for calculation of Śīghrocca, mandocca and pāta for iṣṭa dina. However, for pāta, the result will be deducted from 12 rāśi, because movement of pāta is in opposite direction of graha.

Note - When it is unnecessary to explain in more detail, the author has just referred to quotation from previous authorities - mainly sūrya sidhānta or siddhānta śiromaṇi. Sometimes quota-

tions have been given for comparison or contradiction on important points.

Verses 24-25 - Calculation of guru varṣa - calculate bhagaṇa of guru as before and add 3 (bhagaṇas) Multiply the sum by 12 and add their rāśis lapsed and add 2 again. Divide this sum by 60 and add 1 to the remainder which indicates guru varṣa counted from Prabhava etc.

Notes: (1) Secret of guru varṣa has been explained in chapter 21 of this book.

(2) Guru takes about 12 years to move around sun and about 1 year to cover 1 rāśi. Hence guru varṣa (time in a rāśi with medium speed) is similar to saura varṣa (time of 12 rāśis or complete bhagaṇa) Guru varṣa is called saṃvatsara of 361.02672 sāvana days which is smaller by 4.23203 days from saura varṣa and bigger by 1.02672 days from sāvana varṣa of 360 days.

(3) 60 years are needed to complete 5 revolutions of guru and 2 revolutions of Śani. Thus a cycle of 60 years has been adopted for saṃvatsara of guru. This is the active life period of a man.

(4) Guru varṣa are listed in verses 32-46. Vārāhamihira in Vṛhatsaṃhitā has assumed the beginning of saṃvatsara-cakra from 35th saṃvatsara Prabhava, instead of the first vijaya. However, the calculation method given here will start guru, samvatsava from the 13th 'vikrama', for start of first rāśi. Thus one complete round of 12 rāśis in 12 samvatsaras is considered complete at beginning of guru motion. This is only a convention. Same result could have been obtained by calculating rāśi

of madhyama guru and count the samvatsara from 13th.

(5) Symbolic formula

(a) Madhya guru = B bhagaṇa + R rāśi + A aṃśa etc.

(b) Total saṃvatsara = (B+3)x12+R+2 = S

(c) S/60 = s+r (remainder 0 to 59)

(d) r+1 is 1 to 60 samvatsara counted from prabhava.

(6) Samvatsara for Ist rāśi-completed R=0, B=O

n = (r+1) counted from 35th samvatsara

$$= r + 35 = \frac{S}{60} = \frac{(B + 3) \times 12 + R + 3}{60} + 35$$

$$= \frac{12\,B + R + 38 + 35}{60} = \frac{38 + 35}{60} = 13 \text{ remainder}$$

Verse 26 - Elapsed part of guru varṣa - (Omit bhagaṇa and rāśi from madhyama guru). Multiply aṃśa by 12 and add its 1/330 part which indicates elapsed days of saṃvatsara. (gata dina). (Deduct it from 361.02672 to find remaining days i.e. bhogya dina)

Explaination - 30° of rāśi - 361.027 days

$$1^\circ = \frac{361.027}{30} = 12 + \frac{1.027}{30} \text{ days}$$

$$1^\circ \; 12 \times \left(1 + \frac{1.027}{360}\right) = 12 \times \left(1 + \frac{1}{330}\right) \text{ approx.}$$

Verse 27 - If in a Cāndra varṣa, madhyama guru does not move to different rāśi, it is called adhivatsara. (Guru varśa is 7 days bigger then Cāndra varṣa and it may not complete 1 rāśi in that period.

Verse 28 : If with sphuṭa gati guru crosses two rāśis in a saura varṭa, then it is called lupta varṣa (saṃvatsara) (Normally guru will touch 2 rāśis every saura varṣa which is only 4 days bigger) unless both years start almost at sometime within 4 days gap. However, if its true motion is faster, and years start almost same time, it may touch the third rāśi also at end of saura varṁa)

Verse 29 - If in a saura varṣa, guru in its sphuṭa motion goes to next rāśi at higher speed (aticāra), and does not return to the same rāśi, that year is called mahācāra kāla. This year is as bad and inauspicious as a lupta saṃvatsara. (In this year also sphuṭa motion is faster than madhyama gati, not compensated by reverse motion. But guru may not cross into 3rd rāśi, if its saṃvatsara does not start with saura varṣa).

Verses 30-31 : 60 Bārhaspatya varṣa contain 12 Bārhaspatya yuga (of 5 years each).

Divide current number of bārhaspatya years by 5, add 1 to the result. Sum is guru yuga starting from Acyuta etc. Within the yuga, the years are named according to remainder as 'saṃ', pari, idā, 'anu' and 'id' vatsaras. Their adhipatis are agni, sūrya, candra, brahmā and Śiva respectively.

Comments : This classification of vatsaras was done in vedāñga jyotiṁa. In one yuga of 19 years, there were five types of years. The years starting from Ist to 6th lunar tithi was called saṃvatsara. Years starting (solar) from next block of 6 candra tithis were called pari, idā, anu and id vatsaras respectively. In a yuga of 19 years, there were 5 years of samvatsara type. Subsequently in yajur

jyotiṣa, a yuga was of 5 years, each of the 5 vatsaras occuring once. Same names have been adopted for bārhaspatya yugas also.

Verses 32-46 : Names of bārhaspatya yugas, varṣa and good or bad years -

Yuga (adhipatis)	years	Śubha(s) or Aśubha (A)
1. Viṣṇu (Viṣṇu)		
	1. Prabhava	
	2. Vibhava	
	3. Śukla	all Śubha
	4. Pramada	
	5. Prajāpati	
2. Bārhaspatya	6. Aṅgirā	S
(Bṛhaspati)	7. Śrīmukha	S
(First yuga	8. Bhānu	A
according	9. Yuvā	S
to our method	10. Dhātā	A
of calculation)		
3. Śākra	11. Īśvara	S
(Śākra)	12. Bahudhānya	S
	13. Pramada	A
	14. Vikrama	A
	15. Vṛṣa	S

(Guru will cross vṛṣa rāśi, when vṛṣa saṁvatsara will start).

Yuga (adhipatis)	years	Śubha(s) or Aśubha (A)
4. Pāvakīya	16. Citrabhānu	
(vahni)	17. Subhānu	
	18. Tāraṇa	all aśubha
	19. Pārthiva	
	20. Vyaya	
5. Tvāṣṭra	21. Sarvajit	S
(Tvaṣṭā)	22. Sarvadhārī	S

	23. Virodhī	A
	24. Vikṛti	A
	25. Khara	A
6. Ahirbudhnya	26. Nandana	S
(Ahirbudhnya)	27. Vijaya	S
	28. Jaya	S
	29. Manmatha	A
	30. Durmukha	A
7. Paitṛka	31. Hemalambī	S
(Pitara)	32. Vilambī	S
	33. Vikārī	A
	34. Śārvarī	A
	35. Plava	A
8. Vaiśva	36. Śokakṛta	S
(Viśvedeva)	37. Śubhakṛta	S
	38. Krodhī	A
	39. Viśvāvasu	A
	40. Parāvasu	A
9. Cāndra	41. Plavaṅga	A
(Niśāpati)	42. Kīlaka	A
	43. Saumya	S
	44. Sādhāraṇa	S
	45. Virodha kṛta	A
10. Aindrānala	46. Paridhāvī	S
(Indra and	47. Pramāthi	S
Agni)	48. Ānanda	S
	49. Rākśasa	A
	50. Anala	A
11. Āśvina	51. Kapila	A
(Āśvinī	52. Kāla	A
kumāra)	53. Siddhārtha	S
	54. Raudra	A
	55. Durmati	A
12. Bhāgya	56. Dundubhi	A
(Bhaga)	57. Rudhirodgārī	A
	58. Raktākśa	A

59. Krodhana A
60. Kśaya A

Verse 47 - Sūrya and Candra complete their full bhagaṇas in a mahāyuga or in a pādayuga. Hence their madhyamāna can be calculated even from ahargaṇa for mahāyuga or for any pādayuga also.

Verse 48 - Another short method of finding ahargaṇa is described below. It is not a fault for being a repetition, as great poets like Śri Harṣa also have adopted such practice.

Verse 49 : Multiply years since beginning of creation by 12 and add completed months from caitra śukla pratipadā. Keep it in two places. At one place multiply it by 1,00,00,000 and divide by 32,53,55,104. Add the quotient to result in second place. Multiply the result by 30 and add complete days passed after amāvāsyā. Keep it again at two places. At one place multiply it by 1,00,00,00,000 and divide by 63,90,97,35,058. Deduct quotient from quantity in second place. Result will be ahargaṇa from beginning of creation counted from midnight of Laṅkā.

Derivation of Formula

Saura varṣa x 12 = saura māsa

Completed Cāndra māsa from caitra pratipadā is assumed equal to saura māsa. This approximation does not affect the result as the remainders found in calculation of adhimāsa or kśayatithi are not used.

Total saura māsa x 30 = saura dina.

Cāndra tithi after amāvāsyā are similarly assumed equal to saura dina.

No of adhimāsa

$$= \text{No. of sauramāsa (s)} \times \frac{\text{Adhimāsa in a kalpa}}{\text{Saura māsa in a kalpa}}$$

$$= \frac{1,59,33,36,000}{51,84,00,00,000} = S \times \frac{1,00,00,000}{32,53,55,104} \text{ approx.}$$

This is added to sauramāsa to get cāndra māsa.

cāndra māsa x 30 = cāndra tithi

Kśaya tithis till iṣṭa day

$$= \frac{\text{No. of sauradina till iṣṭa day (D)}}{} \times \frac{\text{Kśaya tithi in kalpa}}{\text{Sauradina in kalpa}}$$

$$= D \frac{25,08,22,52,000}{15,55,20,00,00,000} = D \times \frac{1,00,00,00,000}{63,90,97,35,058} \text{ approx.}$$

We keep the significant digits same, so the approximation is sufficient for knowing integral numbers of adhimāsa or kśaya tithi.

Verse 50 : For calculating ahargaṇas from kali beginning, the same procedure will be followed. However, 4 zeros from the multipliers will be removed and 4 last digits of divisions (5104 and 5058) also will be taken out. Kaliyuga started on śukravāra; so days will be counted from friday.

Note : Kaliyuga = 1/10 yuga 1/10,000 kalpa. Hence 4 less no. of digit are required for approximation. Thus multiphers and divisors each are divided by 10,000.

Verse 51 - Kalpa bhagaṇa is multiplied by 1811 and divided by 4000 to get bhagaṇa at the end of dvāpara. If the madhyama graha calculated from kaliyuga first day to iṣṭa day is added, madhya graha from beginning of kalpa is obtained.

Derivation : Total yugas in a kalpa = 1,000

Total yugas upto dvāpara ėnd

6 manus x 71 = 426 yuga

7 sandhyā x satyayuga = $\frac{7 \times 4}{10} = \frac{14}{5}$ yuga

Satya + Treta + dvāpara = $\frac{4 + 3 + 2}{10} = \frac{9}{10}$ yuga

Time in creation = $\frac{79}{20}$ yuga (to be deducted)

Hence total yuga upto dvāpara end is

$$426 + 27 + \frac{14}{5} + \frac{9}{10} - \frac{79}{20} = 453 - \frac{1}{4} = \frac{1811}{4}$$

$$\frac{\text{Bhagaṇa at dvāpara end}}{\text{Kalpa bhagaṇa}} = \frac{\text{Yuga at dvāpara end}}{\text{Yuga in a a kalpa}} = \frac{1811}{4000}$$

Verses 52-55 : Position of graha at kali beginning (midnight of 17/18 February 3102 B.C. at Laṅkā) are given below in viliptā (seconds).

Maṅgala	12,41,568	Candra mandocca	4,34,160
Budha śīghra	1,13,724	maṅgala mandocca	4,56,840
guru	82,620	Budha mandocca	8,13,240
Sukra śīghra	1,49,040	guru mandocca	6,01,020
Śani	11,91,024	Śukra mandocca	2,35,548
Sūrya mandocca	2,83,176	Śani mandocca	8,97,480
Candra pāta	7,14,788	guru pāta	2,55,960
Maṅgala pāta	1,04,328	Śukra pātā	1,96,020
Budha pāta	1,06,271	Śani pāta	3,25,620

At the time of writing Siddhānta Darpaṇa, kali year 4970 end has been taken as reference year (karaṇābda). Deduct this number from the number of years passed since kali. Add 12 zeros to the right and divide by 2,73,77,85,151. The result will be gata dina from somavāra. Ahargaṇa will be from end day of sphuṭa meṣa saṅkrānti (year 1869 A.D.).

Deduction : This is calculation of sāvana dina in a solar year.

In a kalpa of 4,32,00,00,000 solar years, no. of sāvana dina is 15,77,91,78,28,000.

So, sāvana dina in iṣṭa year (D)

$$= \frac{15,77,91,78,28,000}{4,32,00,00,000} \times \text{no of years (y)}$$

$$\text{or } D = y \times \frac{15,77,91,78,28}{4,32,00,00} = \frac{10^{12}}{2,73,77,85,151}$$

First day of karaṇābda was monday. This will be ahargaṇa till completion of year on meśa saṅkrānti of madhyama sūrya.

Verse 56 - Normally madhyama sūrya enters meṣa, 3 days after entry of sphuṭa sūrya. So this third day after sphuṭa meṣa saṅkramaṇa, 1 ahargaṇa or main day of pañcāṅga is taken. Therefore, madhyama graha is to be calculated for previous day of madhya meṣa saṅkrānti or on 2nd day of entry of sphuṭa sūrya in meṣa. Then difference of grahagati for 1 day is to be added for madhyama graha of iṣṭa dina.

Verse 57 - There are different practices in different countries. Some pañcāṅgas take the entry of sphuṭa sūrya in meṣa. Many pañcāṅgas take caitra śukla pratipadā as Ist day. After madhyama saura varṣa end, karaṇābda (4970 kali or 12-4- 1869 A.D.) started. Author has given madhyamānas of dhruva (rāśi at the beginning of year), ucca, pāta etc. That day was somavāra (monday) and spaṣṭa sūrya had just entered meṣa at sunrise.

Verse 58 - Now madhyama dhruva (mean constants) for graha, mandocca, śīghrocca, pāta etc. are stated for somavāra day before karaṇābda at

time of sunrise at lankā (0° meridian through ujjain at equator)

Verse 59-69 - Table of Karaṇābda dhruva - (in rāśi / aṃśa / kalā / vikalā / parā)

(For 12-4-1869, Laṅkā sun rise)

Ravi	11/28/15/20/46
Candra	0/3/20/29/53
Maṅgala	5/1/24/17/25
Budha Śīghrocca	10/18/14/9/2
Guru	0/3/45/1/21
Śukra Śīghrocca	11/13/41/42/12
Śani	7/18/12/17/24
Ravi mandocca	2/18/47/54/0
Candra mandocca	10/22/34/59/4
Maṅgala mandocca	4/7/1/42/13
Budha mandocca	7/16/4/10/16
Guru mandocca	5/17/17/0/15
Śukra mandocca	2/5/39/38/29
Śani mandocca	8/9/19/44/10

Pāta dhruva of candra are corrected for reverse movement (bhacakra Śuddhi is substraction from 12 rāśis)

Canḍra pāta (Rāhu)	3/21/19/18/28
Maṅgala pāta	0/28/51/23/4
Budha pāta	0/29/17/28/58
Guru pāta	2/11/3/15/59
Śukra pāta	1/24/3/31/0
Śani pāta	3/0/13/27/24
Ketu pāta	9/21/19/18/28

Verse 70 : The dhruva above have been calculated according to proportion of kalpa

bhagaṇa. Candra pāta is called Rāhu, 6 rāśi or 180^0 away from that is ketu pāta.

Verse 71 : Method to calculate mandocca and pāta for past days has already been described. Mandocca and pāta for a particular year can be calculated by this method. Multiply iṣṭa varṣa by kalpa bhagaṇa and divide by 2,00,000 which will tell the position in liptā etc.

$$\text{Derivation} - \frac{\text{Iṣṭa varsa}}{\text{Kalpa varṣa}} = \frac{\text{Iṣṭa bhagaṇa}}{\text{Kalpa bhagaṇa}}$$

$$\text{or Iṣṭa bhagaṇa} = \text{Iṣṭa varṣa} \times \frac{\text{Kalpa bhagaṇa}}{\text{Kalpa varṣa}}$$

$$= \text{Iṣṭa varṣa} \times \frac{\text{Kalpa bhagaṇa} \times 360 \times 60 \text{ liptā}}{4{,}32{,}00{,}00{,}000}$$

$$= \text{Iṣṭa varṣa} \times \frac{\text{Kalpa bhagaṇa}}{2{,}00{,}000} \text{ liptā}$$

Verse 72 : Add this result to karaṇābda dhruva (deduct from pāta) to get iṣṭa graha, ucca, pāta etc. Alternatively, this can be calculated from annual motion (hāra) also.

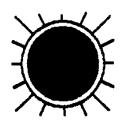
Verse 73-74 - Hāra (annual motion) in liptā is obtained by dividing kalpa bhagaṛa by 2,00,000. Multiply elapsed years after karaṇābda (gata varṣa) and add to dhruva to get ucca, graha etc.

Verse 75 - Table of pāta hāra -

Ravi mandocca hāra 599	Guru mandocca hāra 248
mangala mandocca hāra 645	Śukra mandocca hārā 359
Budha mandocca hāra 488	Śani mandocca hāra 2857
Mañgala pāta hāra 671	Budha pāta hāra 362
Guru pāta hāra 1818	Śukra pāta hāra 212
Śani pāta hāra 367	

Verse 76 - (Normally all astronomers assume that mandocca and śīghrocca move from west to

east). Author says mandocca of maṅgala, budha and śani and śīghrocca of Budha moves in both directions. This will be discussed while calculating true motion (graha sphuṭa)

Verses 77-78 : While praying to lord Jagannātha in end, author states position of nīlācala (Purī temple). It is 284 yojana north of equator on sea coast and 184 yojana east from Indian 0° longitude (Ujjain).

Chapter - 4

CALCULATION AT DIFFERENT PLACES

Scope - In chapter 3, madhya graha etc were calculated for Laṅkā. In this chapter, calculations will be done for any place on earth.

Mathematical Notes and definitions -

(1) Trigonometrical ratios-

$\angle ACB = \theta$, $\angle ABC$ is a right angle

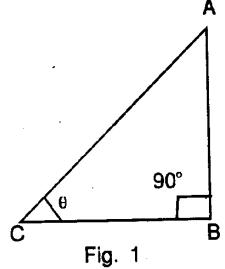
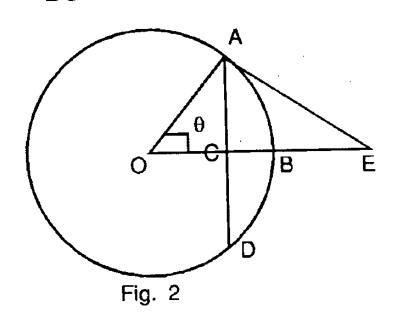
Then the following ratios depend only on the value of angle θ, and not on the lengths of the sides of triangle. By definition these ratios are -

Fig. 1

$$\text{Sin } \theta = \frac{AB}{AC} \qquad \text{Cot } \theta = \frac{1}{\tan \theta}$$

$$\text{Cos } \theta = \frac{BC}{AC} \qquad \text{Sec } \theta = \frac{1}{\cos \theta}$$

$$\text{Tan } \theta = \frac{AB}{BC} \qquad \text{Cosec } \theta = \frac{1}{\sin \theta}$$

Fig. 2

(2) **Indian Terms** - To avoid decimals, a circle of circumferance 21,600 units, i.e. radius of 3438 units is taken. One unit of circumference is equal to 1 kalā, then 21,600 kalā = 360° = 1 revolution.

We draw OA and OB, two radii

such that $\angle AOB = \theta$

Jyā of $\angle\theta$ is AC = R sin θ

or sin θ x 3438 kalā

AC is half of the chord AD which is like string of bow shaped arc ABD. Hence its name is Jyārddha or Jyā in short.

OC is koṭijyā =R Cos θ = 3438 x Cos θ

Tangent on A, meets base OB at E.

AE/OA = tan θ or AE = OA. tan θ = R tan θ

Hence this ratio is called tangent or tan in short. In sanskṛta it is called sparśa jyā. OE pierces like arrow, hence called chedjyā. OE = OA sec θ = R sec θ (sec is short of secant), Complement of angle θ i.e. $90°-\theta$ is called koṭi of the angle. Thus koṭi jyā = jyā of koṭi,

koṭi sparśa jyā = sparśa of koṭi

and Koṭi chedajyā = chedajyā of koṭi

In sanskṛt another ratio is defined, called utkrama jyā which is CB = R (1-cos θ).

(3) Ratio of circumference to diameter is fixed and is called π (a greek letter, pronounced as 'pāi') in modern mathematics. It is a transcendental number which cannot be expressed by any exact number. It can be expressed as non-recurring non-terminating decimal number to any desired

approximation. Values upto 1,00,000 decimal places have been published. Calculation was on computer by the formula

$$\pi = 24 \tan^{-1} \frac{1}{8} + 8 \tan^{-1} \frac{1}{57} + 4 \tan^{-1} \frac{1}{238}$$

$\tan^{-1}A$ is an angle θ such that θ = tan A. It can be expressed as an infinite convergant series when A is smaller than 1.

22/7 and 355/113 are rough practical approximations of π correct upto 2 and 6 places of decimal respectively. If paridhi is expressed in kalā, radius is 3437 3/4 kalā approximately, which is same as 1 radian angle. (1 radian is an angle made by arc equal to radius)

Mādhāva of Saṅgamagrāma (kerala) in 13th century used infinite series to calculate value of π up to 30 places and sine table upto 9 places. Value of π up to 30 places have been expressed in a verse by him (read with kaṭapayādi notation) -

गोपी भाग्य मधुव्रात श्रृंगिषो दधि संधिगाः

खल जीवित खातावा गल हात रसन्धराः ।

Accordingly,

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

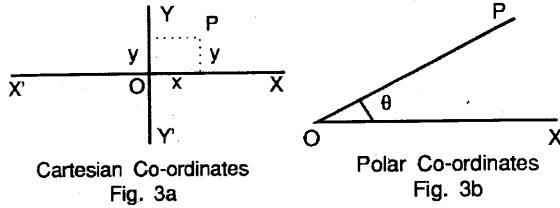
= 3.14,15,92,65,35,89,75,43,23,84,52,64,33,83,279 ---

(4) **Yojana** - Yojana is a measure of length as explained in the first chapter. Siddhānta darpaṇa takes yojana of 1600 hasta = 24,000 feet or 7.3152 kms approx. (if 1 hasta is taken as 18"). It takes diameter of earth as 1600 yojana then it is about 4.94 miles approximately (hand will be about 19.6").

(5) Longtiude, Latitude and sphuṭa paridhi - Study of sides and angles on a sphere is subject of spherical Trigonometry. It is called gola pāda in jyotiṣa.

To know position of a point in space by measuring its angle or distance from fixed point and lines is the basis of coordinate geometry (or cartesian geometry in the name of Rene de-Cartes of France, the originator). In a plane, two systems are used to indicate location of a point.

Cartesian Co-ordinates
Fig. 3a

Polar Co-ordinates
Fig. 3b

In both systems, O is the fixed point called origin and a line through it OX is called X axis. In cartesian coordinates, another line OY perpendicular to OX (in counter clock wise direction) is called Y axis. In cartesian coordinate location of a point P is indicated by its distance x from θ along axis (x coordinate) and distance y in direction of y axis (y coordinates). Distance in the direction OX′ and OY′ are negative. (Figure 3 a).

In polar coordinates, location of a point P is indicated by its distance r (always positive) from origin O and the angle θ made by OP with OX in counter clockwise direction. (r,θ) indicate position of any point in space (Figure 3b)

Conversion from one system to other is not difficult.

$r^2 = x^2 + y^2$ $\quad\quad$ $x = r \cos \theta$

$\theta = \tan^{-1} y/x$ $\quad\quad$ $y = r \sin \theta$

For example, if Bhubaneswar be origin, then location of Puri can be indicated in cartesian coordinates as

40 kms south (x coordinate)

35 kms east (y coordinate)

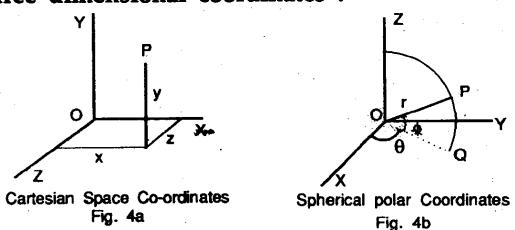
In polar coordinates - 53 kms away (*r*) in direction of 40° (θ) from south towards east.

In a plane, two quantities called coordinates are needed to locate a point. In space, 3 quantities are needed - so it is called 3 dimensional space. In theory of relativity, time is considered fourth dimension. An event in world is indicated by 3 space and 1 time coordinates. Hence world is called 4 - dimensional space time continum.

For example, a hill top in Puri can be specified by its height from mean sea level, in addition to two coordinates of plane.

Three dimensional coordinates :

Cartesian Space Co-ordinates
Fig. 4a

Spherical polar Coordinates
Fig. 4b

Cartesian space coordinates are measured along mutually perpendicular X,Y,Z axis. If a right hand screw is rotated from X direction to Y direction, it will move in Z direction. The distances of any point P from origin O along the three axis are called (x,y,z) coordinates.

In spherical polar coordinates, distance OP of P from origin is r coordinate. Angle θ between plane of z axis and OP with X axis is second coordinate. In the plane, elevation of OP from XY plane (with line OQ) is called ϕ. θ takes values from 0 to 2 π or 360°. ϕ takes values from - 90° to + 90° or can take any value. This system is more useful for spherical geometry and astronomy.

Conversion formula -

$r \sin \Phi = Z, \; r \cos \Phi \cos \theta = x,$

$r \cos \Phi \sin \theta = y$

In astronomy, only two angle coordinates are used. For places on earth, the distance from centre is fixed as radius of earth (r coordinate). OZ is line from centre to north pole. Angle θ is measured from prime meridian (great circle or plane passing through north pole and Greenwich (London)). In India, prime maridian was assumed through Ujjain as a reference. Φ is the angle with equator plane (XY plane). In popular terms θ Coordinate is called longitude (- 180° to + 180° and Φ coordinates is called latitude (- 90)° (south) to + 90° (north). Positive direction of longitude is called east, and negative direction west).

In astronomy, a second frame of reference is also used. This is fixed with refrence to stars which don't move. Planet's movement is observed with reference to stars. Zodiac or rāśi vṛtta is path of apparent motion of stars in which coordinates θ is measured from 0° to 360°. Deviation from this plane is called vikśapa or Śara. (-90° to +90°).

For calculation of eclipse etc, frequently we need to convert the figures from equatorial coordinates to zodiac coordintes. This is called dṛk Karma.

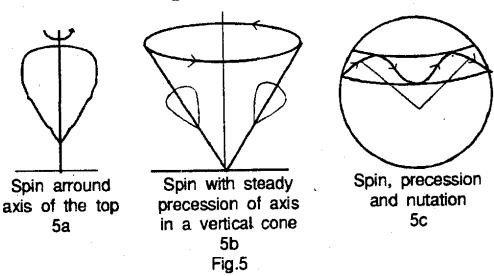
Sphuṭa paridhi of earth, at any point is circumference of circle on earth's surface parallel to equator (latitude) circle or simply called a parallel of particular degree.

(6) Motion of a top and earth's motion

Spin arround axis of the top 5a

Spin with steady precession of axis in a vertical cone 5b

Spin, precession and nutation 5c

Fig.5

A top rotating fast along its axis stands vertical on a rough surface due to gyroscopic stability. Its lower end is fixed due to friction with earth and it moves away from vertical position and falls due to gravity in the end.

Spin (figure 5a) - Rotation of a top about its axis is called spin. When top is rotating very fast, its axis is vertical and its appears stationary.

Precession (fig 5b) - Precession is conical motion of the axis of top. Upper point of the axis makes a circle about the vertical direction.

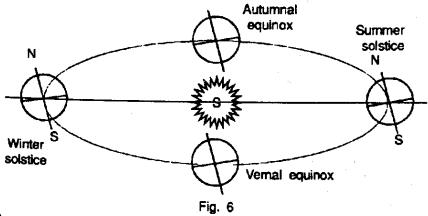
Nutation - When motion of top becomes slower, its axis falls further away from vertical and rises again atternatively. In steady precession, upper point of the top makes a horizontal circle on a sphere. In nutation it moves in a wave like

path between two horizontal circles on the sphere as in fig. 5c.

(7) **Rotation of earth around its axis** - Motion of earth around its axis is completed in one day and causes day and night. Due to that the sphere of stars in sky appears to make a daily rotation from east to west. This is spin motion of a top.

Axis of earth is inclined at angle of about 23-1/2° from perpendicular to the plane of ecliptic (i.e. plane of earth's orbit round the sun). Due to that the sun appears either north or south of the equator. During summer season in north hemisphere, it will be perpandicular to earth's surface at noon time at some place between equator and 23-1/2° north (Tropic of cancer)

When the plane containing vertical to ecliptic and earth's axis contains sun, inclination of sun towards north or south is maximum. These points opposite to each other are called summer and winter solstice. In summer solstice, axis is directly inclined towards sun, and sun is perpendicular to tropic of cancer (23 1/2°)

Fig. 6

At two points on orbit, 90° away from place of maximum inclination, the axis of earth is inclined side ways and not towards sun. Then sun rays are perpendicular on equator (i.e. on plane con-

taining ecliptic and arth's axis). On such points, day and night are equal. 'Nakta' means night in sanskrit, it is called noct in greek. Equinox means equal day and night. On one of equinox points, sun goes from south to north hemisphere. This is called vernal equinox. The other point is called autumnal equinox. Northward motion of sun is called uttara - ayana and southward motion is dakṣiṇāyana. Both ayanas, make one hāyana, a complete year.

Precession of axis - At present, earth's axis towards north is directed to pole star (Dhruva Tārā). So pole star appears to be fixed. Axis is moving like precession of a top in conical motion due to two reasons - (1) Earth is not spherical, it has bulge at equator due to centrifugal force of rotation (2) Orbit of moon is inclined to earth's orbit at about 5° angle which creates unequal pull at different ends of bulge. To some extent, inclination of other planetary orbits also affects the axis.

Practical effect of precession of axis is that, points of equinoxes move slowly westwards. If solar year is counted by motion relative to fixed stars, start of seasons shifts slowly. 1° change of equinox, i.e. 1 day change of season occurs in about 72 years. One month change is in about two thousand years.

In western astronomy, solar year is counted from equinox to equinox. Position of vernal equinox is taken as 0° meṣa. Difference between vernal equinox, and static meṣa 0° of Indian astronomy is called Ayanāṁśa. For determining day length, rising period of rāśis etc, position of sun from

equinox position is taken. From that position; spherical triangle is completed. Since equinox moves backward (to west), ayanāṃśa is added to sun position. It is called sāyana sun or any other planet.

REFERENCES

1. For trigonometry, any school text book can be referred like S.L. Loney's Trigonometry.
2. Cartesian geometry of two dimensiouns can be found in any college text book, e.g by Loney or by Śānti Nārāyaṇa. Geometry of 3 dimensions can be found in book by R.J.T. Bell.
3. Results of spherical trigenometry can be found in text books by Todhunter or by Gorakh Prasad.
4. Transformation of axis can be found in books of classical mechanics or foundations of vector/tensor analysis. Differential geometry of Weatherburn or by Shanti Narayan can be referred for space curves, surfaces and polar coordinates.
5. Polar coordinates/transformation of axis are explained in classical mechanics also. M.Sc/Hons level text books also discuss motion of top. The following books may be referred.

   Classical Mechanics - by Goldstein.

Principles of Mechanics - by Synge & Griffith

Mechanics - by Simon

Earth's top tike motion has been discussed in detail in motion of top (4 vols) by W. Sommerfield & Felix Klein

Translation of the text (Chapter 4)

Verse 1 - I (author) will describe in short the various measurements of earth. In second half of the book, these will be discussed in detail.

Verse 2 - Average diameter of earth (madhyavyāsa) is 1600 yojanas. Multiply this by 10,800 and divide by 3,438. You get the paridhi (circumference) described in 3rd verse.

Verse 3-4 - Paridhi at centre (equator) is 5,026/10 yojana. Jyā of 90° is taken as 3438 kalā. Hence, sphuṭa bhū-paridhi is obtained by multiplying, madhya paridhi by lamba jyā of the place and dividing by 3438. Otherwise, this madhya paridhi can be multiplied by 12 and divided by viṣuva karṇa.

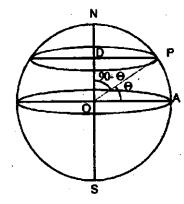



Fig. 7

Derivation - (1) NS is line joining north and south pole. O is centre. The circle perpendicular to NS line is called sphuṭa bhū paridhi. Largest circle passes through centre 0, at point A and is called equator. Sphuṭa paridhi at point P is to be calculated.

OA = OP = radius R of earth

Paridhi at centre is $2 \pi R = C$

Latitude of place P is $\angle POA = \theta$ (Akśamśa)

Lamba amśa = $90° - \theta = \angle POD = \Phi$

For circle of sphuṭa paridhi at P, $r = DP = OP \sin \Phi$

or $r = R \sin \Phi$

Circumference $= 2 \pi r = 2 \pi R \sin \Phi$

$$= C \sin \Phi = C. \frac{R \sin \Phi}{R} = C \times \frac{\text{Lamba jyā}}{3438}$$

(2) Second method is based on measurement of palabhā explained in Tripraśnādhikāra. On Viṣuva saṃkrānti, sun rays are perpendicular on equator, i.e. paralled to OA. At point P, a pole PR is kept vretical of 12 unit lengths. Its shadow PC on horizontal surface is palabhā and RC is Pala Karṇa or viṣuva karṇa.

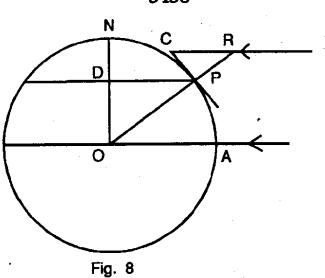



Fig. 8

In Fig 8, OPR is straight line, RC | | OA or RPC and ODP are similar.

$$\text{Hence } \frac{RC}{OP} = \frac{PR}{PD} \text{ or } \frac{\text{Viṣuva Karṇa}}{R} = \frac{12}{r}$$

$$\text{or } r = \frac{12R}{\text{Viṣuva Karṇa}} \text{ hence the result.}$$

Verse 5 - Laṅkā, Rohitaka, Avanti, Kurukśetra etc. are on the prime meridian line (Pradhāna mādhyandina rekhā) which passes through both merus.

Note (1) Rekhā is a straight line in a plane but it is arc of a great circle in a sphere (the circle passing through centre of sphere, which is greatest). Like straight line of a plane, it is the shortest distance between two points, and doesn't change the direction.

(2). This verse means same as verse 62 in madhyamādhikāra of Sūrya-siddhānta and represents the convention of treating the longitude through Ujjain as reference line (0° longitude). At present, the meridian passing through Greenwich is 0° meridian.

(3) According to historical traditions, 'Polanaruā' (meaning Paulastya nagara) in present Śrī Laṅkā was the capital of Laṅkā. However, for astronomical purpose, Laṅkā is the imaginary point of intersection of longitude through Ujjain and equator (i.e. middle point of that line between south and north pole). Laṅkā is nearest land mass near the point; hence it is called Laṅkā (presumption)

(4) Location of original Kurukśetra is not known. If present Rohataka (a district headquarter in Hariyāṛā) is taken as Rohitaka, then it is 8 pala east from madhya rekhā. Hence, Bhāskarācārya has not indicated it on madhya rekhā. He says that this line touches regions like Kurukśetra etc.

Verses 6-9 - Deśāntara is the east west distance between two places with same akśāṃśa on sphuṭa bhū paridhi (local latitude circle perpendicular on polar axis or parallel to equator).

Multiply this deśāntara yojana by 60 and divide by spaṣṭa bhūparidhi. Alternately, multiply

by viṣuva karṇa in liptikās and divide by 60, 314. You will get deśāntara in daṇḍa etc.

All days, months and years start with mid-night at Laṅkā i.e. from midnight at places on mādhyandina rekhā. If a place is east from rekhā, add the deśāntara (ghaṭī) to get the midnight time at that place, from which day, months will start at that place. If the place is west from rekhā, deśāntara is to be deducted.

Derivation - (1) Earth rotates with uniform speed around its axis or in the direction of bhūparidhi. Complete rotation of bhūparidhi takes 60 daṇḍa or 1 day. Thus by ratio and proportion -

$$\frac{\text{Deśāntara in daṇḍa}}{\text{Deśāntara in yojana}} = \frac{\text{60 daṇḍa}}{\text{spaṣṭa bhūparidhi}}$$

or Deśāntara daṇḍa =

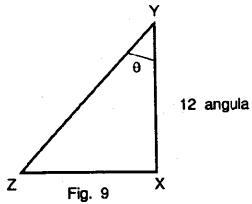
$$\frac{\text{Deśāntara yojana (east west distance)}}{\text{sphuṭa bhūparidhi}}$$

(2) Viṣuva Karṇa - Palabhā is length of the shadow of a vertical stick (cone or Śaṅku) at noon on a day when day and night are equal. Height of śaṅku is 12 aṅgula.

Viśuva karṇa or pala-karṇa is the length of hypotenus, i.e. distance from tip of 12 angula śaṅku to the tip of shadow.

Palabhā or pala karṇa gives a measure of the angle of latitude (akśāṃśa) as sun is vertically above equator on viṣuva day (when day and night are equal)

In Figure 9, X is a place on akśāṃśa $\theta°$. Angle of sun rays at mid day will be 0° at equator, so

Fig. 9

it will be $\theta°$ at latitude $\theta°$ (Derivation 2 after verse 4, Fig 8)

i.e. $\angle XYZ = \theta$ (akśāṃśa)

XY is Śaṅku of 12 aṅgula (units) of length. XZ is palabhā and YZ is palakarṇa.

Sphuṭa bhūparidhi = $2\pi r$ (r = sphuṭa Trijyā)

= $2\pi R \cos\theta$ (R = radius of earth)

$$= \text{Bhūparidhi} \times \frac{XY}{YZ}$$

$$= \frac{\text{Bhūparidhi} \times 12 \text{ aṅgula}}{\text{Palakarṇa aṅgula}}$$

$$\text{Deśāntara daṇḍa} = \frac{\text{deśāntara yojana} \times 60}{\text{sphuṭa bhūparidhi}}$$

$$= \frac{\text{deśāntāra yojana} \times 60}{\text{Bhūparidhi} \times 12} \times \text{palakarṇa}$$

$$= \frac{\text{deśāntara yojana} \times \text{palakarṇa in liptā}}{60314}$$

(As per verse 2, bhūparidhi x 12 = 5026/10 yojana x 12 = 60314 yojanas)

Verses 10-11 - Some astronomers opine that day starts everywhere from the sunrise at Laṅkā. Due to that confusion, the author decides that at any place the lord of vāra will be ruling from sunrise at that place for period of 60 daṇḍas.

Verse 12 : Bala (power) of yāma and yāmārdha is not connected to siddhānta (astronomy) it is

useful for phalita (astrology only). So it is not discussed here.

Verse 13-14 - Bhaskarācārya (and his followers) assumes start of all (motion of planets, day etc.) from sunrise at Laṅkā. Thus the ahargaṇa according to his theory is different from other theories. This separate ahargaṇa (of Bhāskara) doesn't give position of planets as they are actually seen, hence it is not followed in this book.

Note - Bhāskara ahargaṇa will give correct position of planets for sunrise at Laṅkā only. Since day length is different for different latitudes, sunrise will be at different times on same longitude also. But midnight will be at same time on the whole longitude, hence it gives correct result.

Verse 15 - Method of finding midnight position of planets at iṣṭa (desired place) - Multiply deśāntara kāla (in daṇḍa) of the place with dainika gati of graha and divide by 60. Add the result to the graha at Laṅkā at midnight if the place is west from Laṅkā. (Since earth rotates in east direction, midnight will be later in a place to the west and in the extra time, the graha will move further). Deduct, if the place is towards east.

Verse 16 - Alternately, difference in grahagati can be obtained by multiplying dainika gati with deśāntara yojana and dividing by sphuṭa bhūparidhi.

Note : Deśāntara ghaṭī of a western place is the time taken by earth to reach midnight position for that place. Alternate method follows from methods of findig deśāntara ghaṭī (vrse. 9).

Verse 17-21 - Old method of finding longitude. - calculate the time of pūrṇa (full) candra grahaṇa (lunar eclipse) at madhya rekhā (prime meridian through Laṅkā or Ujjain).

(Note - Exact time of Pūrṇa grahaṇa is the time of unmīlana (when moon starts emerging from shadow).

By observation, see the actual time of Pūrṇa grahaṇa at your place. The difference in time is deśāntara kāla.

If the place is west from Ujjain, then the time found by observation (dṛk-siddha or vedha) will be less than calculated time (i.e. eclipse will be at same time, but corresponding local time will come later at western place). For places east of Ujjain, observed time will be more.

Time difference can also be calculateed on basis of sparśa (when moon starts entering the shadow) or mokśa (when moon completely emerges from shadow).

To find deśāntara yojana, multiply it (deśāntara kāla) by sphuṭa paridhi and divide by 60 (already explained in verse 9).

To calculate graha at iṣṭa time, multiply the dainika gati of graha by iṣṭa kāla and divide by 60. Add the result to graha at midnight at the place.

Notes : (1) Time difference (in daṇḍa) from Laṅkā midnight is due to two components - (1) difference between midnight times at the place and at Laṅkā (2) Time lapsed after midnight of the place at desired time.

Dainika gati of graha is movement in 60 daṇḍa (1 day). Hence movement in iṣṭa daṇḍa is

$$\frac{\text{Dainika gati} \times \text{iṣṭa daṇḍa (kāla)}}{60}$$

components of iṣṭa kāla are added or substracted as explained before.

(2) Candra grahaṇa is due to covering of moon by shadow of earth, both of which are at one place. Thus there is no parallax and it is seen similar from all positions. But Sūrya grahaṇa is by obstruction of sun's vision by moon (at 1/400 of the distance). Their relative directions are seen different from different places, (called parallax), hence sūrya grahaṇa starts at different places at different times. Hence only candra grahaṇa can be used for comparison of midnight times.

(3) Terms of grahaṇa

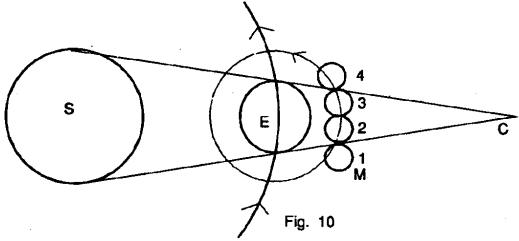



Fig. 10

In fig 10, S is sun, E is earth and M is orbit of moon. C is shadow cone of earth due to rays from sun. 1,2,3,4 are successive positions of moon.

1. position of moon touching the shadow - at sparśa kāla.

2. position of moon when it has just entered completely in shadow - Nimīlana or saṃmīlana (meaning closing of eyes) kāla

3. Position of moon about to emerge from shadow; unmīlana (opening of eyes) kāla
4. position of moon when it has just emerged completely from shadow - Mokśa kāla

Grahaṇa will be discussed more completely in chapters on candra and sūrya grahaṇa

(4) Other methods of finding longitude - Now very accurate watches are available and any event can be observed with telescope more accurately. In observing candra grahaṇa, there will be difference of 2-3 minutes in observation by different persons. Eclipse of satellites of jupiter occurs daily. It is observed through telescope and compared with time given in nautical almanc. This will give accurate longitude.

Alternatively, two watches are to be tallied with local times of places, whose longitudes are to be compared. They can be tallied with sunrise or preferably at midnight time. Then by telephone, the local time of the two places can be compared. The time difference will be deśāntara kāla. Now T.V. and radio announce Indian standard times (mean time at 82°30′ east of greenwich). Local mean time can be found by correcting local true time with time equation (fixed for particular days of solar year or sun position). From that time difference, difference with 82°30′ longitude can be known.

(5) Time can also be known accurately by movement of stars during night. This is particularly useful for sea journeys in a clear night. Since, method of finding longitude was known since remote past in India, long journey in sea was

possible. Due to difficulty in knowing time in absence of watches, this method could be known in western astronomy only in 1480 A.D. after which Cobumbus could undertake his journey, in 1492 in pursuit of sea route to India from Spain. Finding latitude is easy through palabhā, discussed in more detail in Tripraśnādhikāra.

Verses 22-24 : By above corrections for deśāntara kāla, we get the graha for nirakśodaya kāla (sunrise time at equator at same longitude). Due to difference in akśāṃśa (north south distance) from Laṅkā, cara saṃskāra is needed, because sunrise times are different for different places on same longitude due to akśāṃśa.

From sphuṭa ravi (sun) krānti (true inclination of sun from vertical in north south direction i.e. inclination from vertical at noon), find cara daṇḍa (time in daṇḍa by which day-half is longer than normal day half of 15 daṇḍa). Multiply it with dainika gati of graha and divide by 60. If sun (sāyana) is in six rāśi from tulā to mīna, add the result to the position of graha. If sāyana sun is in meṣa to kanyā, then deduct the result. For finding graha at the time of sun set, do the reverse process.

Notes (1) This part (chapter 1 to 4) is madhyamādhikāra, dealing with mean position of planets. Nothing has been so far discussed, as to how, true (sphuṭa) position of planets can be found. Sphuṭa krānti of sun can be found only at moon time by direct observation. By comparison with previous dayś krānti, it can be calculated for sunrise time (3/4 of the difference of 1 day krānti will be added to previous noon figure, to find krānti at sunrise).

2. Meṣa to kanyā - 1st six rāśis are in north hemisphere and other six are in south (sāyana rāśis to be more accurate). When sun is in southern hemisphere, days will be smaller in north hemisphere compared to night. Hence sunrise will be later and sunset earlier than equator (where day night are always equal) Thus graha will move for more time at sunrise compared to sunrise at equator, difference of motion will be added.

3. Cara is variation of day from 30 ghaṭikā, caradala is half of cara. In short cara is used for caradala which is directly calculated. Jyā of cara (angular difference in earth's rotation) is called cara jyā.

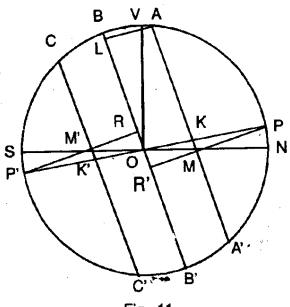



Fig. 11

4. Explaination of cara - (difference in day length) O is the place for which it is to be found out for how long, a graha will be above horizon. NOS is North south line (kśitija rekhā) POP′ is the north south line at equator (∠PON is equal to akśāṃśa of O).

NPVSP′ is yāmyottara vṛtta, i.e. the vertical circle in the plane of longitudinal circle (great circle passing through north pole and vertical at place O).

A planet in krānti vṛtta appears to move daily in a vertical circle at equator in east west direction. Its diameter BOB′ is perpendicular to north south line P′O P at equator. This circle is called ahorātra vṛtta (only diameter is seen in perpendicular plane). Corresponding to point O, the planet rises in the east goes upto B, highest point in sky (south from vertical in north hemisphere) and sets in west again at O. Motion from O to B′ and back to O are not visible as these are below the horizon. Both motions OBO or OB′O take 12 hours each.

CK′C′ is the diameter of ahorātra vṛtta (diurnal circle) of a planet in south hemisphere. At equator, it is visible for motion K′C K′ for half the day i.e. 12 hours. However, at place O, it rises only at point M′ and is not visible for period K′ to M′ (in 12 hours) which is called cara.

Time for K′C = 30 ghaṭī (12 hours)

For K′M′ in morning and M′K′ in evening, sun (or a planet) will not be visible above horizon.

Thus length of the day is 30-2 K′M′

A MA′ is diameter of ahorātra vṛtta of a planet in north hemisphere.

Krānti of planet corresponding to AA′ is AB (north) and corresponding to CC′ it is BC (south)

Carakāla is time corresponding to movement between KM or K′M′ (called kśitijyā or kujyā)

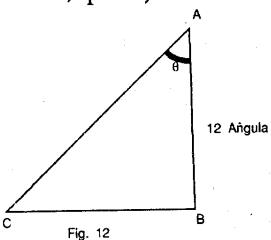
Radius of ahorātra vṛtta is called dyujyā ('Dyu' means light)

Carajyā of planet is projection of kujyā on viśuvavṛtta BOB′. It is OR′ for north krānti and OR for south krānti.

Angle made by carajyā (length of circumference) at the centre expressed in prāṇa is called caraprāṇa or carakhaṇḍa.

5. Methods of calculating carajyā

(Chapter 6-104, p-352)

Fig. 12

AB is cone at a place with akśāmśa $\theta°$. It is kept vertical on day of equinox at noon time. Since sun rays are perpendicular to equator or that day, it will make angle $\theta°$ with AB.

BC is shadow at that time (figs 12)

$\angle BAC = \theta$

Length of AB is 12 aṅgula as per convention. BC is palabhā.

$$\tan\theta = \frac{BC}{AB} = \frac{\text{Palabhā}}{12} \quad \text{-----(1)}$$

Now according to figure 11 in para(4), BA is north krānti. $\angle BOA$ is angle of krānti (angle not shown)

AL is krānti jyā ($AL \perp OB$)

$AL = OK$

Now $\angle KOM = \theta$ = akśāṁśa

$$\text{Tan}\,\theta = \frac{KM}{OK} = \frac{\text{Kśitijyā}}{\text{Krāntijyā}} \quad \text{.......(2)}$$

From (1) $\text{Tan}\,\theta = \dfrac{\text{Palabhā}}{12}$

$$\text{so, Kśitijyā} = \frac{\text{Krāntijyā} \times \text{Palabhā}}{12} \quad (3)$$

PKO and PMR′ (grand circles) are both perpendicular on AK and BO. Due to similarity of spherical triangles (as in plane triangles)

$$\frac{AK}{KM} = \frac{BO}{OR'}$$

$$\text{or carajyā } OR' = \frac{BO \times KM}{AK} = \frac{\text{Kśitijyā} \times \text{Trijyā}}{\text{dyujyā}}$$

(6) The difference in planet motion at sunrise is calculated by proportion of motion in carakhaṇḍa compared to dainika gati in 60 daṇḍa.

Verse 25 - The value of cara daṇḍa for a particular sphuṭa surya previous year will be same for the equal rāśi of madhyama sūrya this year (exactly same for equal sphuṭa sūrya). This approximate equality is used for checking the results obtained through palabhā. By taking this value of cara daṇḍa, there will be negligible error.

Verses 26-30 - Bhujaphala saṃskāra - Now, I tell about another saṃskāra (correction) in madhya graha based on nirakśa lagnamāna and ayanāṃśa etc. Mid-night calculated from madhya ravi is different from midnight of sphuṭa ravi. Difference between sphuṭa and madhyama ravi is called bhujaphala and correction for that is needed.

Add ayanaṃśa to madhyama ravi, find manda bhujaphala, multiply it by udayāsu (time of rising of rāśi in prāṇa) of the rāśi at equator (nirakśa) and divide by 1800. Multiply the result by dainika gati of graha and divide by asu of madhya ravi sāvana dina. The result in kalā etc is to be added or substracted from madhya graha for bhujaphala saṃskāra. (There are 21659 asus in a madhya sāvana dina).

For correction in śīghraphala, mandocca of candra or bhujāntara of rāhu, reverse is done. (positive bhujaphala is to be substracted or vice versa)

Notes (1) Manda bhujaphala is neither explained nor method of finding it has been described in madhyamādhikāra (chapters 1 to 4).

Manda bhujaphala is the correction to graha raśi due to its unequal speeds which is slowest at mandocca. (Since sphuṭa graha is closer to mandocca than madhya graha, it is termed as attraction of mandocca).

Real motion of earth E is in an ellipse around sun S at one of the focus. The farthest point E on far side of major axis is the slowest point called mandocca. (It is manda = slow and highest = Ucca.) E_2 is closest to sun called the nīca point. Middle points of the orbit E_1 and E_3 are not at right angle to direction of major axis but towards mandocca position (apparent attraction towards it).

Apparent elliptical motion of sun around earth

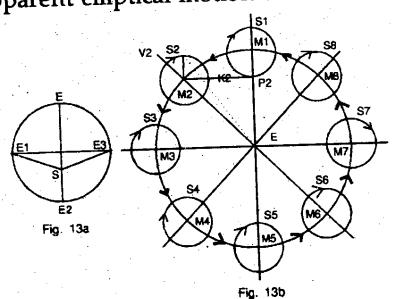



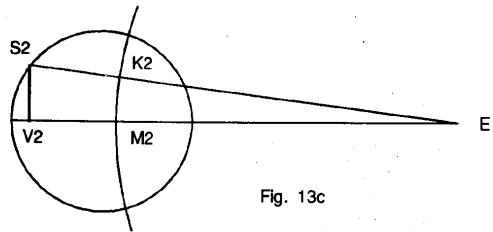
Fig. 13a

Fig. 13b

is explained by combination of two circular movements. Fig. 13(a) is real orbit of earth round sun

Fig 13b indicates apparent positions of sun calculated by combination of two circular motions. E is earth around which madhyama sūrya M is moving in a circle in anticlockwise direction. 8 positions are indicated as M_1, M_2 ---M_8. Sphuṭa graha S is rotating in a smaller circle (manda paridhi) in opposite direction. Both complete the rotation in equal time. Corresponding positions of sphuṭa graha are indicated by S_1, S_2 - - - S_8.

At position 2 for example $\angle S_2 M_2 V_2 = \angle M_1 E M_2$ as speeds of madhya graha and manda graha are equal. Apparent position K_2 on kakśā vṛtta is sphuṭa graha. $M_2 K_2$ is called manda phala. $S_2 V_2$ perpendicular on manda trjyā is called manda bhuja phala (fig 13 c) which is almost equal to mandaphala as mandaparidhi is very small compared to madhyaparidhi. M_2V_2 is koṭiphala.

Fig. 13c

Mandaphala and bhujaphala is negative in Ist semicircle after mandocca. (it is to be substracted from madhya graha). In 2nd semicircle it is positive.

Kakśā vṛtta is 360° or 21,600 kalā.

Manda paridhi is expressed in angle in proportion to length of kakśā vṛtta.

$$\text{Sin } \angle S_2 M_2 V_2 = \text{Sin } \angle M_1 E M_2$$

$$\text{or } \frac{S_2 V_2}{S_2 M_2} = \frac{M_2 P_2}{E M_2}$$

$$\text{Bhujaphala } S_2V_2 = M_2P_2 \times \frac{S_2M_2 \text{ (Manda Trijya)}}{EM_2 \text{ (kakśā Trijyā)}}$$

$$= \text{Bhuja jyā} \times \frac{\text{mandaparidhi (sphuṭa)}}{360}$$

$$\text{Koṭiphala} = \text{Koṭijyā} \times \frac{\text{mandaparidhi}}{360}$$

Mandaparidhi also changes slightly, because, earth is not at centre of orbit, but on one side at the focus.

(2) Udaya kāla of different rāśis is calculated in chapter 6-121. Due to oblique direction of rāśis with equator (24° or 23°27′ more accurately), the time taken by different rāśis to rise is different. As we move away from equator this inclination with local horizontal plane increases. Difference in rising time of rāśis becomes more. However, total time of rising of all rāśis will be same as Nākśatra dina for all places. The rising time of rāśis for Ist to 6th rāśis is same as that of 7th to 12th rāśis in reverse order. At equator, position of Ist to 3rd rāśi is same as 6th to 4th rāśi (symmetric for sāyana rāśi), hence their rising times are same. For difference in start of mid night at equator, only the rising times at equator are needed. A comparison of traditional rising times based on Sūrya siddhānta and modern values is given below-

Sāyana rāsi				Sūrya siddhānta Parama Krānti 24°			New observations Parama Krānti 23°27′		
				Asu	Pala	Minutes	Asu	Pala	Minutes
Meṣa	Kanyā	Tulā	Mīna	1670	278	111	1675	279	111.7
Vṛṣa	Siṁha	Vṛścika	Kumbha	1795	299	120	1794	299	119.6
Mithuna	Karka	Dhanu	Makara	1935	323	129	1931	322	128.7

At other places, udayāsu of rāśi is lessened by carāsu. It is added for rāśi 4 to 9.

(3) Since Udayāsu is calculated for nākśatra dina and dainika gati is calculated as per sāvana dina of 21659 asu.

$$\text{gati in 1 asu} = \frac{\text{dainika gati}}{21{,}659}$$

$$\text{gati in udayāsu} = \frac{\text{udayāsu} \times \text{dainika gati}}{21{,}659}$$

Hence correction for maṇḍa bhujaphala

$$= \frac{\text{maṇḍa bhujaphala} \times \text{gati in udayāsu}}{1800}$$

because udayāsu is for rise of 30° i.e. 1800 kalā.

Verse 31-32 - Alternate method for bhujāntara saṁskāra

$$\text{Bhujāntara} = \frac{\text{Ravi maṇḍa bhujaphala} \times \text{dainika gati}}{21{,}600}$$

= mandabhuja phala ÷ (21,600 ÷ dainika gati)

It will be added or substracted as before.

Note - In this formula, different rates of rising of rāśis and difference between nākśatra dina and sāvana dina are ignored.

Verse 33 : After, bhujāntara saṁskāra, I am telling the method of udayāntara saṁskāra which is due to difference between madhyama ravi in krānti vṛtta and imaginary madhya ravi in nāḍīvṛtta (in plane of equator).

Verses 34-37 - For this purpose (for udayāntara saṁskāra) make the madhyama ravi sāyana (add ayanāṃśa). Find the bhukta asu of that rāśi (part of udayāsu of rāśi in proportion to lapsed degrees in that rāśi). Add the udayāsu of previous rāśi starting from meṣa. Then calculate the kalā of sāyana ravi and substract from Ist result.

Multiply the difference by the dainika gati of graha (in liptā) and divide by 21,659 (as dainika gati is for sāvana dina of 21,659 asu) The result is udayāntara phala. Substract the result from madhya graha, if ravi is in sama or even pāda (2nd or 4th quadrant) and add if ravi is in viṣama pāda (Ist or 3rd quadrant - 0 to 90° or 180° to 270° from mandocca). For correction in pāta or ucca, do the reverse.

Notes : (1) Madhyama ravi + Ayanāṁśa = Sāyana madhyama ravi = S

Bhukta asu for S = rising times for rāśis from 0° to S

1 asu time = time for movement of 1 kalā at equator

Hence Bhukta asu of S = Its kalā at equator = E

Kalā of S = S′

Correction for observation in plane of equator

= E-S′ in kalā equivelent to asu time.

Difference in madhyama graha =

$$(E - S) \times \frac{\text{Dainika gati}}{21{,}659}$$

as 1 day is of 21659 asu (sāvana dina)

This difference is negative for Ist and 3rd quadrant i.e. Sāyana ravi is more in krānti vṛtta than in nāḍīvṛtta. At 90° and 270° they are equal, and no correction is needed.

(2) This is effect of transformation of coordinate axis from ecliptic to equator, because time is measured by movement along equator (asu is 1′ movement).

Verse 38-39 - The three saṃskāra (cara, bhujāntara and udayāntara) can be made to sphuṭa graha also instead of applying it to madhya graha. Then we will use sphuṭa graha instead of madhya graha in all places. Once the saṁskāra has been done to sphuṭa graha, it is not to be applied again to madhya, mandocca and śīghrocca because these results are used to calculate sphuṭa graha.

Note : The sanskāras are for difference in time measurements and not due to madhya or sphuṭa graha, hence correction to any value can be done. In short, correction time difference between madhya and sphuṭa is negligible.

Verses 40-41 - If we take asu arising out of mandaphala of ravi while making bhujāntara saṁskāra, then udayāntara karma is done from sāyana madhyama ravi.

When we take asu equal to kalā of mandaphala of ravi then udayāntara will be done from sāyana ravi before bhujāntara sanskāra, both are to be done separately.

Note : (1) In taking asu equal to mandaphala, it is already converted to value in equator plane; hence separate udayāntara saṁskāra is not necessary.

(2) A review of all corrections - (a) Deśāntara sañskāra - It is due to different times of sunrise which is earlier in east. Hence time in east is more counted from sunrise or midnight. At present reference is not 0° longitude only. Every country has fixed reference time according to time zone from 0° longitude through Greenwich. Thus Indian standard time is standard time for 82°.30′ east of

greenwich, i.e. 5-1/2 hours more. Correction for local standard time is done for difference in deśāntara (longitude) Since 360° rotation of earth in 24 hours, 1° rotation is in $\frac{24 \times 60}{360}$ = 4 minutes. Hence 4 minutes time is added for each degree longitude towards east.

Local standard time - Indian standard time

= (longitude - 82°30′) in degrees x 4 minutes

(b) Cara sañskara - Midnight or midnoon is same for all places in a longitude. When time is measured from midnight (in hour system), then no correction is needed. However, in India, sāvana dina is counted from sunrise which is different at different latitudes. Difference in day length increases as we move away from equator. In practice we do not correct the time, but find the time of sun rise. Time of sunrise depends on position of true (sphuṭa) sāyana sun which is fixed for a particular day of a solar year like christian era. It also depends on latitude of the place. Thus date wise charts are prepared for sunrise time at different longitudes (in local mean time). at 1° or 10° intervals. It can be calculated from krānti of that day noted from pañcāñga or calculated from sāyana ravi.

Difference between true time and sunrise time - both counted from midnight gives iṣṭa kālā in Indian system.

(c) Bhujāntara sañskāra - This is due to difference in standard time and true time - both. Local standard time is calculated on the assumption that each day is of 24 hours. Day length is made

of two components. To move from 1 nakśatra to that nakśatra again it takes 23 hours 56 minutes due to earth's daily motion. Meanwhile, sun also moves about 1° ahead due to orbital movement of earth in same direction (360° in about 360 days). To cover that distance more earth takes about 4 minutes more (360° is covered in about 24 hours). Thus nākśatra dina is 23 hours 56 minutes = 21,600 asu and sāvana dina is 24 hours = 21,659 asu. Difference is 59 asu = about 4 minutes (= 60 asu).

While nākśatra dina is fixed, extra 4 minute component varies and each sāvana dina is not 24 hours exact. But the watches are calculating 24 hours for each day according to standard time. The standard or mean time and true or solar time start together at sāyana meṣa sañkrānti, 23 March., when day and night are equal. Around 24th April when sun is at farthest (mandocca is at nirayana meśa 10° or sāyana 32°), sun is slowest. So days are smaller than 24 hours after 23rd March. By taking 24 hours for each day clock time is slower than true time. This addition in clock time to get true time accummulates for about 6 months upto 14 minutes. Then it is negative correction and again both times tally on 23rd March next year.

Effect of 4 minutes shorter nākśatra dina is that a particular lagna (e.g. meṣa) will start 4 minutes earlier on next day. Effect of difference in true time and standard time is that sun will be at top most position at true noon not at local mean noon (12 hrs local standard time). This is also called correction due to time difference, or velāntara sañskāra. The formula for knowing difference in true and standard time is called time equation.

This difference depends only on sun's position (indicating bhujāntara) or the day of solar year.

(d) Udayāntara sañskāra - This is negligible and is not necessary when bhujāntara is measured in asus. In modern astronomy also, this is included in time equation.

Verse 42 - Multiply akśāmśa kalā by bhūparidhi and divide by 21,600. Then we get the distance of place from nirakśa (equator) towards north or south on the yāmyottara vṛtta (longitude line).

Note - Bhūparidhi covers 21,600 kalā. Akśāmśa kalā is north south distance from equator in kalās. Thus the distance from equator is calculated because 1 kalā is same on longitude line or equator.

Verse 43-45 - To save enormous labour in calculating graha, I am giving 'padaka' of sūrya etc. like Kōcanācārya. ('Kocannā' was an astronomer of Andhra Pradesh who had prepared charts for easy calculation. These charts were popular in south Orissa also at the time of author).

Ahargaṇa is given for years 1,2 ------, 10,20,---, 100, 2000-----, thousands, lākhs, ten lākhs, crores and ten crores. These start from madhyama sūrya at meṣa sañkramaṇa. Vāra Śuddhi has been done in this. To calculate the ahargaṇa, add the figures .given in table and divide by 7. If correct vāra doesn't come, then add or substract 1 for tally with vāra.

Verses 46-51 - By this method, ahargaṇa for first day of pañjikā is calculated. Graha is calculated for that ahargaṇa from their respctive padaka (tables). In this addition, we take figures upto 5

divisions from rāśi (parā). By this, graha gati can be calculated for up to 1 arbuda (10^8) days.

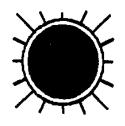
After writing padaka of graha and ucca etc, their dhruva (starting position) at beginning of kali and beginning of Karaṇābda (standard year for start of calculation by author-meṣa saṃkrānti of 1869) are written. Also write the dainika gati of graha, ucca and pāta. Write bhujāntara, cheda (part) mandocca hāra (part), pātahāra and deśāntara kalā etc. In the 73 tables, while adding rāśi etc of graha, multiples of 12 rāśi (1 revolution) are deducted. When calculation is from kali beginning, we get madhyama graha etc for Laṅkā midnight. If calculation is from karaṇābda; then value is for sunrise. For madhyamāna of candrapāta (rāhu), the angles are deducted from dhruva. Result is deducted from complete revolutions.

Verses 52-55 - After obtaining madhyamāna, deśāntara samskāra etc. are done. Then grahasphuṭa is done with help of table of khaṇḍaphala. As a siddhānta grantha, the tables should have been given after their related text. But at the time of printing (in 1899) all charts were given in appendix.

Verse 56 - For convenience in grahasphuṭa, I (author) have given phala, dhruva, gati etc in chart for 1 to 10^8 days. After calculating graha sphuṭa according to charts, you may not observe the graha in same position. Then correction is to be made by seeing dainika gati, dhruva padak etc. in second part of this book.

Verse 57 - May the Lord Jagannātha reside in my heart who is worshipped by Kubera's friend

Śiva at Nīlācala situated at akśāṁśa 4/27 (palabhā) and deśāntara 8434 viliptās which are 4/19 palabhā and 9138 viliptā according to new calculations.

Verse 58 - Thus, ends this fourth chapter written by Śrī Candraśekhara born in renowned royal family of Orissa. Siddhānta Darpaṛa is for tally of calculation and observation and education of students. Padaka charts have been given for fast calculation.

B. SPHŪṬĀDHIKĀRA

Scope - This part deals with finding true position of planets. So far we have calculated methods of finding mean position, which assumes constant average speeds of planets, to a first approximation. This part contains two chapters. Chapter 5 discusses true motion of planets. Chapter 6 deals with special corrections to moon's motion and accurate pañcāṅga on that basis.

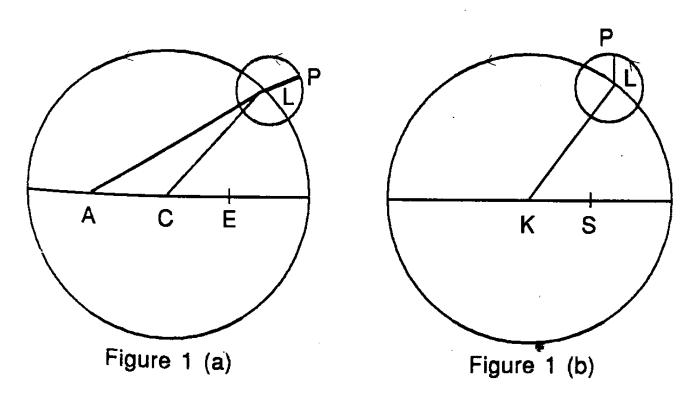
Chapter - 5

TRUE PLANETS

(Making grahas sphuṭa)

General Introduction

(1) Concepts of Plenetary motion

Figure 1 (a) Figure 1 (b)

Ptolemy

Ė is earth, P is planet Line A L turns with mean angular velocity of the planet round the sun. The line LP Turns with the mean angular velocity of earth. Length AC = CE is specified for each planet Length LP is related to the earth sun distence (for outer planets)

Copernicus

S represents the sun and P the planet The line KL turns with the angular velocity of the planet round the sun, while LP turns at twice the rate. The length KL and KS is specified for each planet. (Not drawn to scale)

From the data collected over centuries, apparent circular motion of planet with some loops and retroacting motion were detected, where it was difficult to find a pattern. But Hipparchus (100-120 BC) and Ptolemy (85-165 AD) were able to describe it on the basis of epicyclic moitons. As explained in diagram of Ptolemy (Fig 1a) planets moved in circles whose centre moved on some other circle round the earth, centre of this circle was slightly different depending on changes in the velocity of planet.

This was successful in predicting the future position of planets, but was unable to reveal any law of nature. Copernicus modified the pattern with similar construction; (Fig 1b) but with sun at rest in which patterns were easier to detect. Based on this construction, Kepler (1571-1642) framed 3 laws–

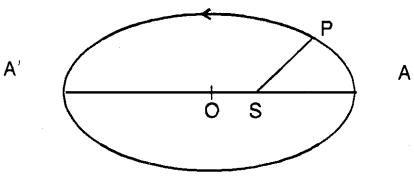



Figure 2 - Keplarian Orbit

Line SP joining sun S to a planet P sweeps out equal areas in equal time intervals (rule 2). P moves on an ellipse with S at focus. (rule 1). OA is semi major axis and ratio OS/OA is called eccentricity.

Third law is that square of time period T of revolution of a planet is proportional to cube of its mean distance from sun.

These laws led Newton to prove that all matters attract each other with a force proportional to inverse square of the distance between them. Together with plausible assumption that force is proportional to masses of attracting matter, it formed his theory of gravitation.

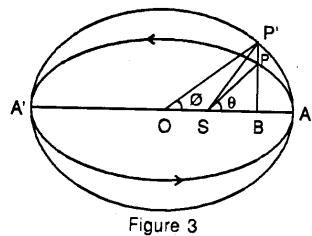
However, the method of calculation of planetary position remains the same. In both the methods, we calculate the direction and distance of planet from sun (heliocentric position). Then on basis of earth's distance and direction from sun, we calculate the direction of planet from earth (geocentric position.) Heliocentric position is only a mathematical necessity. Actual observation is always from earth, equal to geocentric position.

(2) **Calculation of planets in Western astronomy** - Calculation of sun's position is simplest. We calculate position of apsis (nearest point on major axis -- Indian method starts with farthest point) mean anomaly (angle with apsis) and position of vernal eqinox from which longitudes are measured. In a solar calender, sun's revolution is almost equal to year and position, longitude and latitude of sun depend on date of calender with minor corrections.

Moon's orbit has perturbations due to attraction of sun and other planets. Movement of its node is faster (Due to its nearness to earth and effect of sun, parallax etc, its accurate calculations for eclipse is needed. First, we derive the formula for calculation.

To know the true position - (1) A planet in its etiptical orbit with sun at focus is calculated to know its direction and distance from sun. (Heliocentric position)

(2) Position of earth is calculated from sun. From its direction and distance we calculate

Figure 3

direction and distance of planet from earth (Geocentric position).

Explaination of Anomalis (Fig 3) - APA' is eliptical orbit of a planet and S is the attracting sun at a focus. Revolution of the planet is counted from position A when it is closest to sun (near end of major axis) After 'd' days planet is seen at position P, ∠ASP is manda kendra = θ (True anomali).

Auxiliary circle is drawn on diameter AA'. PB is perpendicular on major axis, on extension it meets anxiliary circle on P'. ∠AOP' = φ is called eccentric anomaly. θ and φ are measured in length of arcs (radian measure). If daily mean motion of planet is n radians, then $2\pi/n$ is the period of

revolution. If daily motion is always n, then the planet after 'd' days with be 'nd' which is called mean anomaly (madhyama manda kendra). This will be true anomaly, if speed of planet (angular) is constant. According to second law of Keplar --

$$\frac{\text{AreaASP}}{\text{Area of ellipse}} = \frac{\text{d}}{\text{Time of revolution}} = d \,/\, \frac{2\pi}{n} = \frac{dn}{2\pi}$$

$$\frac{\text{Area ASP}}{\text{Area ASP}'} = \frac{\text{area of ellipse}}{\text{area of circle}} = \frac{b}{a} = \frac{\pi ab}{\pi a^2}$$

where a and b are semi major and semi minor axis.

$$\text{so } \frac{\text{Area ASP}}{\text{Area of ellipse}} = \frac{\text{Area ASP}'}{\text{Area of circle}} = \frac{\text{Area ASP}'}{\pi a^2}$$

But Area ASP′ = Area AOP′ – Area SOP′

$$= \frac{a^2\Phi}{2} - \frac{BP' \times OS}{2} = \frac{a^2\Phi}{2} - \frac{a\sin\Phi ae}{2}$$

$$= \frac{a^2}{2} (\Phi - e\sin\Phi)$$

Where e = eccentricity (cyuti) of ellipse.

$$\text{Hence } \frac{dn}{2\pi} = \frac{\text{Area ASP}}{\text{Area of ellipse}} = \frac{a^2}{2} \frac{(\Phi - e\sin\Phi)}{\pi a^2}$$

or $dn = \Phi\text{-}e \sin \Phi$ ....(1)

This is relation between mean anomaly and eccentric anomaly.

Relation between true anomaly and eccentric anomaly-Polar equation of ellipse is

$$SP = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

As per difinition of ellipse

SP =e x distance of P from directrix

=e x distance fo B from directrix (1 to major axis)

=e (distance from centre to directrix − centre to B)

$$= e \left(\frac{a}{e} - OB\right)$$

$$= e\left(\frac{a}{e} - a \cos \Phi\right) = a - e\, a \cos \Phi$$

or, radius vector (karṇa) = $a (1 - e \cos \phi)$---(2)

So, $\frac{a(1 - e^2)}{1 + e \cos \theta} = a(1 - e\cos\Phi)$

Or, $1 + e \cos \theta = \frac{1 - e^2}{1 - e \cos \Phi}$

or, $e \cos \theta = \frac{1 - e^2}{1 - e \cos \Phi} - 1 = \frac{(e \cos \phi - e)}{1 - e \cos \Phi}$

or, $\cos \theta = \frac{\cos \Phi - e}{1 - e \cos \Phi}$

$$\mathrm{Tan}^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{1 + \cos\theta} = \frac{1 - e\cos \Phi - \cos \Phi + e}{1 - e \cos \Phi + \cos \Phi - e}$$

$$= \frac{1 + e}{1 - e} \cdot \frac{1 - \cos\Phi}{1 + \cos \Phi}$$

$$= \frac{1 + e}{1 - e} \cdot \tan^2 \frac{\Phi}{2}$$

$$\text{or} \quad \tan \frac{\theta}{2} = \sqrt{\frac{1 + e}{1 - e}} \cdot \tan \frac{\Phi}{2} \;\; \ldots\ldots. \;(3)$$

Equations (1), (2) and (3) can be used to find manda kendra (True anomaly), manda karṇa (distance from sun) and d (time in days) from A (perihelion-nearest point).

For practical purpose, these equations are not convenient. For calculation on basis of average velocities which are known accurately, equation (3) needs to be expanded in a power series of small e as coefficient of sines of average position.

Equation (3) can be re-written on basis of formula of Trigonometry

$$\tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i\,(e^{i\theta} + e^{-i\theta})}, \quad i = \sqrt{-1}$$

where e = base of natural logarithm

$$= 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + -----$$

To differentiate it from eccentritity, we write is E then (3) becomes

$$\frac{E^{\frac{i\theta}{2}} - E^{\frac{-i\theta}{2}}}{i\,(E^{\frac{i\theta}{2}} + E^{\frac{-i\theta}{2}})} = \sqrt{\frac{1+e}{1-e}} \cdot \frac{E^{\frac{i\phi}{2}} - E^{\frac{-i\phi}{2}}}{i\,(E^{\frac{i\phi}{2}} + E^{\frac{-i\phi}{2}})}$$

$$\text{or} \quad \frac{E^{i\theta} - 1}{E^{i\theta} + 1} = \sqrt{\frac{1+e}{1-e}} \cdot \frac{E^{i\Phi} - 1}{E^{i\Phi} + 1}$$

Adding 1 to each side and substracting from 1, then dividing

$$E^{i\theta} = \frac{\sqrt{1+e}\,(E^{i\Phi} - 1) + \sqrt{1-e}\,(E^{i\Phi} + 1)}{\sqrt{1-e}\,(E^{i\Phi} + 1) - \sqrt{1+e}\,(E^{i\Phi} - 1)}$$

$$= \frac{E^{i\Phi}(\sqrt{1+e} + \sqrt{1-e}) + \sqrt{1-e} - \sqrt{1+e}}{-E^{i\Phi}(\sqrt{1+e} - \sqrt{1-e}) + \sqrt{1-e} + \sqrt{1+e}}$$

$$= \frac{E^{i\Phi} - p}{1 - pE^{2\Phi}} \quad \text{where } p = \frac{\sqrt{1+e} - \sqrt{1-e}}{\sqrt{1+e} + \sqrt{1-e}}$$

$$\text{or } E^{i\theta} = E^{i\Phi} \cdot \frac{1 - p\,E^{-i\Phi}}{1 - pE^{i\Phi}}$$

Taking logarithm of both sides,

$$i\theta = i\Phi + \log\,(1 - pE^{-i\Phi}) - \log\,(1 - pE^{i\Phi})$$

$$= i\Phi + (E^{i}\phi - E^{-i\Phi}) + \frac{p^2}{2}(E^{2i\Phi} - E^{-2i\Phi})$$

$$+ \frac{p^3}{3}(E^{3i\Phi} - E^{-3i\Phi}) + \ldots.$$

or

$$\theta = \Phi + \Phi + \frac{1}{i}(E^{i\Phi} - E^{-i\Phi}) + \frac{p^2}{2i}(E^{2i\Phi}\ E^{-2i\Phi})$$

$$+ \frac{p^2}{3i}(E^{3i\Phi} - E^{-3i\Phi}) + \ldots\ldots\ldots$$

or

$$\theta = \Phi + 2p \sin \Phi + \frac{2p^2}{2} \sin 2\Phi + \frac{2p^3}{3} \sin 3\Phi + \ldots$$

or $$\theta = \Phi + 2\left(p \sin \Phi + \frac{p^2}{2} \sin 2\Phi + \frac{p^3}{3} \sin 3\Phi \ldots\right)$$

(4)

Equation (4) needs to be expressed as a series in mean velocity n and d which are easily determined.

For this, we use Taylor's infinite series based on Lagranges mean value theorem of differential calculus. This is written as

$$f(x + h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \ldots \infty$$

let $y = x + h f(y)$ .... (5)

Then $F(y) = F[x+h f(y)]$. Then by Taylor's Theorem,

$$= F(x) + h \cdot f(x) \cdot F'(x) + \frac{h^2}{\lfloor 2} \frac{d}{dx} [f(y)^2 \cdot F'(x)]$$

$$+ \frac{h^3}{\lfloor 3} \cdot \frac{d^2}{dx^2} [f(y)^3 \cdot F'(x)] + \ldots$$

$$+ \frac{h^n}{\lfloor n} \frac{d^{x-1}}{dx^{n-1}} [fy)^n F'(x)] \qquad + \ldots\ldots (6)$$

From equation (1), $dn = \Phi - e \sin \Phi$

or $\Phi = dn + e \sin \Phi = m + e \sin \Phi$ where $m = dn$

This is in form of (5) where $y = \Phi$, $x=m$, $h = e$

Let $F(\Phi,) = \Phi,,$ then $F(m) = m$ and $F'(m) = 1$

Hence form (6)

$$\Phi = M + e \sin m .1 + \frac{e^2}{\lfloor 2} \frac{d}{dm} (\sin^2 m . 1)$$

$$+ \frac{e^2}{\lfloor 3} \cdot \frac{d^2}{dm^2} (\sin^3 m.1)$$

$$+ \frac{e^4}{\lfloor 4} \cdot \frac{d^3}{dn^3} (\sin^4 .m .1) + \frac{e^5}{\lfloor 5} \frac{d^4}{dm^4} (\sin^5 m .1) + \ldots$$

Expansion of Sin ^{n}m is given by

(when n is even)

$$\sin^n m = \frac{1}{2^{n-1} (-1)^{n/2}} [\cos nm - n \cos (n-2) m$$

$$+ \frac{n(n-1)}{\lfloor 2} \cos (n-4) m - \frac{n(n-1)(n-2)}{\lfloor 3}$$

$$\cos (n-6) m + \ldots]$$

When n is odd, then

$$\sin^n m = \frac{1}{2^{n-1} (n-1)^{\frac{n-1}{2}}}$$

$$[\sin nm - n \sin (n-2) m + \frac{n(n-1)}{\lfloor 2} \sin (n-4) m$$

$$\frac{n(n-1)(n-2)}{\lfloor 3} \sin (n-6) m + \ldots\ldots]$$

Hence $\frac{d}{dm} (\sin^2 m) = \frac{d}{dm} \frac{(1 - \cos 2m)}{2} = \sin 2m$

$$\frac{d^2}{dm^2} (\sin^3 m) = \frac{d^2}{dm^2} \left(\frac{3 \sin m - \sin 3m}{4} \right)$$

$$= \frac{d}{dm} \left(\frac{3 \cos m - 3 \cos 3m}{4} \right)$$

$$= \frac{-3 \sin m + 9 \sin 3m}{4} = \frac{3}{4} (3 \sin 3m - \sin m)$$

$$\frac{d^3}{dm^3}(\sin^4 m) = \frac{d^3}{dm^3}$$

$$\left\{\frac{1}{2^3(-1)^2}(\cos 4m - 4\cos 2m + \frac{2 \times 3}{\lfloor 2} \times 1)\right\}$$

$$\frac{1}{8}(4^3 \sin 4m - 4 \times 2^3 \sin 2m) = 4\,(2 \sin 4m - \sin 2m)$$

$$\frac{d^4}{dm^4}(\sin^5 m)$$

$$= \frac{d^4}{dm^4}\left\{\frac{1}{2^4(-1)^2}(\sin 5m - 5 \sin 3m + \frac{5 \times 4}{2} \sin m)\right\}$$

$$= \frac{1}{16}(5^4 \sin 5m - 5 \times 3^4 \sin 3m + 10 \sin m)$$

$$\frac{d^5}{dm^5}(\sin^6 m) = \frac{d^5}{dm^5}\left\{\frac{1}{2^5(-1)^3}(\cos 6m - 6 \cos 4m\right.$$

$$\left. + \frac{6 \times 5}{\lfloor 2} \cos 2m - \frac{6 \times 5 \times 4}{\lfloor 3} \times 1)\right\}$$

$$= \frac{1}{32}(6^5 \sin 6m - 6 \times 4^5 \sin 4m + 15 \times 2^5 \sin 2m)$$

Hence $\Phi = m + e \sin m$

$$+ \frac{e^2}{2} \sin 2m + \frac{e^3}{\lfloor 3} \times \frac{3}{4}(3 \sin 3m - \sin m)$$

$$+ \frac{e^4}{\lfloor 4} \times 4\,(2 \sin 4m - \sin 2m) + \frac{e^5}{\lfloor 5} \times \frac{1}{16} \times$$

$$(5^4 \sin 5m - 5 \times 3^4 \sin 3m + 10 \sin m) + \frac{e^6}{\lfloor 6} \cdot \frac{1}{32} \times$$

$$(6^5 \sin 6m - 6 \times 4^5 \sin 4m + 15 \times 2^5 \sin 2m) + \ldots$$

Separating sin m, sin 2 m ...etc.

$$\Phi = m + (e - \frac{e^3}{8} + \frac{e^5}{192}) \sin m + (\frac{e^2}{2} - \frac{e^4}{6} + \frac{e^6}{48})$$

$$\sin 2m + (\frac{3e^3}{8} - \frac{27e^5}{128}) \sin 3m + (\frac{e^4}{3} - \frac{4e^6}{15}) \sin 4m$$

$$+ \frac{125e^5}{384} \sin 5m + \ldots \quad (7)$$

Next quantities contains powers of e^6 or more hence they are very small and left out (e is very small because orbit is almost circular with very small eccentricity)

Equation (1) can be also written as

$e \sin \Phi = \Phi\text{-}m$

or $\sin \Phi = \dfrac{\Phi - m}{e}$

From (7), this becomes

$$\sin \Phi = (1 - \frac{e^2}{8} + \frac{e^4}{192}) \sin m + (\frac{e}{2} - \frac{e^3}{6} + \frac{e^5}{48}) \sin 2m$$

$$+ (3\frac{e^2}{8} - \frac{27e^4}{128}) \sin 3m + \left(\frac{e}{3} - \frac{4e^5}{15}\right)$$

$$\sin 4m + \frac{125e^4}{384} \sin 5m + \ldots . \quad (8)$$

Now expansion of Sin 2 Φ, sin 3 Φ --- are to be obtained

Now in equation (6), take F (Φ) = sin 2 Φ, then F(m) = Sin 2m and F′ (m) = 2 sin 2 m

Hence equation (6) becomes -

$$\sin^2 \phi = \sin 2m + e \sin m \times 2 \cos 2m$$

$$+ \frac{e^2}{\lfloor 2} \frac{d}{dm} (\sin^2 m \times 2\cos 2m) + \frac{e^3}{\lfloor 3}$$

$$\frac{d^2}{dm^2} (\sin^3 m \times 2 \cos 2m) + \frac{e^4}{\lfloor 4} \frac{d^3}{dm^3}$$

$$(\sin^4 m \times 2\cos 2m) + \frac{e^5}{\lfloor 5}\frac{d^4}{dm^4}(\sin^5 m \times 2\cos 2m) + \ldots$$

In this, $\sin m \times 2\cos 2m = \sin 3m - \sin m$

$$\frac{d}{dm}(\sin^2 m \times 2\cos 2m) = \frac{d}{dm}\left(\frac{1-\cos 2m}{2} \times 2\cos 2m\right)$$

$$= \frac{d}{dm}(\cos 2m - \cos^2 2m) = \frac{d}{dm}\left(\cos 2m - \frac{1+\cos 4m}{2}\right)$$

$$= 2\sin 4m - 2\sin 2m$$

$$\frac{d^2}{dm^2}(\sin^3 m \times 2\cos 2m)$$

$$= \frac{d^2}{dm^2}\left(\frac{3\sin m - \sin 3m}{4} \times 2\cos 2m\right)$$

$$\frac{d^2}{dm^2}\left(\frac{3\sin m\cos 2m - \sin 3m\cos 2m}{2}\right)$$

$$\frac{d^2}{dm^2}\left[\frac{3}{4}(\sin 3m - \sin m) - \frac{1}{4}(\sin 5m + \sin m)\right]$$

$$\frac{d^2}{dm^2}\left\{\frac{1}{4}(3\sin 3m - 4\sin m - \sin 5m\right\}$$

$$= \frac{1}{4}(-3^2\sin 3m + 4\sin m + 5^2\sin 5m)$$

$$\frac{d^3}{dm^3}(\sin^4 m \times 2\cos 2m) = \frac{d^3}{dm^3}$$

$$\left\{\frac{1}{8}(\cos 4m - 4\cos 2m + 3)\, 2\cos 2m\right\}$$

$$= \frac{d^3}{dm^3}\left\{\frac{1}{8}(2\cos 4m \,.\, \cos 2m - 4 \times 2 \,.\, \cos^2 2m\right.$$

$$\left. + 6\cos 2m)\right.$$

$$= \frac{d^3}{dm^3}\left\{\frac{1}{8}(\cos 6m + \cos 2m) - \frac{4}{8}(1 + \cos 4m) + \frac{6}{8}\cos 2m\right\}$$

$$= \frac{d^3}{dm^3}\left\{\frac{1}{8}(\cos 6m - 4\cos 4m + 7\cos 2m - 4)\right\}$$

$$= \frac{1}{8}(6^3 \sin 6m - 4^4 \sin 4m + 7 \times 2^3 \sin 2m)$$

so $\{\sin 2\Phi = \sin 2m + e(\sin 3m - \sin m)$

$$+\frac{e^2}{2}(2\sin 4m - 2\sin 2m)$$

$$+\frac{e^3}{\lfloor 3}\ \frac{1}{4}(25\sin 5m - 27\sin 3m + 4\sin m)$$

$$+\frac{e^4}{\lfloor 3}\cdot\frac{1}{8}(216\sin 6m - 256\sin 4m + 56\sin 2m) + \ldots\ldots$$

$$= (-e + \frac{e^3}{6})\sin m + (1 - e^2 + \frac{7e^4}{24})\sin 2m$$

$$+(e - \frac{9e^3}{8})\sin 3m + (e^2 - \frac{4e^4}{3})\sin 4m + \frac{25e^3}{24}\sin 5m + \ldots.$$

In equation (6), now take $F(\Phi) = \sin 3\Phi$, then $F(m) = \sin 3m$ and $F'(m) = 3\cos 3m$, then it becomes -

$$\sin 3\Phi = \sin 3m + e\sin m \times 3\cos 3m +$$

$$\frac{e^2}{\lfloor 2}\cdot\frac{d}{dm}(\sin^2 m \times 3\cos 3m)$$

$$+\frac{e^3}{\lfloor 3}\cdot\frac{d^2}{dm^2}\left\{\sin^3 m \times 3\cos 3m + \ldots\right\} + \ldots\ldots$$

$$= \{\sin 3m + \frac{3e}{2}(\sin 4m - \sin 2m)$$

$$+\frac{e^2}{3}\cdot\frac{3}{4}\ \{(5\sin 5m - 6\sin 3m + \sin m)$$

$$+ \frac{e^3}{6} \cdot \frac{3}{8} (36 \sin 6m - 48 \sin 4m + 12 \sin 2m) + \ldots$$

or,

$$\sin 3\Phi = \frac{3e^2}{8} \sin m - (\frac{3e}{2} - \frac{3e^2}{4}) \sin 2m + (1 - \frac{9e^2}{4}) \sin 3m$$

$$(\frac{3e}{2} - 3e^2) \sin 4m + \frac{15e^2}{8} \sin 5m + \frac{9e^3}{4} \sin 6m + \ldots.$$

Similarly $\sin 4\Phi = \sin 4m + e \sin m \times 4 \cos 4m$

$$+ \frac{e^2}{2} \cdot \frac{d}{dm} (\sin 2m \times 4 \cos 4m) + ---$$

$$= \text{Sin } 4m + 2e (\sin 5m - \sin 3m) + \frac{e^2}{2} (6 \sin 6m$$

$$- 8 \sin 4m + 2 \sin 2m) + \ldots\ldots\ldots\ldots$$

or $\sin 4\Phi = e^2 \sin 2m - 2e\sin 3m + (1-4e^2) \sin 4m + 2e \sin 5m + \ldots\ldots\ldots$

$$\text{Sin } 5\Phi = \text{Sin } 5m + \frac{5}{2}e (\text{Sin } 6m - \text{Sin } 4m) + --$$

$$= -\frac{5e}{2} \sin 4m + \sin 5m + \frac{5e}{2} \sin 6m + --$$

Value of p can be known in terms of e by expanding with binomial theorem also (Taylor's theorem is not needed)

$$p = \frac{\sqrt{1+e} - \sqrt{1-e}}{\sqrt{1+e} + \sqrt{1-e}} = \frac{1 - \sqrt{1-e^2}}{e}$$

$$= \frac{1}{e} [1 - (1 - e^2)^{1/2}]$$

$$\frac{1}{e} (\frac{e^2}{2} + \frac{e^4}{8} + \frac{e^6}{16} + \ldots.)$$

$$\text{or } p = \frac{e}{2} + \frac{e^3}{8} + \frac{e^5}{16} + \ldots.$$

$$p^2 = \left(\frac{e}{2} + \frac{e^3}{8} + \frac{e^5}{16}\right)^2 = \frac{e^2}{4} + \frac{e^4}{8} + \frac{5e^6}{64}$$

$$p^3 = \left(\frac{e}{2} + \frac{e^3}{8} + \frac{e^5}{16}\right) \left(\frac{e^2}{4} + \frac{e^4}{8} + \frac{5e^6}{64}\right) + ...$$

$$= \frac{e^3}{8} + \frac{3e^5}{32} + \frac{9e^7}{128} + ...$$

$$p^4 = \left(\frac{e^2}{4} + \frac{e^4}{8} + \frac{5e^6}{64}\right)^2 = \frac{e^4}{16} + \frac{e^6}{16} + ...$$

$$p^5 = \left(\frac{e^4}{16} + \frac{e^6}{16}\right) \left(\frac{e}{2} + \frac{e^3}{8} + \frac{e^5}{16}\right) = \frac{e^5}{32} + ...$$

Now equation (4) can be written as

$$\theta = m + \left(e - \frac{e^3}{8} + \frac{e^5}{192}\right) \sin m + \left(\frac{e^2}{2} - \frac{e^5}{6} + \frac{e^6}{48}\right)$$

$$\sin 2m + \left(\frac{3e^3}{8} - \frac{27e^5}{128}\right) \sin 3m + \left(\frac{e^4}{3} - \frac{4e^6}{15}\right)$$

$$\sin 4m + \frac{125e^5}{38} \sin 5m + ...$$

$$+ 2 \left\{\left(\frac{e}{2} + \frac{e^3}{8} + \frac{e^5}{16}\right) \left[\left(1 - \frac{e^2}{8} + \frac{e^4}{192}\right) \sin m\right.\right.$$

$$+ \left(\frac{e}{2} - \frac{e^2}{6} + \frac{e^5}{48}\right) \sin 2m$$

$$+ \left(\frac{3e^2}{8} - \frac{27e^5}{118}\right) \sin 3m + \left(\frac{e^3}{3} - \frac{4e^5}{15}\right)$$

$$\sin 4m + \frac{125e^5}{384} \sin 5m + ...]$$

$$+ \frac{1}{2} \left(\frac{e^2}{4} + \frac{e^4}{8} + \frac{5e^6}{64}\right) \left[\left(-e + \frac{e^3}{6}\right)\right.$$

$$\sin m + \left(1 - e^2 + \frac{7e^4}{24}\right) \sin 2m$$

$$+\left(e - \frac{9e^3}{8}\right) \sin 3m + \left(e^2 - \frac{4e^4}{3}\right) \sin 4m + \frac{25\, e^3}{24} \sin 5m + ..]$$

$$+ \frac{1}{2} \left(\frac{e^3}{8} + \frac{3e^5}{32}\right) \left[\frac{3e^2}{8} \sin m - \left(\frac{3e}{2} - \frac{3e^3}{4}\right) \sin 2m\right.$$

$$+ (1 - \frac{9e^2}{4}) \sin 3m$$

$$+ (\frac{3e}{2} - 3e^2) \sin 4m + \frac{15e^2}{8} \sin 5m + \frac{9e^2}{4} \sin 6m + ...]$$

$$+ \frac{1}{4} (\frac{e^4}{16} + \frac{e^6}{16}) [e^2 \sin 2m - 2e \sin 3m +$$

$$(1 + 4e^2) \sin 4m + 2e \sin 5m]$$

$$+ \frac{1}{5} \frac{e^5}{32} [- \frac{5e}{2} \sin 4m + \sin 5m + \frac{5e}{2} \sin 6m] \Big\}$$

Terms beyond e^6 and sin 6 m have been left out as they are neglible. Collecting the multiples of sin m, sin 2m ---etc.

$$\theta = m + (2e - \frac{e^3}{4} + \frac{5e^5}{96}) \sin m + (\frac{5e^2}{4} - \frac{11e^4}{24} + \frac{17e^6}{192})$$

$$\sin 2m + (\frac{13e^3}{12} - \frac{43e^5}{64}) \sin 3m + (\frac{103e^4}{96} - \frac{451e^6}{480})$$

$$\sin 4m + \frac{1097e^5}{960} \sin 5m \quad ....(9)$$

Actual equation for knowing heliocentric true position -

Equation (9) is the main equation from which helocentric position of planets are calculated from their mean speeds and eccentricity of orbits. This is called manda karṇa in Indian system. For example, in case of Jupiter, e = 0.048254, hence e^2 = 0.0023284, e^3 = 0.0001124, e^4 = 0.0000054, e^5 and higher powers are very small and can be neglected for calculation of 1″ accuracy.

For Jupiter -

θ = m + (0.0 96508 - 0.0000281) sin m

+ (0.002 9 106 - 0.0000025) sin 2m

$+ 0.0001218 \sin 3m + 0.0000058 \sin 4m + \text{--}$

or $\theta = m + 0.0964799 \sin m + 0.002\,9081 \sin 2m$

$+ 0.0001218 \sin 3m + 0.0000058 \sin 4\,m + \text{---}$

(10)

If the sines are expressed in kalā or vikalā in Indian system, then the value of θ will come in kalā or vikala and this will be manda phala of guru from centre of sun. If they are expressed in fractions, the terms after m will be in radian. To convert them in kalā or vikalā, they are to be multiplied by 3437.75 or 206265.

Equation for any planet can be obtained by putting its eccentricity e in equation (9). The eccntricities are given in end of this section.

Helocentric distance -

Manda karṇa (Heliocentric distance of planet) is

$SP = a(1\text{-}e \cos \Phi)$

Putting $F(\Phi) = 1 - e \cos \Phi$, $F(m) = 1\text{-}e \cos m$, $F'(m) = e \sin m$, in equation (6), Taylor's series gives

$$1\text{-}e \cos \Phi = (1\text{-}e \cos m) + e \sin m \frac{d}{dm} (1\text{-}\cos m) +$$

$$+ \frac{e^2}{\lfloor 3} \frac{d}{dm} (\sin^2 m \times e \sin m)$$

$$+ \frac{e^3}{\lfloor 3} \cdot \frac{d^2}{dm^2} (\sin^3 m \times e \sin m) + \ldots\ldots\ldots$$

$$= 1\text{-}e \cos m + \frac{e^2}{2} - \frac{e^2}{2}\cos 2m + \frac{3e^3}{8}\cos 3m - \frac{e^4}{3}\cos$$

$$4m + \frac{e^4}{3}\cos 2m$$

$$= (1 + \frac{e^2}{2}) - e(1 - \frac{3e^2}{8})\cos m - \frac{e^2}{2}(1 - \frac{2e^2}{3})\cos 2m$$

$$- \frac{3e^3}{8}\cos 3m + \ldots\ldots$$

Hence, radius (karṇa)

$$= a\,[\,(1 + \frac{e^2}{2}) - e(1 - \frac{3e^2}{8})\cos m - \frac{e^2}{2}(1 - \frac{2e^2}{3})$$

$$\cos 2m - \frac{3e^3}{8}\cos 3m] \ldots\ldots\ldots\ldots\ldots (11)$$

Semi major axis (smallest+largest distance),/ 2 of Jupiter a is 5202.8 hencee equation of its redius is

5202.8 [(1+0.0011642) – (0.048254 – 0.0000421) cosm

– (0.0011642 – 0.0000018) cos 2m – 0.0000421 cos 3m)]

= 5202.8 (1.0011642 – 0.00482119 cos m - 0.0011624 cos 2m - 0.0000421 cos 3m)

= 5208.86-251.06 cosm - 6.05 cos 2m - 0.22 cos 3m

Semi major axis has been expressed as ratio of earth's mean distance from Sun which is taken as 1000

Parameters of planetary orbit

Constants for earth -- $a_{\oplus} = 1.4959787 \times 10^{11}$ metres, $\oplus$ is symbol for earth, a is semi major axis Time period of revolution $T_{\oplus} = 3.1558150 \times 10^7$ sec.

Mass $m_\oplus$ = 5.976 x 10^{24} kg, Moment $M_\oplus$ = 2.66 x 10^{40} kg m^2/sec

Eccentricity $e_\oplus$ = 0.0167

Orbits of other planets

Planet	a in $a_\oplus$	Period Years	Mass (in $m_\oplus$)	Moment (in $m_\oplus$)	inclination of orbit	Eccentricity e
Mercury	0.3871	0.24	5.6 x 10^{-2}	3.4 x 10^{-2}	7° 0' 14"	0.2056
Venus	0.7233	0.62	8.1 x 10^{-1}	7.0 x 10^{-1}	3° 23' 39"	0.0068
Mars	1.5237	1.88	1.1 x 10^{-1}	1.3 x 10^{-1}	1°51' 0"	0.0934
Jupiter	5.2028	11.87	3.2 x 10^2	7.6x10^2	1° 18' 21"	0.0484
Saturn	9.5389	29.46	9.5 x 10^1	2.9x10^2	2°29"25"	0.0557
Uranus	19.18	84.01	1.5 x 10^1	6.4x10^1	0°46" 23"	0.0472
Neptune	30.06	164.8	1.7x10^1	9.5x10^1	1°46' 28"	0.0086
Pluto	39.44	247.6	2.0 x 10^{-3}	1.2 x 10^{-2}	17°8' 38"	0.2486

Conversion of Orbital distance to ecliptic distance Equation (10) gives the distance (angular) of planet in its orbit from its nīcha (perihelion) or closest position. If the orbit of planet would have been in same plane as earth's orbit (or plane of

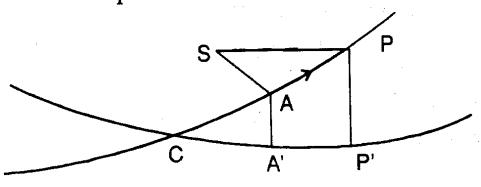



Figure 4 - Inclination of orbit with ecliptic

ecliptic), this would have been its distance in ecliptic also. But every planet's orbit is at an angle with ecliptic which is its parama śara (maximum distance from ecliptic). This inclination is given in the chart above. There is no inclination for earth's orbit (or sun) because it is measured from this orbit only.

In fig 4, PC is orbital ellipse and CP′ is the ecliptic. S is centre of sun and A is perihelion (nīca) of the planet. P is true position. PP′ is perpendicular on ecliptic, hence it passes through pole of ecliptic. Then ASP is orbital true anomaly (Kakśā spaśṭa kendra) and SP is spaṣṭa karṇa. AA′ is perpendicular on ecliptic and also passes through its pole. Distance A′P′ along ecliptic is the ecliptic true anomaly (krānti vṛttīya spaṣṭa kendra).

For theoretical calculation, it is easier to find out relation between CA and CA′ or CP and CP′. But in practice, we need to know only the minor correction to orbital distance to know ecliptic distance.

This correction or difference between orbital distances from pāta C (intersection point of orbit and ecliptic) is called pariṇati.

Nīca pariṇati = CA - CA′

Planet pariṇati = PA-P′A

PP′ is instantaneous or iṣṭakālika śara, ∠PCP′ is parama śara (equal to maximum angular distance from ecliptic), PC is distance from pāta to graha or vipāta graha. ∠PP′C is right angle, hence PCP′ is a spherical right angle triangle. **From** Napier's laws -

(1) $\text{Sin } (90^\circ - CP) = \cos (PP') \times \cos CP'$

(2) $\text{Sin } PP' = \cos (90^\circ\text{-}PCP') \times \cos (90^\circ\text{-}CP)$

(3) $\text{Tan } PP' = \text{Sin } CP' \times \tan PCP'$

(4) $\text{Tan } CP' = \cos (PCP') \tan CP$

$\text{Sin } (CP\text{-}CP') = \text{Sin } CP. \cos CP' - \cos CP. \sin CP'$ ---(12)

From formula (3),

$$\sin CP' = \frac{\tan PP'}{\tan PCP'}$$

Formula (4), $\dfrac{\sin CP'}{\cos CP'} = \cos (PCP') \tan CP$

$$\therefore \cos CP' = \frac{\sin CP'}{\cos (PCP') \tan CP}$$

$$\{ = \frac{\tan PP'}{\tan PCP'} \times \frac{\cos CP}{\cos PCP' \sin CP}$$

$$= \frac{\tan PP'}{\sin PCP'} \times \frac{\cos CP}{\sin CP}$$

so,

$$\sin (CP - CP') = \sin CP \frac{\tan PP'}{\sin PCP'} \times \frac{\cos CP}{\sin CP} - \cos CP \frac{\tan PP'}{\tan PCP'}$$

$$= \frac{\tan PP \times \cos CP}{\sin PCP'} - \frac{\cos CP \times \tan PP'}{\tan PCP'}$$

$$= \frac{\tan PP' \times \cos CP}{\sin PCP'} [1 - \cos PCP']$$

$$= \frac{\sin PP'}{\cos PP'} \times \frac{\cos CP}{\sin PCP'} \times \text{vers sin } PCP'$$

From formula (2), $\dfrac{\sin PP'}{\sin PCP'} = \sin CP$

Hence sin (CP-CP′)

$$= \frac{\sin CP \times \cos CP'}{\cos PP'} \times \text{vers sin } PCP'$$

Parama śara of all planets except Budha is less than 3.4° hence their iṣṭakālika śara will be still smaller. Hence Cos PP′ ≅ 1. Then

Sin (CP-CP′) = Sin CP cos CP × vers sin PCP′ = 1/2 sin 2 CP. V sin PCP′, or, sin (Pariṇati) = 1/2 X versed sin of parama śara X Sin (2 × vipāta graha) --- (13)

Equation (13) gives correction to find position of planet in krānti vṛtta.

Geocentric position

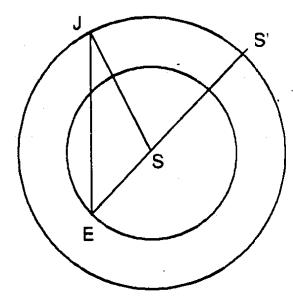



Figure 5 - Geocentric position of planets

To find the direction and distance of planet from earth, we have to know the position of earth itself. Position of earth also can be known from equation (9) like other planets. Position of Sun from earth is opposite to earth from sun direction i.e. 180° away.

Śīghra kendra is difference of ecliptic spaṣṭa kendra and position of sun from earth.

In Figure 5, S, E and J are positions of sun, earth and Jupiter. ESS′ is direction of Sun from earth (both centres). S′ is its position in ecliptic. S′SJ is śīghra kendra of Jupiter. ∠ESJ =180°-S′SJ and in ΔEJS, two sides ES, SJ and angle between them is known. Then EJ, ∠ SEJ and ∠EJS also can be known. From trigonometry

$$\tan \frac{SEJ - SJE}{2} = \frac{SJ - SE}{SJ + SE} \tan \frac{SEJ + SJE}{2}$$

Here ∠ S EJ + ∠ SJE = ∠S′ SJ = śīghra kendra

$$\therefore \tan \frac{SEJ - SJE}{2} = \frac{SJ - SE}{SJ + SE} \tan \frac{\text{śīghra Kendra}}{2}$$

From this difference of angles ∠SEJ and ∠SJE can be known. Their sum (śīghra kendra)) is already known. By adding these and dividing by 2 we get ∠SEJ which is angle between Jupiter and Sun as seen from earth. This is called Ināntara (Ina=Sun).

Distance of Jupiter from Earth JE is śīghra karṇa.

$$\frac{JE}{\sin \angle ESJ} = \frac{JS}{\sin \angle SEJ} \quad \text{by sin ratios}$$

But sin ∠SEJ = Sin (180°-∠SEJ) = ∠JSS′ = Sin (Śīghra kendra)

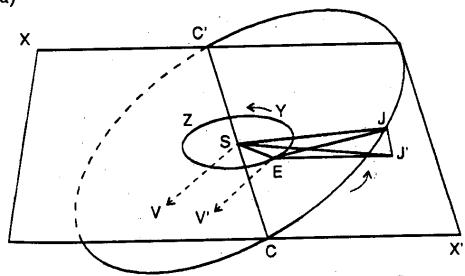



Figure 6 - Sighra Kendra in ecliptic

Hence, Śīghra karṇa JE = Sin of Śīghra kendra × manda karna × $\frac{1}{\sin \text{(ināntara)}}$ ---(14)

In fig 6, XX′ is ecliptic plane which contains earth's orbit EYZ. Orbit of Jupiter is CJC′ which cuts ecliptic on C and C′. C is north pāta and C′ south pāta. S,E and J are true positions of Sun, earth and jupiter.··JJ′ is perpendicular on ecliptic plane. V is point of vernal equinox (north pāta of ecliptic and equator planes). ∠JSJ′is heliocentric inclination of Jupiter, ∠VSJ′ is longitude of planet (angle in ecliptic plane between vernal equinox and planet - seen from sun. JEJ′ and V′EJ′ are geocentric inclination and longitude of jupiter. SV I I EV′. ∠VSE is heliocentric longitude of earth, hence ∠VSE + 180° is geocentric longitude of sun.

(Δ SEJ′ is same as Δ SEJ of figure 5) Śīghra karṇa and ināntara in equator are EJ′ and ∠SEJ′. True śīghra karṇa and ināntara are EJ and ∠SEJ;

$$EJ = \frac{EJ'}{\cos \angle JEJ'} \quad \text{--(15)}$$

∠ J E J′ is very small, hence is cosine is almost 1.

This is only a rough outline of calculation of palnetary positions in modern astronomy.

There are prturbations in positions of earth due to effect of moon, jupiter and venus (others negligible) Similarly prerturbations occur in other planets also for which corrections are necessary. There is slight change in eccentricity and positions of pāta also which cause other corrections. The corrections to the orbit of moon are more important because it has largest effect on earth's tides, climates, calender and eclipse etc.

(3) Tables of Sun -

Precession of equinoxes - According to Newcomb, rate of general precession in longitude per tropical year of 365.2422 days is 50″. 2564+0″.02223 (t/100) + 0″.0000026 $(t/100)^2$

where t is measured in tropical years from 1900.0 AD.

Annual rates of precession per sidereal year of 365.25636 days is 50″.258 35 + 0″.02223 (t/100) + 0″.00000026 $(t/100)^2$

In Julian year of 365.25 days, precession is 50.″25747+0″.02223 (t/100) + 0″.0000026 $(t/100)^2$

In Indian system, initial point from which longitude is measured is a fixed point of ecliptic with respect to stars. In modern astronomy, it is the point of vernal equinox. Distance from fixed initial point of vernal equinox point is called Ayanāṃśa.

To fix initial point accurately, star spica (αvirginis) has been assigned a nirayana (from fixed point) longitude of 180°. Since the star also has some small motion, its longitude of epoch time is taken when fixed point and vernal equinox point were together with sun on it.

This epoch of 0° Ayanāṃsa (0° sun also) was on 285 AD, March 22, 17h 48m E.T. or 21h 27m IST. That was beginning of śaka era 207, Samvat era 342 and Kaliyuga era 3386. Julian day on March 22 noon was 1825325 and kali elapsed days at midnight was 1236770. The day was sunday. Mean sun (both tropical and sidereal) was 0°0′0″ and (Mean moon - Mean sun) = 351°.67. Thus it was also a new moon day. In Besselian fictitious year, epoch was

$$285\ \frac{79.994}{360} = 285.2222 \text{ A.D.}$$

The epoch is 1614.7778 years before 1900.0 AD. From 0° Ayanāmśa of this epoch to ayanāṃśa at 1900 AD, January 0.813 i.e. 19h 31m ET is 22°27′43″.51. Thus Ayanāmśa from 1900.0 AD is

$$A = 22°27'43''.\ 51+50''.\ 2564t+1''.\ 1115\left(\frac{t}{100}\right)^2+0''.0001\left(\frac{t}{100}\right)^3$$

This is formula in tropical years. In sidereal years, it is

$$A = 22°27'43.''40+50''.25835t+1''.1115\left(\frac{t}{100}\right)^2 + 0''.0001\left(\frac{t}{100}\right)^3$$

In Julian year, formula is

$$A = 22°27'43''.\ 40+50''.\ 2575t+1''.\ 1115\left(\frac{t}{100}\right)^2 + 0''.0001\left(\frac{t}{100}\right)^3$$

Daily rate of precession in 1900 AD = 0″.137597

If time is taken from 285AD epoch, formula in tropical years is

$$A=49''.8981t+1''.1073\ (t.100)^2 + 0.0001\ (t/100)^3 + \text{----}$$

There are simllar formula for sideral and Julian years.

Position of star Spica of 180° long in 285 AD -

In 1950.0 AD its position was

R.A. = 200° 38′19″.6, Declination = -10°54′3″.4

Annual proper motion $\Delta\alpha$ = -0″.039 $\Delta\delta$ = -0″.033

Tropical longitude = 203° 8′36″3 latitude -= -2°3′2″.8

Sidereal long = 179°.58′59″.7 (Ayanāṃśa 23°9′36″.6)

Annual proper motion in ecliptic system is

$\Delta\lambda$ = - 0″.0232, $\Delta\beta$ = -0″. 0449

Due to slow motion of plane of ecliptic, longitudes and latitudes of fixed stars undergo changes. Annual rates are as follows -

$\Delta\lambda = \pi \cos(\lambda - \prod) \tan\beta$ $\qquad$ $\Delta\beta = -\pi \sin(\lambda - \prod)$

In 1950 AD	In 285 AD
$\prod$ = 0″.4708	0″.4824
λ=203° .4′, β = −2° 3′,	λ = 180° 0′ , β =− 1°56′
$\prod$ = 174°24′ (Trop)	159°12′
Hence $\Delta\lambda$ = - 0″. 0147	− 0″. 0151
$\Delta\ \beta$ = −0″ . 2283	−o″ . 1713

Average value of $\Delta\lambda$= -0″.0149, $\Delta\beta$ = - 0″.1998

Proper motion - 0″.0232, - 0″.00449

Total anual variation $\Delta\lambda$= -0″.0381, $\Delta\beta$ = −0″.2447

In 1665 years (1950-285 AD), total variation in longitude is - 63″.4, in latitude - 6′47″.4. Then nirayana longitude in 285.22 AD is 180°0′3″.1 and latitude is − 1°56′15″.4 Thus at epoch, its nirayana longitude was 180° approx.

Obliquity of Ecliptic to the equator -

$\in$ = 23°27′8″.26-46″. 845T - 0″.0060T^2 +0″.001837T^3

where T = Julian century of 36525 days from 1900.0AD E.T.

Rate of variation per century is

$$\frac{d\in}{dT} = -46''.845 - 0''.012\ T + 0''.00549\ T^2$$

When T^3 term has appreciable value, century figures need some correction. Then putting $T = Tc + \frac{t}{100}$

(Tc = completed centuries, t = extra years)

$\in$ = 23°27′8″.26-46″ 8457 Tc - 0″.006T_c^2 + 0.00183Tc^3

$$+ (-0''.00651\ Tc + 0''.00549\ T_c^2) \times \frac{t}{100}$$

Mean Longitude of Sun - (L) - Epoch is 1900 AD, Jan 0.0 ET. i.e. 0h0′4″.4 universal time, T = Julian centuries of 3 6525 ephemeris days from epoch. According to Newcomb, sun's mean tropical longitude, freed from aberrations is

L=279°12′13″.88 + 129602768″.13 T+1″.089 T^2

Motion in a century of 36525 ephemeris day is 12960 2768.″13 = 360° x 100+27″.6813 x 100
= + 46′8″.13

Daily motion is 0°59′8″.3304074

If Tc is completed century, t = remaining years, d = extra days,

$L = 277°12'13''.88 + (46'8''.13))\ Tc + (59'8''.330)$

$d + 1''.089\ Tc^2 + 2.178\ Tc \times \frac{t}{100}$

Sidereal or Nirayana Mean Sun (L') is

$L' = 256°44'30''.48 + 129597742''.38T - 0''.0225T^2$

$-0.''0001T^3$

Motion in a century = 360° x 100–22".5762 × 100 = -37'37".62

Daily motion = 35488". 192 80988 = 0°59'8".1928098

Sun's Perigee (=II) and Mean anomaly (=g))

Trop II = $281°13'14''.92 + 6189''.03\ T + 1''.63T^2 + 0''.012T^3$

Sid II' = $258°45'31''.52 + 1163''.\ 28\ T + 0.''52\ T^2 + 0''.012T^3$

Motion of II' per century = 19'23".28, per year = 11".63, per day = 0".0318

Mean anomaly of the earth or the sun

= g = L - II or L' - II'

$g = 357°58'58''.96 + 129596579.''\ 10\ T - 0''.541T^2 - 0''.012T^3$

Daily motion = 0°.9856002670 = 3548".160961

Hence the period = 365.2596413 ephemeris days.

Mean anomaly M in days is obtained by dividing g by daily motion and adding a constant of 5.37018 days

$M = 3.32376 + 36525\ T - 0.0001525T^2 - 0.0000034T^3$

36525 days = Period x 100 - 0.96413 days.

Mean Elongation of the Moon in days

Brown's Moon

= $263°50'45''.48+1732564379''.31T - 4''.08T^2 + 0''.0068T^3$

Newcomb's sun = $279° 12'13''.88 + 129602768''.13T + 1''.089T^2$

D=Moon - sun

= $344°38'31''.60+1602961611''.18T - 5''.169T^2 + 0''.0068 T^3$

Daily motion of D = 43886".697089

Period = 29.53058867 days

Converting into days

$D = 28.27079 + (\text{period} \times 1236+25.192399)\ T -- 0.0001178T^2 + 0.000000155T^3$

Venus and Sun - Mean Tropical Venus is

$341°57'57''.49 + 210669162''.88T + 1''.1148T^2$

V = Venus – Sun

= $62°45'43''.61+81066394.''75T + 0''.0258T^2$

Daily motion of V = 2219".4769, period = 583.921373 days. Converting into days

$V = 101.8004 + (\text{period} \times 62 + 321.87487)\ T + 0.0000116T^2$

Sun and Jupiter - Mean Tropical Jupiter is

$238°0'27''.69 + 10930687.15T + 1''.205T^2$

J = Sun - Jupiter = $41° 11'46''.19 + 118672080''.98T - 0''.116\ T^2$

Daily motion of J = 3249".064503

Period = 398.884048 days

Convereting into days, we get

J = 45.6458 + (Period x 91 + 226.55163) T 0.000036T^2

Nodes of Moon - Tropical longitude of the node is

Ω = 259°12′35″.11-6926911″.23T+7″.48T^2 + 0″.008T^3

- Ω = 100°47′24″.89+6926911″. 23T-7″.48T^2 - 0″.008T^3

Daily motion = 190″.63412, Period = 6798.36327 days converting into days expression for - Ω and adding a constant of 0.818 days, No. of days N since tropical longitude of moon's mean node was zero is

N = 1904.177+ (period x 5 + 2533.1835) T - 0.003924T^2 - 0.000042T^3

Julian day Number :

Pope Gregory introduced in 1582 AD year of 365.2425 days by omitting 10 days (Oct. 5 to Oct. 14)) from calender. Before that, there was leap year in every 4 years. In Gregorian calender, 97 leap years come in 400 years. Years divisible by 4 or centuries by 400 are leap yeears of 366 days. Normal year is of 365 days.

Julian days are numbered serially from Jan 1, 4713 B.C., Monday at Greenwitch mean noon.

Besselian Fictitious year begins when the tropical mean sun is 280°0′20″.5 or the same unaffected by aberrations is 280°0′0″. Notation like 1900.0 AD. is used for this year.

Let K = time from beginning of Besselian year upto beginning of calender year i.e. Jan 0, oh E.T. for common year or Jan 1, 0h E.T. for leap year.

Day from beginning of ficitious year = Day of year + K

$K = -0°48'6''.6 + 129\ 602768''.13T + 1''.089\ T^2$

Daily motion = 3548″ 3304074

Period of length of Tropical solar year = 365.24219878 days

K in days = - 0.8135 + (perod x 100 + 0.780122) T + $0.000307T^2$

Inequalities of long period in mean longitude

$\delta L = +6''.40 \sin (231°.19 + 20°.20T) + (1''.882 - 0''.016T) \times \sin (57°24 + 150°.27T) + 0''.266 \sin (31°.8 + 119°.0T) + 0''.202 \sin (315°.6 + 893°.3T)$

First term has a period of 1782.2 years (century variation of 20°.2) i.e. 1° in 3548 days.

$\delta L = +6''.40 \sin [(\text{AD year} - 755.5) \times 0.202)]$

Equation of Centre: e = eccentricity of orbit, g = mean anomaly (written as m in derivation of formula).

Equation of centre

$$(2e - \frac{e^3}{4}) \sin g + (\frac{5}{4}e^2 - \frac{11e^4}{24}) \sin 2g + \frac{13}{12}e^3$$

$$\sin 3g + \frac{103e^4}{96} \sin 4g$$

Here, e = 0.016,751, 04-0.000,041,80T-$0.000,000,126T^2$ = 0.016, 75104-0.000,041,80 (T+.00 $301T^2$)

Multiplying by 206264.8 we get

$e = 3455''.150 - 8''.621\ (T + 0.003T^2)$

So equation of centre = + 6910'.057 Sin g + 72'.338 sin 2 g + 1".054 sin 3 g + 0".018 sin 4g - 17".240 (T + $0.003T^2$) sin g – 0".361T sin2g.

Perturbations to Sun -

Action of Moon - Longitude of sun (or earth in opposite direction) is the longitude from centre of mass of earth and the moon. This is called geometric longitude. The origin is to be transferred to centre of earth.

Radius of earth is taken as unity, Π' and Π are horizontal parallaxes in seconds of arc of moon and sun respectively, β' are β are their latitudes. Distance of mass centre from earth centre in direction of moon

$$= \frac{206265}{82.30\,\Pi'} = \frac{2506.3}{\Pi'}$$

(Ratio of earth mass to moon mass is 81.30 adopted in 1968)

$$\Delta L = 2506.3 \times \frac{\Pi}{\Pi'} \operatorname{Cos}\beta' \sin (D\text{-}O)$$

$$\beta = 2506.3 \times \frac{\Pi}{\Pi'} \operatorname{Sin}\beta'$$

Substituting numerical values -

ΔL = + 6". 44 sin D - 0".42 sin (D-g')

Sun's latitude β = + 0".58 sin U, or + 6". 44 sin β' or roughly 0.11 x moon's latitude in seconds

U = Mean moon - lunar node

Δ log R = + 0.0000134 Cos D.

Action of other planets

Action is calculated in terms of Q = difference in heliocentric latitudes of the planet and earth.

Due to elliptical shape the deviation due to planets also depends on g and (W′-W) where

g = mean anonaly of earth (i.e. of sun)

W′ = Longitudes of the planet's perihelion, W = perihelion of earth

T is in 100 years from 1850 AD then

$K' = W'-W = 29°5'55''-18'40''T$ (Venus)

$K'' = W''-W = 232°56'11'' + 7'18''T$ (Mars)

$K''' = W''' - W = 271°33'16'' - 6'33''T$ (Jupiter)

Century variations of these quantities are very small, and they can be considered as constants for 1000 years or more

g = hehiocentric lat. of earth – W

g′ = hel. long. of the planet – W′ (e.g. for venus))

= planet - k′-W

= (Planet-earth) + (earth - W) - K′

= Q + g - K′

Perturbations due to venus (Approx Newcomb formula)

Pert = + 4.84 sin Q - 5.53 sin 2 Q-0.67 sin3Q - 0.21 sin 4Q - 0.12 sin (2Q+g) - 2.50 sin (g+12°-2Q)

- 1.56 sin (g+12°-3Q) + 0.14 sin (g+12°-4Q)

- 1.02 sin (2 g + 40°-3Q) - 0.15 sin (2 g+40°-4Q)

+0.12 sin (2g+40°–5Q) – 0.15 sin (3g+56°–5Q)

Corresponding formula given by Le-Verrier is

Pert = + 4.91 sin Q-5.61 sin 2 Q-0.67 sin 3Q - 0.21 sin 4 Q - 2.52 sin (g-2Q + W-90°) - 1.58 sin (g-3Q+w-90°)

For first approximation, calculation is based on Q only, then for M = g+5°.29 it is calculated

Perturbations due to Jupiter - Newcomb formula is

Pert = + 7.21 sin (Q-1°5′)-2.73 sin (2 Q - 0°15′)

- 0.16 sin (3 Q + 4°51′) + 2.60 sin (Q+g-84°46′)

- 1.61 sin (2 Q + g - 22°.6) - 0.56 sin (3 Q + g + 87°2)

- 0.16 sin (g-Q+20°.1) - 0.21 sin (3 Q + 2 g + 77°)

First three terms according to Le verrier are

+ 7.′20 sin (Q-1°5′) - 2.73 sin (2 Q-18′) - 0.16 sin (3Q + 5°)

These terms are tabulated for Q, then for Q and M.

Perturbations due to Mars - Newcomb formula is

Pert = + 2.04 sin (2Q+15′) + 0.27 sin (Q-0°.6)

- 1.77 sin (2 Q+g-36°16′) - 0.58 sin (4Q+2g+84°)

- 0.50 sin (4 Q + g - 47°) - 0.43 sin (3 Q + g - 47°.7)

Aberrations - Correction in longitude due to aberration of light in earth's atmosphere is

- 20″.50 - 0″.34 cos g

Nutation

Tropical longitude is calculated from mean equinox of the date. Correction due to nutation is to be made in tropical longitude, but, not necessary for nirayana longitudes.

Solar nutation = - 1″.27 sin 2L+0″.13 sin g – 0.05 sin (3L+79°)

Lunar nutation = - 17″.23 sin Ω + 0″.21 sin 2Ω

Principal term of the lunar nutation is slowly increasing at the rate of 0″.17 per thousand years.

In the obliquity of ecliptic,

Solar nutation = + 0.″55 cos2L + 0.02 cos (3L+79°)

Lunar nutation = + 9″.21 cosΩ - 0″.09 cos 2Ω

Here L and Ω are the tropical mean longitudes of the Sun and the lunar node respectively.

Radius Vector

Radius vector is expressed in terms of mean distance of earth from sun. Mean distance is expressed by Gauss formula based on Keplar's third law

$a^3n^2 = k^2 (1+m)$

where k is Gaussian gravitational constant = 3548″.187607

m = mass of earth and moon, taking sun mass as unity

n = observed sidereal mean daily motion of earth.

a = mean distance from sun to mass centre of earth and moon.

Value of k is based on sidereal period of 365.256898 days of earth considered as particle

without mass or of 365.256344 days with adopted value of mass.

With Newcomb's value of m = 1 ÷ 329390 and n = 3548″.19282, we get log a = 0.000,000,013.

Long term effect of attraction of inner planets is equivalent to an increase in mass of sun, to balance it, radius vector a increases. Observed daily motion n remains constant.

Elliptic term of radius vector is (equation 11)

$$R = a\ [1 + \frac{e^2}{2} - (e - \frac{3}{8}e^{3})\cos g - (\frac{1}{2}e^2 - \frac{1}{3}e^4)\cos 2g$$

$$- \frac{3}{8}e^3 \cos 3g - \frac{1}{3}e^4 \cos 4g\ ]$$

$$\text{and } \log R = \log a + \log(1 + \frac{e^2}{2}) - M\ [\ (\frac{e^2}{4} - \frac{5}{32}e^4)$$

$$+ (e - \frac{3}{8}e^3)\cos g + (\frac{3}{4}e^2 - \frac{11}{24}e^4)\cos 2g$$

$$+ \frac{17}{24}e^3 \cos 3g + \frac{71}{96}e^4 \cos 4g\ ]$$

where M is the modulus of common logarithm = 0.434294.

Taking value of e for 1900,

R = 1.000, 140,5 - 0.016, 749, 2 cos g - 0.000,140,3 cos 2 g

- 0.000,001, 8 cos 3 g - 0.000,000,7 T + 0.000,04,18 $(T+0.003T^2)$ cos g + 0.000,000, 7T Cos 2 g

Log R = 0.000,030,6 - 0.007,274, 1 cos g - 0.000,091, 4 cos g - 0.000,0015 cos 3 g - 0.000,000,15 T + 0.000,018,14 $(T + 0.003T^2)$ cos g + 0.000,000,46 T cos 2 g.

Terms free of T are value for 1900 A.D. Terms with T are secular variation.

Effect of planets on radius vector -

Due to venus = - 1″.12 cos Q + 3″.25 cos (2Q+7′) + 0″.50 cos (3 Q - 1°5) + 0″.18 cos (4Q-2°.4) + 0″.08 cos (5Q-3°)

Log = - 2″.36 cos Q + 6.84 cos (2Q+7′) + 1.05 cos (3Q-1°5) + 0″.38 cos (4Q-2°.4) + 0.16 cos (5 Q-3°)

Jupiter = + 3″.36 cos (Q-1°6′) - 1″.91 cos (2Q--13′) - –0.13 cos (3 Q + 4°31′)

Log = 7.07 cos (Q-1°6′) - 4.03 cos (2Q-13′)-0.28 cos (3Q+4°21′)

Due to Moon = + 6″.35 cos D or log = + 13.36 cos D.

Sun's semi diameter and horizontal parallax -

At unit distance, apparent semi diameter of sun = 961″.18 and horizontal parallax = 8″.794. At any distance, Semi – diamter = 961″.18 /R = 16′1″.18+16″.10 cos g+0″.27 cos 2 g

Parallax = 8″794/R = 8″.79+0″.15 cos g.

For calculation of eclipse, allowance of 1″.55 is made for irradiation. Then true semi diameter at unit distance is 959″.63

Reduction of Rt ascension and declination

λ = tropical longitude of the sun, α = right ascension,

δ = declination, ε = true obliquity of ecliptic to equator,

$$\sin \delta = \sin \lambda \ \sin \alpha \text{ and } \frac{d\delta}{d\varepsilon} = \sin \alpha$$

$$\tan \alpha = \tan \lambda . \cos \varepsilon$$

$$\text{or } \alpha = \lambda - (\tan^2 \frac{\varepsilon}{2} \sin 2\lambda - \frac{1}{2} \tan^4 \frac{\varepsilon}{2} \sin 4\lambda$$

$$+ 1/3 \tan^6 \frac{\varepsilon}{2} \sin 6\lambda)$$

$$\frac{d\alpha}{d\varepsilon} = -\frac{1}{2} \sin 2\alpha \tan \varepsilon = -0.2168 \sin 2\alpha$$

Sidereal Time - Sidereal time at any instant is defined to be west hour angle of the First point of Aries (Vernal equinoctical point) from the upper meridian of the place.

Sidereal time at mean noon (i.e. 12h local mean time) on any day is the right ascension of the fictitions mean sun, which is defined to be the tropical mean sun at moment as affected by mean aberration.

At mean midnight, sidereal time is 12h (i.e. 180°) + R.A. of fictitions mean sun for the moment.

Sidereal time at Greenwich mean midnight = $6h6^m\ 47^s,558 + 8640184^s.542\ T + 0^s.0929T^2$

where T is Julian centuries of 36525 days from 1900 AD, Jan 0, 0^h or E.T.

Motion in a century = $100^d + 0^h3\ m4^s.542$

Motion in a day = $3^m\ 56^s.5553605$

Equation of Time = Local apparent time - Local mean time

Local apparent noon = 12^h L.M.T - equation of time (E)

At 0^h E.T., Equation of time = Apparent sidereal time - Apparent R.A. of sun.

E = R.A. of mean sun - R.A. of true sun

Both are affected by aberration and nutation. True sun is also affected by perturbation. Omitting aberration and nutation from both sides, only perturbation λ remains in true sun.

True Sun = L + equation of centre

$$E = L - (L + \text{Eqn of c}) + \tan^2 \frac{\varepsilon}{2} \sin 2\lambda$$

$$- \frac{1}{2} \tan^4 \frac{\varepsilon}{2} \sin 4\lambda + \frac{1}{3} \tan^6 \frac{\varepsilon}{2} \sin 6\lambda$$

- effect of perturbation in longitude

Equation of centre in seconds of time is

+ (460.67 - 1.149T) sin g + 4.82 sin 2g+0.07 sin 3g

Value in arc is, $\tan^2 \frac{\varepsilon}{2}$ = 0.0430836-0.0000491 T

In seconds of time, $\tan^2 \frac{\varepsilon}{2}$ = 592.44–0.675 T

So equation of time (in seconds of time) is

= - (460.67-1.149 T) sin g - 4.82 sin 2g - 0.07 sin 3g + (592.44 - 0.675 T) sin 2λ - 12.76 sin 4λ + 0.36 sin 6λ - $\frac{1}{15}$ perturbation in longitude.

4. Equation for other planets

Basic constants of Mercury

Mean longitude, L for 3200 BC, Jan 0.5 epoch is

$L = 49°.677936 + 538106654''.8\ T - 1''.084T^2$

L for 1900 AD epoch is 173°.303523 (51 centuries - 13 days)

Mean anomaly, g for 3200 BC is

$g = 53°.107661 + 538101055.04T - 0''.024T^2$

g for 1900 AD is 98°.169610

Argument of latitude, U is for 3200 BC

$U = 62°.977228 + 538102388''.05T - 0''.458\ T^2$

U for 1900 AD is 136°.609863

Constants for venus -

Mean longitude L for 3200 BC (-51 centuries + 13 days)

$= 285°.18561 + 210669162''.88T + 1''.1148T^2$

L for 1900 AD is 341°.97032

Mean anomaly g for 3200 BC is

$g = 223°.83111 + 210664093.95\ T + 4''.63\ T^2$

g for 1900 AD = 214°.34622

Argument of latitude U for 3200 B.C. is

$U = 252°.31206 + 210665923''.42T + 0''.3612T^2$

U for 1900 AD is 266.59425

Constants for Mars - for 3200 BC

$L = 33°.370172 + 68910117''.19\ T - 1''.1184T^2$

$g = 152°99708 + 68903493.19T - 0''.651T^2 - 0''.0192T^3$

$U = 23^\circ.923117 + 68907340.''7\ T - 1''.1234T^2 - 00''.00192T^3$

For 1900 AD epoch constants are

$L = 292^\circ 416147, \quad g = 318^\circ.\ 387964$

$U = 242^\circ.\ 918470^\circ$

Constants for Jupiter - For 1900 AD are

$L = 238^\circ.0496 + 10930687''.148T + 1''.20486T^2 - 0''.005936T^3$

$U = 138^\circ.60587 + 10927049''.24T + 0''.06314T^2 + 0''.024704T^3$

$g = 225^\circ.32833 + 10924891''.286T + 2''.59772T^2 + 0''.06314T^3$

Long period inequality in longitude L is

$E = (1186''.618572 - 0''.0347004\ t + 0''.000033372t^2)\ \mathrm{Sin}C - 12''.013596\ \sin 2C$

where t is number of years from 1800 A.D.

$C = 95^\circ.8814 + 0^\circ.38633184\ t + 0^\circ.0000351\ t^2$

Constants for Saturn - for 3200 BC are

$L = 147^\circ.9623 + 4404635''.581T - 1''.16835T^2 - 0''.021T^3$

$g = 156^\circ.74269 + 4397585''.284T - 1''.80655T^2 - 0''0376T^3$

$U = 79^\circ.704558 + 4401492''.0785T - 1''.7162T^2 - 0''.0019T^3$

Constants for 1900 AD are

$L = 260^\circ.46036,\ g = 172^\circ.74219,\ U = 152^\circ.43062$

Notes : (1) Newcomb's formula has been corrected by Rossi for Mars. There are other perturbations for planets also which have not been written.

(2) Moon's motion in detail will be discussed in the next chapter.

(3) From these constants, equations of centre and radius vector can be obtained.

(4) Value of eccentricity also changes. For the present century, values given in chart can be taken as constants.

(5) From these equations, constants are tabulated for centuries. Then by ratio, they are fixed for specific years.

(6) Equations of centre and radius vector give true positions from the constants of year.

(7) Longitudes and latitudes are reduced to ediptic.

(8) Heliocentric longitude and latitude are converted to geocentric values.

Let S he longitude of Sun, R its radius vector

H is heliocentric longitude of planet, b its latitude

r is radius vector from sun of planet,

x is geocentric longitude, y is latitude

then $\tan P = \frac{r \cos b \sin (H - S)}{R + r \cos b \cos (H - s)}$, $x = S + P$

$$\tan y = \frac{r \sin b \sin P}{R + r \cos b \sin (H - S)}$$

5. **References** - (1) Any text book on modern coordinate geometry, Trigonometry can be referred for these formula.

(2) Dynamics of planetary motion has been explained in Dynamics of Rigid bodies by A.G. Webster.

(3) Derivation of formulas can be referred in books on spherical trigonometry. Inportant books are (1) Spherical Trigonometry by Gorakh Prasad, Pothishala, Allahabad - 2, (2) A hand book of Practical Astronomy by R.V. Vaidya, Payal Prakashan, Nagpur (3) Astronomy by G.V. Ramachandran, Tiruchirpalli (4) Practical Astronomy by Schroeder, published by Werner Lauries, London (5) Astronomy by R.H. Baker - D van Nostrand, East West Edition (6) Celestial Dynamics by W. Smart - Longman, Green (4) Astronomical charts were published by Simon Newcomb in 1899 and 1906 but they are out of print. Nautical Almancs published by govt. of India, specially Ist edition of 1958 can be referred. Tables of sun have been published by Sri N.C. Lahari from Calcutta in 1993 (revised edition.)

Translation of the text

Verse 1 - Scope and definition - The position in which graha in seen from earth is to be found by calculation. This process is called Sphuṭīkaraṇa of graha or making is sphuṭa. The graha give results according to the position they are seen. Hence method for making a graha sphuṭa is being explained.

Verses 2-5 - Reasons of planetary motion - Celestial sphere containing graha and nakśatra revolves around earth once in a day from east to west due to attraction of a wind (Pavana) named Pravaha rotating round earth. This is called daily motion.

Graha move in opposite direction from west to east compared to stars (or nakśatras) with slow speed according to their own enregy. This is called natural speed of a planet (svābhāvika gati). There

are deviations from this average speed of planets under influence of ucca (śīghra and manda).

(According to Sūrya siddhānta) Invisible forms of kāla like śīghrocca, mandocca and pāta residing in celestial sphere are reasons of planetary motion.

Notes (1) Daily moiton is due to rotation of earth around its axis. Earth appears fixed to us and stars rotate in opposite relative motion. Reason of earth's rotation is due to initial conditions of its formation, now it continues due to inertia. That inertia is assumed to be 'Pravaha', an imaginary force in vacuum. This is similar to assumption of ether for propogation of light in vacuum.

(2) True position of a planet is closer to mandocca (farthest point in elliptical orbit) compared to its mean position. Due to that reason attraction by mondocca in seen. Similar is case with śīghrocca.

(3) Pāta is point of intersection of planetary orbit with ecliptic due to its inclination. Hence, pāta appears to repulse a graha away from ecliptic.

Verses 6-9 - Nature of motion - Mean sun moves around earth between nakśatra and orbit of grahas. Other planets like mars are in orbit round mean sun and along with it, they also revolve round earth. Hence (mean) sun is called attractor of all. From dainika gati of Mañgala, Bṛhaspati and śani - dainika gati of ravi is more and they are attracted by ravi. Hence ravi is called śīghrocca of these planets.

Compared to Budha and śukra, speed of ravi is slower and it always remains between them. Hence Budha and Śukra are called their śīghrocca.

Notes (1) Outer planets are almost in same direction from earth as from sun. Minor correction is due to position of sun from earth.

(2) Inner planets are within a small distance from sun which is their average position. First correction for their true position, is due to their own motion. Hence they are own śīghrocca.

Verses 10-16 - Slow, fast and reverse motion - Planets in successively farther orbits from sun are - budha, śukra, maṅgala, bṛhaspati and śani. Hence their angular speed appears progressively slower from earth (if linear speed in orbit is assumed to be same).

Like ravi, moon also is rotating round the earth, but from very close distance. Hence angular speed of moon is largest, though its linear speed (in yojanas etc) is small.

Budha and śukra are close to ravi, compared to earth. Therefore, they are seen with ravi after 12 rāśi (full rotation), as well as, after 6 rāśi (half rotation).

Maṅgala, bṛhaspati and śani are farther from ravi - compared to earth. Hence, they appear together with ravi at 12 rāśi difference and in opposite direection at 6 rāśi difference.

When earth is in one direction of ravi, and star planets (tārā graha) maṅgala, budha, guru, śukra and śani are in opposite direction - then the graha appears to move in forward direction (mārgī gāti)

If a tārā graha and earth are in the same direction of ravi, then the graha appears to move in reverse direction (vakrī gati) due to difference between mean and śīghra speeds. (Figures 7, 8)

Notes (1) Explaination of forward and reverse speeds - Figure 7 indicates relative speeds of earth

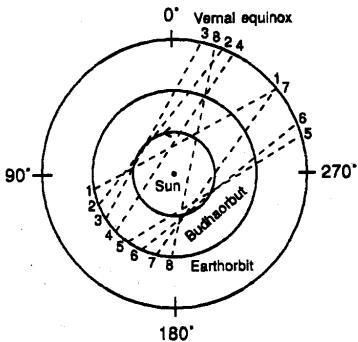



Figure 7 - Foreward and reverse speeds of inner planet (Budha)

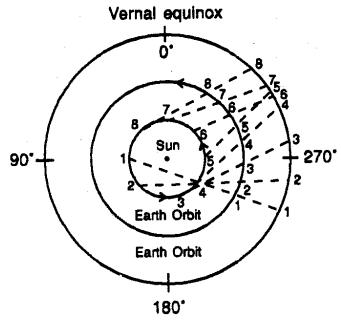



Figure 8 - Foreward and reverse speeds of outer planet (Mars)

and an inner planet Budha. Figure 8 compares earth's motion with an outer planet mars. Numbers 1, 2 -----8 indicate successive simultaneous positions of the planets in their orbits. Ecliptic is a much bigger circle whose 0° stars' from vernal equinox. All the circles are in same plane and the movements are in positive direction (anticlock wise). Line of

sight from position 1 of earth to simultaneous position of planet 1 on a point on ecliptic is marked 1. Point 1, 2 --- 8 on ecliptic are apparent directions of the planets seen in ecliptic after regular intervals. Speeds in inner orbits are faster.

Figure 7 shows that 1,2,3 positions indicate forward movement of Budha. After point 3, budha comes on same side of sun as earth and moves backwards in positions 4 and 5. Between 5 and 6, it is almost stationary before moving forward again to positions 7 and 8.

Similarly, positions 1 to 5 in figure 8 indicate forward motion of mars. At 6, it moves back wards when earth and mars both are on same side of sun. Actually ecliptic circle is at almost infinite distance and position 7 also is in backward direction from 6. But due to small construction of ecliptic it looks forward. From position 8, planet again moves forward.

(2) Fast and slow speeds - Angular speed is arc length divided by radius in unit time. Hence for same arc length in unit time, angular speed will be less for large radius. Thus farther planets will look slower. In siddhānta text, this was considered only reason for slow speed. But linear speed also becomes slower as explained by Keplar's law (to balance lesser gravitational pull).

Verses 17-18 - Five tārā graha revole round the sun at constant distance in east direction. They are attracted by their mandocca and śīghrocca ravi. They always remain in bha-cakra (ecliptic circle).

Earth is in centre of bha-cakra, ravi is at the centre of five tārā graha. Hence at full circle (12

rāśi) or half circle difference, earth is in same line as sun and the planet.

Verse 19-22 : At 3 rāśis after cakra or cakrārdha (90° after 360° or 180° - i.e. 90° or 270°) i.e. at the end of odd quadrant, difference between planet's direction and sun's direction is maximum. Hence śīghra paridhi is different at the end of odd and even quadrants (0°, 360°).

Circle of nakśatras (bhakakśā) is 360 times away from its centre earth, compared to distance of sun's orbit from earth.

Division by this ratio (hāra) 360 into degrees of 1 revolution (360°) we get 1° which is difference between śīghra paridhi at the end of odd and even quadrants.

I (author) wil explain the difference between nīca and ucca paridhi. This can be seen direectly by observation, so presumption is not necessary.

Explainations (1) Difference in śīghra paridhi at end of odd and even quadrants is due to elliptical shape of planetary orbits. The difference depends on eccentricity of the orbit. It will be explained later on while explaining motion on śighra and manda paridhis.

(2) The assumption is that difference in points of observation causes difference in paridhis. At 90° or 270° śīghra kendra, difference is maximum due to śīghra motion. Difference in heliocentric and geocentric position will be angle made by radius of sun orbit at distance of star circle. Difference of 1° will be observed from 57.3 times the distance. This assumption will be correct if stars orbit is considered 60 times away compared to earth's orbit.

This figure has been accepted by Āryabhaṭa, Sūrya siddhānta and all others. 360 times the distance will give less than 1°/6 difference. Reasoning is wrong in both ways - about reason of difference in śīghraparidhi and about the angle of difference.

(3) Siddhānta darpaṇa has taken sun's distance about 11 times the figure accepted in classical siddhāntas (based on 72,000 yojana diameter in Atharva veda is stead of 6,500 yojana in siddhānta). He has increased distance of stars further 6 times. Even after increase of about 66 times, it is still highly under estimated. Even the nearest star (4.4 light years) is 2,80,000 times the distance of sun. Rohiṇī at 14 lakh times and svātī about 87 lakh times the distance are other nearest stars.

Verses 23-27 - Attraction of ucca - Planets starting from ravi (all) are attracted by gods named their mandocca by chord of air (invisible force of attraction to an imaginary point mandocca). Hence true planets (spaṣṭa graha) are always deviated towards mandocca from their mean position (madhyama graha).

Planets with slow motion of own and attracted by their mandocca, also move under influence of pravaha from east to west (due to daily motion of earth).

(From Sūrya siddhānta) - Planets being always (day and night) under attraction of their ucca, move in different ways - sometimes east or west. When ucca is in east semicircle of the planet, ucca pulls the planet towards east. When ucca is in western semi circle, it pulls towards west. Direction of ucca on a circle is always same, but it is called east in

one semi circle and west in the other. When the planets move towards east under attraction of ucca, the deviation is positive and in west it is negative. (Normal motion of planets relative to stars is towards east hence deviation in same east direction is added and in opposite direction, substracted.).

Explaination - (1) Actual orbits of ravi and candra are elliptical round the earth (relative motion of ravi). In such an orbit earth is not at centre but on a focus. Thus centre of planetary motion is deviated towards farther end of major axis called mandocca. It is called so because at this position candra is farthest and hence slowest (ucca and manda). To a first approximation mean planet moves in a circle round earth at the focus. Next approximation assumes motion in an eccentric circle with centre at centre of the elliptic orbit.All the points of this circle are thus deviated from corresponding position of madhya graha towards mandocca. Hence mandocca appears to attract the planet. More accurately, mandocca doesn't attract because in such case, speed of the planet will be increasing in that direction. It should be maximum at mandocca. But it is only a deviation or displacement towards mandocca. This will be explained mathematically while computing the corrections.

(2) If we rotate along a circle, after reaching mandocca, the movement will be away from it upto 180° difference. In remaining half it will be towards mandocca. Since attraction is always towards mandocca, 0° to 180° circle after it is considered negative or western deviation.

Verses 28-33 - Vikśepa and pāta - Orbit of planets starting from moon (except ravi) are at an angle with the krānti vṛtta (i.e. apparent orbit of ravi). It meets kranti vṛtta at the points, where the circles bisect each other (being great circles of a sphere). Half of the orbit is north of krānti vṛtta (upward direction of right hand screw rotating in orbit direction). Other half is south. At pāta, the planets are on krānti vṛtta; Away from pāta they are deflected towards north or south slight from Krānti vṛtta. Hence pāta is considered reason of north or south deflection called vikśepa (or śara). Śara is distance of perpendicular from graha on Kranti vṛtta, measured in kalā or minutes of angle). Foot of perpendicular is manḍa sphuta graha.

(From Sūrya siddhānta) - When planet is ahead of its northern pāta by 0° to 180°, it is deflected northwards. (Hence this pāta is called northern pāta or pāta, in short). Then pāta is behind the graha and called in west. Pāta in east (or before the graha) deflects it towards south.

Budha and śukra revolve with sun (being in inner orbit). Their śīghrocca is the planet itself. So pāta east from śīghrocca causes south śara. In other half of orbit it is north śara.

When pāta is with graha or 180° away, graha is on one of the pāta points and hence on the krānti vṛtta. There is no śara in that position. When difference between planet and pāta is 90° or 270° (end of odd quadrants), śara is maximum (parama).

Mean parama śara of planets is given below (compard with modern values given in introduction)

Planet	Śara in Kalā	Degree	Modern Value
Candra	309	5°9′	5°8′42″
Maṅgala	111	1°51′	1°51′0″
Budha	164	2°44′	7°0′14″
Bṛhaspati	78	1°18′	1°18′21″
Śukra	148	2°28′	3°23′39″
Śani	149	2°29′	2°29′25″

Notes (1) Siddhānta darpaṇa figures are an improvement over previous siddhanta books according to comparative chart given below. Except two inner planets, this compares well with modern values.

(2) Pāta according to other texts

	Sūrya Siddhānta	Brahma sphuṭa Siddhānta & Siddhānta Śiromaṇi	Mahā siddhānta	Ptolemy
Candra	4°30′	4°30′	4°30′	5°0′
Mangala	1°30′	1°50′	1°46′	1°0′
Budha	2°0′	2°32′	2°18′	7°0′
Guru	1°0′	1°16′	1°14′	1°30′
Sukra	2°0′	2°16′	2°10′	3°30′
Śani	2°0′	2°10′	2°10′	2°30′

(3) While figures of siddhānta darpaṇa for moon and outer planets are very accurate, it looks wrong for budha and śukra compared to modern figures. Reason is that the modern figures are heliocentric whereas these are geocentric.

In this figure 9, E is earth centre, S is sun centre and ESA is Krānti Vṛttā, cut by plane perpendicular to budha orbit. This plane cuts

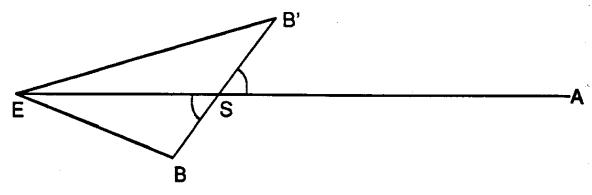



Figure 9 - Pata of inner planet

budha orbit at points B and B'. As seen from sun, budha orbit makes angle BSE or B'SA with kranti vṛtta. From sun this is parama śara. As seen from earth, parama śara is ∠B'ES or ∠BES (almost equal because budha orbit is small).

If ES is taken as 1, then mean radius of budha orbit SB = SB' = 0.3871.

$$\frac{\sin \angle BES}{BS} = \frac{\sin \angle BSE}{BE}$$

$$\text{or } \sin \angle BES = \frac{BS}{BE} \times \sin \angle BSE$$

$$= \frac{0.3871}{1} \times \sin 7^0\ o'\ 14''$$

= .3871 × 0 .1219= 0.0 472

or ∠ B E S = 2° 42'

which is very close to value for budha in this book (2°44')

Relative distance of Śukra is 0.7233 hence its angle θ is given by (as seen from earth)

Sin θ = .7233 x sin 3°23'37"

= .7233 x .0592 = .0428

or θ = 2°27' which is only 1' less than siddhānta darpaṇa.

Verses 34-41 - Types of planetary motion - Motion of planets as seen from earth's surface is called sphuṭagati. Sphuṭa gati is of three types -

forward (prāk or mārgī)) gati, reverse (vakrī) gati and zero motion (śūnya gati).

Forward motion is of five types, reverse motion of two types and one zero speed - these are eight types of motions of planets starting from maṅgala (Ravi and candra have no reverse motion).

From Sūrya siddhānta - Eight gatis are named 1. Vakra 2. Aṇuvakra, 3. Vikalā, 4. manda, 5. mandatara 6. sama, 7. śīghra and 8. Atiśīghra.

Reverse motion starts reducing in later half, it is called vakra. At the start of reverse motion it is increasing. Then it is called anuvakra.

When forward motion is less than mean speed and is still decreasing, it is called mandatara, when it is increasing, it is called manda.

Spaṣṭa gati equal to mean speed is called sama. Spaṣṭa gati more than mean speed and still increasing is called śīghratara. When decreasing it is called śīghra.

Ravi and candra are affected only by mandocca (not śīghrocca). They have only five types of speeds - 1. manda, 2. mandatara, 3. sama, 4. śīghra and 5. śīghratara. Their meanings are explained above

Verse 42 - Sphuṭa method - (From sūrya siddhānta) Now, I tell will respect, the method of making a graha sphuṭa by calculation, where mean planets arrive due to 8 types of speeds at the observed place. (dṛk-tulyatā = calculation equal to observation).

Verses 43-46 - Explaination of arc and its sine - Now, I tell the method of calculating sine and

arc, which is used in many sciences and by knowing which, people get the title of ācārya (doctorate).

As a cloth is interspersed with threads, a gola (sphere or its circles) is also mixed up with sines and versed sines. (sine can be found between any two points of a sphere or circle and hence they are infinite in number).

To find the sine (jyā) of a radius inclined with starting radius at 0°, we make a jyā of same arc in opposite direction. Graha is on top of jyārddha (end of radius vector) with which calculation is made. It is also called jyā in short. Half part of a circle or full revolution (bhacakra) looks like a bow (cāpa). The bisecting line of circle passes through its centre and is called diameter (vyāsa).

Verses 47-54 - Method of Calculating sines - For 3 rāśi (90°)jyā, Koṭijyā have extreme values. Jyā of 3 rāśi passes through centre and is equal to radius. Then it has greatest value, Koṭijyā is distance of this jyā from centre and is 0 here (for 90°).

96th part of a circumference is very small and almost a straight line. Hence it is almost equal to its jyā. Thus 1/8 part of a rāśi lipta (1800) i.e. 1/96 of circle is 225 and is equal to first jyā.

First jyā is first khaṇḍāntara (difference) (i.e. Jyā of 225′ - jyā of 0′). To find the second Khaṇḍāntara (2nd jyā - Ist jyā), Ist jyā is divided by itself and result (1) is deducted from Ist jyā $\left(225 - \frac{225}{225} = 224\right)$. Result is 2nd khaṇḍāntara.

Add 2nd khaṇḍāntara in Ist jyā to get the 2nd jyā (i.e. sine of 2x225′ arc). 2nd jyā is again

divided by Ist khaṇḍāntara. If remainder is more than half of 225 then it is ommitted. Quotient deducted from 2nd khaṇḍāntara gives 3rd khaṇḍāntara.

3rd Khaṇḍāntara addeed to 2nd jyā, we get the 3rd jyā. Similarly jyā piṇḍa (quantity) is divided by Ist jyā and substracted from its khaṇḍāntara to give next khaṇḍāntara. This way we get the jyā of Ist to 24th jyā piṇḍa in liptā. In dividing 6,7,12,15,17,20,21 jyā piṇḍa, reemainder is more than half of divider 225. But we still omit it without adding 1 to quotient because Brahmā had told so to Nārada.

Notes : (1) Nārada Purāṇa also gives a complete summary of astronomy and astrology. In the chapter describing calculation of sines, no such explaination from Brahmā has been given as stated here. However, the stated values have been given which means the same thing. There is no dialogue from Brahmā in the chapter, but he is considered original source of the knowledge.

(2) Increase in sines is proportional to its differential coefficient thus $\frac{d}{dx}$ (sin x) = cos x is proportional to Ist difference (khaṇḍāntara) Change in difference itself; i.e. 2nd diff is proportional to 2nd differential coefficient. Thus 2nd difference =

$$\frac{d^2}{dx^2}(\sin x) = \frac{d(\cos x)}{dx} = -\sin x. \text{ (proportional)}$$

Let 1 part be P = 225′. Hence sin x = Sin nP, n=no. of parts.

Here jyā = R sin x where R is radius equal to 3438 kalā.

First difference = Δ_1 (or Ist khandantara).

$\Delta_1 = \frac{d}{dx}$ (R sin x)$_o$. Sin P = R (Cos x)$_o$ Sin P
= R sinP.

Second difference S = $\frac{d}{dx}$ (R cos x) = $-\frac{R\,\delta\, x\, .\sin x}{R}$

Negative sign means that the first differences (khaṇḍantaras) are decreasing with increasing angle and 2nd differences are proportional to jyā (R sin x). It is to be divided by R to get modern sine.

$$\delta\frac{(R\cos x\, .\, \delta\, x)}{R} = \frac{-R\ \text{sins}\ x\, .\, \delta x^2}{R^2}$$

At x = 90^0 it is equal $\frac{\delta\, x^2}{R} = \frac{225 \times 225}{3438}$

= 14′43″30‴. This has been explained by Sri Ranganātha in his ṭikā on Sūrya siddhānta called Gūḍhārtha - Prakāśikā. But he has taken this as 3438/225 = 15′16″48‴ by mistake. Even this is approximate and correct value is 14′47″

(3) Sri Bāpūdeva Śāstrī has given the following proof of the formula in his English translation of Sūrya siddhānta

Δ_1 = Sin P – Sin 0°

Δ_2 == Sin 2 P - Sin P

Δ_3 = Sin 3 P - Sin 2 P

Δn = - Sin n P - Sin (n-1) P

$\Delta n+1$ = Sin (n+1) P - Sin n P

Then $\Delta_1 - \Delta_2$ = 2 sin P - sin 2 P

== 2 sinP - 2 sin P cos P

= 2Sin P (1 - cos P) = 2 Ṡin P. ver sin P

(versed sine = 1-cosine = utkrama jyā.)

$\Delta_2 - \Delta_3$ = 2 Sin 2 P - Sin P - Sin 3 P

$= 2 \sin 2P - \sin P - (3 \sin P - 4 \sin^3 P)$

$= 2 \sin 2P - 4 \sin P + 4 \sin^3 P$

$= 2 \sin 2P - 4 \sin P (1 - \sin^2 P)$

$= 2 \sin 2P - 4 \sin P . \cos^2 P$

$= 2 \sin 2P - (2 \sin P \cos P) 2 \cos P$

$= 2 \sin 2P (1 - \cos P)$

$= 2 \sin 2P . \text{versin } P$

$\Delta_3 - \Delta_4 = 2 \sin 3P - \sin 2P - \sin 4P$

$= 2 \sin 3P - 2 \sin 3P . \cos P$

$= 2 \sin 3P (1 - \cos P) = 2 \sin 3P \text{ versin} P$

$\Delta n - \Delta_{n+1} = 2 \sin nP - \sin (n-1)P - \sin (n+1)P$

$= 2 \sin nP - 2 \sin nP \cos P$

$= 2 \sin nP (1 - \cos P) = 2 \sin nP \text{ versin} P.$

Adding the above equations, we get

$\Delta_1 - \Delta_{n+1} = 2 \text{ versin } P (\sin P + \sin 2P + \sin 3P + \text{----} \sin nP)$

But $\Delta_1 - \Delta_{n+1} = \sin P + \sin nP - \sin (n+1) P$

Hence, $\sin P + \sin nP - \sin (n+1) P =$

$= 2 \text{ ver sin } P (\sin P + \sin 2P + \text{- - - -} + \sin nP)$

or $\sin (n+1) P = \sin nP + \sin P - 2 \text{ versin } P (\sin P + \sin 2P + \text{---} \sin nP))$

Here $P = 3°45' = 225'$

$\therefore 2 \text{ versin } P = 2 \text{ ver sin } 225' = 2 (1 - \cos 225')$

$2 (1 - 0.9978) = 2 \times 0.0022 = \frac{44}{10000} = \frac{1}{227} = \frac{1}{225}$

approx.

Thus $\sin (n+1) P = \sin nP + \sin P - 1/225 \times (\sin P + \sin 2P + \text{--} \sin nP)$

This is the formula for finding (n+1)th sin from nth sin i..e sin np, First khaṇḍātara is added and sum of pervious sines divided by (225) is substracted.

(4) Bhāskara II has explained that the sines were found by constructing regular polygons of increasing number of sides in a circle.

Āryabhaṭa has indicated the geometrical method for finding sines for 12 divisions of a right angle (7°30′ each) in a circle of radius R = 3438′. (Method is explained by Prof. Kripā Śaṅkara Śukla).

Let figure 10 represent a circle of radius R = 3438′. Divide the quadrant into two at T (45°) each. Trisect TA into TB, BR, RA (15° each), RA into

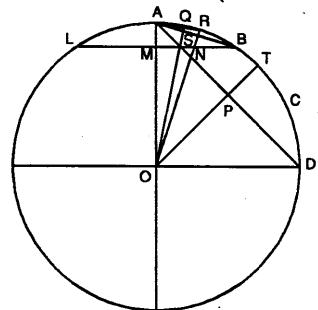



Figure 10 - Geometrical method of sine table

two (RQ, QA, 7-1/2° each). Mark off AL = 30°. Join LB. This is equal to R and denotes chord 60°.

$$R \sin 30° = \frac{R}{2} = 1719'$$

This is the 4th sine, in the $7\frac{1°}{2}$ table to be computed New, from right angled Δ OMB,

$$OM = \sqrt{R^2 - (R/2)^2} = \frac{\sqrt{3}}{2}R = 2978$$

This is R sin 60°, i.e. the eighth R sine

Now from rt angle ΔAMB

$$AB = \sqrt{(R \sin 30^0)^2 + (R \text{ vers } 30^0)^2}$$

$$= \sqrt{(1719)^2 + (460)^2} = 1780$$

This is chord 30°. Half of this i.e. AN, is R sin 15°

Thus R sin 15° = 890′

This is second R sine

Now from rt angle ΔANO

$$ON = \sqrt{AO^2 - AN^2)} = \sqrt{R^2 - (R \sin 15^\circ)^2} = 3321$$

This is R sin 75° = the tenth R sine

Now in rt ΔANR,, where R is mid-point of arc AB, we have

AR

$$= \sqrt{AN^2 + NR^2} = \sqrt{(R \sin 15^\circ)^2 + (r \text{ vers} 15^0)^2}$$

$$\sqrt{890^2 + 117^2} = 898'$$

This is chord 15°. Half of this i.e. AS is R sin 7°30′

Thus, R sin 7°30′ = 449′

This is the first R sine

Now, in rt Δ ASO, OS

$$= \sqrt{R^2 - (R \sin 7^0 30)^2} = 3409'$$

This is 11th R sine for angle 82°30′

Now, R vers 75° = R-R sin 15°, so that

$$\text{chord } 75^\circ = \sqrt{(R \sin 75^o)^2 + (R\text{vers}75^o)^2} = 4186'$$

Half of this 2093′ is R sin 37°30′ (fifth R sine).

Now, R sin 52°30′ = $\sqrt{R^2 - (R \sin 37^o \ 30)^2}$ =2728 (7th R sine)

In semi square AOD, OA = OD=R

so AD = $\sqrt{2}$ R = 4862′

This is chord 90°. Half of this 2431° (i.e. AP) = R sin 45°

In Δ APT, AT = $\sqrt{(R \sin 45^o)^2 + (r \text{ vers } 45^o)^2}$ = 2630′

This is chord 45°. Half of this is R sin 22°30′ = 1315′

(3rd R sine))

R sin 67°30′ = $\sqrt{R^2 - (R \sin 22^o \ 30)^2}$ = 3177′ (ninth R sine).

These are 12 R sines. By finding chord of 7°30′ arc we can find R sines of 3°45′ intervals also.

(5) More accurate method is to calculate sine by infinite convergent series.

$$\text{Sin } \theta = \theta - \frac{\theta^3}{\angle 3} + \frac{\theta^5}{\angle 5} + \ldots\ldots$$

where θ is expresseed in radians (arc/radius) and is betweeen 0° and 90°.

verses 55-66 -- All the verses are quoted from Sūrya siddhānta.

Verses 55 - 60 – These tell the values of 24 R sines at intervals of 3°45′ in kalās. Next verses give values of utkrama jyā = R (1-cos θ)

No.	Arc	R sines	Modern values	Diffe rences	Vers	Diff.	Modern value radius=1
1	225'	225'	224.856	225	7	7	.0022
2	450'	449'	448.749	224	29	22	.0086
3	675'	671'	670.720	222	66	37	.0192
4	900'	890'	889.820	219	117	51	.0341
5	1125'	1105'	1105.109	215	182	65	.05[illegible]1

6	1350'	1315'	1315.666	210	261	79	.0761
7	1575'	1520'	1520.589	205	354	93	.1031
8	1800'	1719'	1719.000	199	460	106	.1340
9	2025'	1910'	1910.050	191	579	119	.1685
10	2250'	2093'	2092.922	183	710	131	.2066
11	2475'	2267'	2266.831	174	853	143	.2481
12	2700'	2431'	2431.033	164	1007	154	.2929
13	2925'	2585'	2584.825	154	1171	164	.3406
14	3150'	2728'	2727.549	143	1345	174	.3912
15	3375'	2859'	2858.592	131	1528	183	.4445
16	3600'	2978'	2977.395	119	1719	191	.5000
17	3825'	3084'	3083.448	106	1918	199	.5577
18	4050'	3177'	3176.298	93	2123	205	.6173
19	4275'	3256'	3255.546	79	2333	210	.6786
20	4500'	3321'	3320.853	65	2548	215	.7412
21	4725'	3372'	3371.940	51	2767	219	.8049
22	4950'	3409'	3408.588	37	2989	222	.8695
23	5175'	3431'	3430.639	22	3213	224	.9346
24	5400'	3438	3438.000	7	3438	225	1.0000

Notes (1) Difference for versed sines are in opposite order and they need not be calculated. From them versed sines are calculated.

(2) Modern values of sin, cos and other ratios are calculated for radius 1. Hence, for calculating Indian sines they are to be multiplied by radius.

(3) Mādhava method for calculation upto 9 decimal places - This has been quoted by Nīlakaṇṭha in his commentary on Āryabhaṭīya. His sentences indicating calculation parameters have been quoted by Śaṅkara in his commentary on Tantra saṅgraha by Nīlakaṇṭha. Original book of

Mādhava is not available. He must have used infinite series and then formed the simplified rules expressed by verses in 'Kaṭapayādi' form.

Method for sines - Place the expressions 0'0"44"', 0'33"6"', 16'5"41"', 273'57"47"', and 2220'39"40"' - five numbers from below upwards. Multiply the lowest by the square of the chosen arc and divide by R^2 (i.e. 2,91,60,000 = 5400^2). Substract the quotient from expression just above. Continue this operation through all the expressions above. The remainder got at last operation is to be multiplied by the cube of the chosen arc and divided by R^3 (i.e., 157,46,40,00,000). Substract the quotient from the chosen arc to get its R sine.

Method for versed sines - Place the six expressions - 0'0"6"',0'5'12", 3'9"37"',71'43"24"', 872'3"5"' and 4241'9"0"' from below upwards. Multiply the lowest by the square of the chosen arc and divide by R^2. Continue the operation through all the opeations above. The last quotient will be the versed sine of the chosen arc. This formula is based on series for sin upto term θ^{11}

Results

No.	Arc.	R sine	sine in decimal	Modern Value
1	225	224'50"22"'	.06540	.06540
2	450	448'42"58"'	.13053	.13053
3	675	670'40"11"'	.19509	.19509
4	900	889'45"15"'	.25882	.25882
5	1125	1105'1"39"'	.32144	.32144
6	1350	1315'34"7"'	.38268	.38268
7	1575	1520'28"35"	.44229	.44228

8	1800	1718'52"24"'	.50000	.50000
9	2025	1909'54"35"'	.55557	.55558
10	2250	2092'46"3"'	.60876	.60876
11	2475	2266'39"50"'	.65934	.65934
12	2700	2430'51"15"'	.70711	.70711
13	2925	2584'38"6"'	.75184	.75184
14	3150	2727'20"52"'	.79335	.79335
15	3375	2858'22"55"'	.83147	.83146
16	3600	2977'10"34"'	.86603	.86603
17	3835	3083'13"17"'	.89687	.89688
18	4050	3176'3"50"'	.92388	.92388
19	4215	3235'18"22"	.94693	.94692
20	4500	3320'36"30"'	.96593	.96593
21	4725	3371'41"29"'	.98079	.98079
22	4950'	3408'20"11"'	.99144	.99144
23	5175'	3430'23"11"'	.99785	.99785
23	5400'	3437'44"48"'	1.00000	1.00000

Verses giving value of constants in kaṭapayādi constants for sine - (with method)

'विस्' 'तुन्न बलः' 'कवीश निचयः' 'सर्वार्थ शीलं स्थिरो', 'निर्विद्धाङ्ग नरेन्द्र रुङ्' निगदितेष्वेषु क्रमात् पञ्चसु । आधस्त्याद् गुणितादभीष्ट धनुषः कृत्या विहृत्यान्तिमः स्याप्तं, शोध्यमुपर्युपर्यथ धनेनैवं धनुष्यन्ततः ॥

Constants and method for versine -

'स्तेनः' 'स्त्री पिशुनः' 'सुगन्धि नगनुद्' 'भद्राङ्ग भव्यासनो'

मीनाङ्गो नरसिंह' ' ऊनधन दृक भूरेव' षटस्वेषु तु ।

आधस्त्याद् गुणिता दभीष्ट धनुषः कृत्या विहृत्यान्तिमः

स्याप्तं शोध्यमुपर्युपर्यथ फलं स्यादुत्क्रमज्यान्त्यजम् ॥'

Sine table in parās -

'श्रेष्ठं नाम वरिष्ठानां' 'हिमाद्रि वेंद भावनः'

'तपनो भानुः सूक्तज्ञो' 'मध्यमं विद्धि दोहनम्' ।१ ।

'धिगाज्योनाशनं कष्टं' 'छन्न भोगाशयाम्बिका' ।

'मृगा हारो नरेशोऽयं' 'वीरो रण जयोत्सुकः' ॥२ ॥

'मूलो विशुद्धो नाळस्य' 'गानेषु विरला नराः'

'अशुद्धि गुप्ता चोरश्रीः' 'शंकु कर्णो नगेश्वरः' ।३ ॥

'तनुजो गर्भजो मित्रं ' 'श्रीमानत्र सुखी सखे' ।

'शशी रात्रो हिमाहारो' 'वेगज्ञ पथि सिन्धुरः' ॥४ ॥

'छायालयो गजो नीलो' 'निर्मलो नास्ति सत्कुले'

'रात्रो दर्पण मभ्रांङ्गं' 'नागस्तुङ्ग नखो बली' ॥५ ॥

'धीरो युवा कथा लोलः' 'पूज्यो नारी जनै र्भगः'

'कन्यागारो नागवल्ली" देवो विश्वस्थली भृगुः' ॥६ ॥

तत्परादि कलान्तास्ता महाज्या माधवो दिताः ।७ ।

Proof of the method for sines -

Mādhava has used the infinite convergent series for sine for θ expressed in radians (between 0 and $\pi/2$)

$$\text{Sin } \theta = \theta - \frac{\theta^3}{\lfloor 3} + \frac{\theta^5}{\lfloor 5} - \frac{\theta^7}{\lfloor 7} + \frac{\theta^9}{\lfloor 9} - \frac{\theta^{11}}{\lfloor 11} + \ldots.$$

(Terms upto θ^{11} have been used for desired accuracy)

$$\text{or } \sin\theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \frac{\theta^7}{5040} + \frac{\theta^9}{3,62,880}$$

$$- \frac{\theta^{11}}{3\,99,16,800} + \ldots\ldots \quad (1)$$

Constants of Mādhava expressed in parā are

(1) 79,94,380 = A_1 (2) 9,86,267 = A_2 (3) 57,941 = A_3 (4) 1,986 = A_4 (5) 44 = A_5 from up to down order.

Let x is the arc length in minutes (kalā).

Then θ converted to degree ($\times$ 180/π) and in minutes becomes x

or $x = \theta \times \frac{180}{\pi} \times 60 = \frac{10800}{\pi}\theta$

At each stage we take its square and divide by R^2. (R = 5400) i.e. multiply by a^2 where a = $\frac{x}{R}$

$$\frac{x}{R} = \frac{10800}{\pi} \times \theta \times \frac{1}{5400} = \frac{2\theta}{\pi} = a \quad (2)$$

After multiplying 5th quantity by a^2 and substracting from 4th, we get $A_4 - a^2 A_5$

Multiply this by a^2 and substract from A_3, we get $A_3 - a^2 A_4 + a^4 A_5$

Multiply by a^2 and substract from A_2, we get $A_2 - a^2 A_3 + a^4 A_4 - a^6 A_5$

Multiply this by a^2 and substract from A_1, we get $A_1 - a^2 A_2 + a^4 A_3 - a^6 A^4 + a^8 A_5$

Multiply by a^3 and substract from arc x = $\frac{10800}{\pi}\theta$ we get

$$x - a^3 A_1 + a^5 A_2 - a^7 A_3 + a^9 A_4 - a^{11} A_5$$

This is value of sin θ in arc length of minutes. To get it in ratio for radius 1 we have to divide it by 10800/π. Then

$$\text{Sin } \theta = \frac{\pi}{10800}(x - a^3 A_1 + a^5 A_2 - a^7 A_3 + a^9 A_4 - a^{11} A_5) \quad \text{-- (3)}$$

First term $= \frac{\pi}{10800} \times x = \theta$ radians

Second term $= \frac{\pi}{10800} \times \left(\frac{2\theta}{\pi}\right)^3 \frac{79, 94, 380}{3600}$

(A is divided by 3600 to convert it in minutes)

$= \frac{\theta^3}{6}$ approx. (taking π = 3.151926 - - -)

Similarly we get all the terms of series (1) from formula (3) by calculations.

(4) Vaṭeśvara has used 96 divisions of a quadrant, each division being 56′15″ of arc. He has given values of R sine and R versine in seconds of arc.

Munīśvara has taken radius length as 191 and at 1° intervals, given the values of R sines upto 4th division of a degree (upto 1/60x60) of a second). Kamalākara and Jagannātha Samrāta both have taken radius of length 60 and given values upto 5th division of a degree. They have taken intervals of 1° and 1/2° respectively.

(5) Direct computation of R sines - Mādhava formula can be used for any angle for calculation upto 9 decimal places. Bhāskara I, Brahmagupta, Vaṭeśvara and Śrīpati have given formulas for direct calculation. All of their formula are equivalent to the following expression.

$$\text{Sin } \theta = \frac{\theta\,(180 - \theta)}{10125 - \frac{1}{4}\theta\,(180 - \theta)}\ ; \theta \text{ in degrees.}$$

While explaining calculation of sine ratios (Jyā - upapatti vāsanā), Bhāskara II, has given two forms of formula which reduce to the same expression.

Proof : In Figure 11, C A is diameter of a

circle of radius R

Arc AB = θ°

and BD = R. sin θ

Area ABC = $\frac{1}{2}$ AB. BC

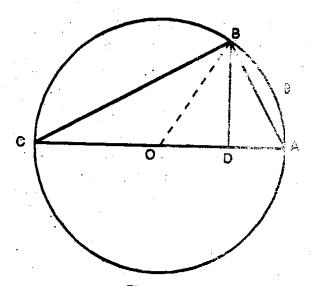
Also, Area ABC = $\frac{1}{2}$ AC. BD

So $\frac{1}{BD} = \frac{AC}{AB \, . \, BC}$

so that $\frac{1}{BD} > \frac{AC}{(\text{Arc AB})\ (\text{arc BC})}$

Let $\frac{1}{BD} = \frac{x\,AC}{(\text{arc AB})\ (\text{arcBC})} + y$

$= \frac{2xR}{\theta\,(180 - \theta)} + y$

Figure 11

or $\frac{1}{R \sin \theta} = \frac{2x\,R + \theta\,(180 - \theta)\,.\,y}{\theta\,(180 - \theta)}$

or $R \sin \theta = \frac{\theta\,(180 - \theta)}{2x\,R + \theta\,(180 - \theta\,)\,y}$ (1)

Putting $\theta = 30^o$, $\frac{1}{2}R = \frac{30 \times 150}{2x\,R + 30 \times 150\,y}$

or $2R + 4500\,y = \frac{9000}{R}$ - - - - (2)

Putting $\theta = 90°$,

$2x\,R + 8100\,y = \frac{8100}{R}$ - - - - (3)

Form (2) and (3) $y = -\frac{1}{4R}$ and $2xR = \frac{40500}{4\,R}$

Hence from (1) $\text{Sin}\,\theta = \dfrac{\theta\,(180 - \theta)}{10125 - \frac{1}{4}\,\theta\,(180 - \theta)}$

Alternate proof :

Let $\sin\lambda = \dfrac{a + b\lambda + c\lambda^2}{A + B\lambda + C\lambda^2}$

where λ is in radians and corresponds to θ degree

Putting $\lambda = 0$, $a = 0$

Putting $\lambda = \pi$, $b + \pi\,c = 0$ So $c = -\frac{b}{\pi}$

Thus $\sin\lambda = \dfrac{b\lambda\,(\pi - \lambda)/\pi}{A + B\lambda + C\lambda^2}$

Since $\text{Sin}\,\lambda = \sin(\pi - \lambda)$,

$$\frac{b\lambda\,(\pi - \lambda)/\pi}{A + B\lambda + c\lambda^2} = \frac{b\lambda\,(\pi - \lambda)/\pi}{A + B\,(\pi - \lambda) + C\,(\pi - \lambda)^2}$$

or, $A + B\lambda + c\lambda^2 = A + B\,(\pi - \lambda) + c\,(\pi - \lambda)^2$

or, $B\,(2\lambda - \pi) = C\,\pi\,(\pi - 2\lambda)$

or, $C = -\frac{B}{\pi}$

Therefore, $\text{Sin } \lambda = \dfrac{b\lambda(\pi - \lambda)}{A\pi + B\lambda(\pi - \lambda)}$

Putting $\lambda = \dfrac{1}{6}\pi$ $(\text{Sin } \lambda = \dfrac{1}{2})$

$$A\pi + B\frac{1}{6}\pi(\pi - \frac{\pi}{6}) = 2b \cdot \frac{\pi}{6}(\pi - \frac{\pi}{6})$$

$$\text{or, } A\pi + \frac{5\pi^2 B}{36} = \frac{10\pi^2 b}{36} \quad (4)$$

Putting $\lambda = \dfrac{\pi}{2}$ $(\text{Sin } \lambda = 1)$

$$A\pi + B\frac{\pi}{2}(\pi - \frac{\pi}{2}) = \frac{b\pi}{2}(\pi - \frac{\pi}{2})$$

$$\text{or } A\pi + \frac{1}{4}\pi^2 B = \frac{1}{4}\pi^2 b \quad (5)$$

From (4) and (5), $B = -\dfrac{1}{4}b$, $A = \dfrac{5\pi b}{16}$

Therefore, $\text{Sin } \lambda = \dfrac{16\lambda(\pi - \lambda)}{5\pi^2 - 4\lambda(\pi - \lambda)}$)

where $\lambda = \dfrac{\pi\theta}{180}$

Verses 67-70 - Jyā of bhuja and koṭi - Madhya graha substracted from mandocca gives manda kendra and from śīghrocca, it gives śīghra kendra. From these quantities, bhuja and koṭi jyā are calculated.

In odd quadrant (viṣama pāda), jyā of passed arc is bhuja jyā and remaining arc gives koṭijyā. In even (sama) quadrant, remaining arc gives bhuja jyā, and passed arc gives koṭijyā. (Quotation of Sūrya siddhānta ends with verse 68).

This means that if the keṇdra (maṇda or śīghra) is less than 3 rāśi (90°), then its jyā is bhuja jyā. If it is between 3 to 6 rāśi, then it is substracted from 6 rāśi. Jyā of the balance arc is bhuja jyā.

When kendra is between 6 to 9 rāśi, we deduct 6 rāśi from kendra. Jyā of the balance arc is bhuja jyā. If kendra is between 9 to 12 rāśi, it is deducted from 12 rāśi. Jyā of balance arc is bhuja jyā. If the quantity, from which deduction is to be done is smaller, then 1 rotation of 12 rāśi (360°) is added to it.

Notes (1) The rule is very simple and needs no explaination, if figure 12 is seen. ANBU is krānti vṛtta in which U is ucca (manḍa or śīghra). The planet moves in anti clock wise direction shown by arrow. M_1, M_2, M_3 and M_4 are positions of the planet in Ist, 2nd, 3rd and 4th quadrants from ucca position. Displacement of the planet along AB line is indicated by perpendiculars from M on AB i.e. parallel to NU line.

Thus in quadrant 1, M_1,U is the passed arc of angle M_1,DU. Its sine is M_1,Y_1, or OX_1, called bhujajyā. Similarly bhujajyā in 2nd, 3rd, and 4th quadrants are OX_2, OX_3, OX_4.

Koṭijyā is jyā of complementary angle (90° - angle) or cosine in modern terms. It is indicated

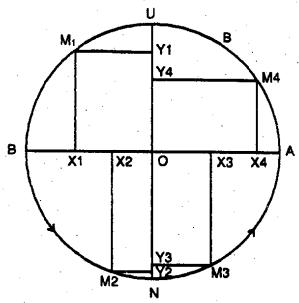



Figure 12 - Bāhu and Koṭijyā

by displacements along UN line. Koṭijyā in the quadrants are OY_1, OY_2, OY_3, OY_4.

(2) Manda Kendra = Mandocca - Madhya graha.

Śīghra kendra = Śīghrocca - Madhya graha

Both are definitions and need no further comment.

Verses - 71-72 - Method of finding jyā and utkrama jyā of any angle - (Quotation from Sūrya siddhānta)

From the chart we get sines and versines, only for angles which are multiples of 3°45′ (225′). Finding values for any intermediate angle is called interpolation. Its method according to Sūrya siddhānta is

$\frac{\text{Required Angle}}{225}$ = Quotient (= completed parts of 225′)

+ remainder (angle lapsed in next part)

Jyā of angle = Jyā of previous part + remainder/225 x (Jyā of current part - Jyā of previous part.)

Notes (1) This is simplest formula based on ratio and propotion i.e. rule of 3 (to find 4th unknown quantity).

$$\frac{\text{Difference in Jyā of fractional part}}{\text{Angle of fractional part}}$$

$$= \frac{\text{Difference in jyā of completed part}}{\text{Angle of 1 part (225′)}}$$

This assumes proportional variation of sine difference. This is called linear variation or linear interpolation.

(2) For small divisions of 3°45′ each; linear formula is sufficient to get accuracy upto a minute.

If divisions are of 10° each, then sine difference doesn't increase proportionate to increase in angle. For such interpolation, Bhāskāra II has given quadratic formula -

$$y = y_0 + \frac{x}{h}(y_1 - y_0) + \frac{x(x-h)}{(2h^2)}(y_2 - 2y_1 + y_0)$$

where h = intervals (10°) at which sines have been calculated; x is increase in angle; y_0, y_1, and y_2 are difference of sine for successive parts.

This formula was first stated by Brahmagupta. Vaṭeśvara has given several forms of the formula. For R sines upto seconds, quadratic formula is needed. Brahmagupta expression for 225′ intervals is -

R sin (225′t+θ′) - - - (where θ′ ≤ 225′, t is an integer)

= sum of t R sine - differences

$$+ \frac{\theta'}{225}\left[\frac{\text{t th R sin diff} + \text{(t + 1)th R sin diff}}{2}\right.$$

$$\left. - \frac{\theta'}{225}\,\frac{\text{t th R sin diff} + \text{(t + 1)th R sin diff}}{2}\right] \quad (1)$$

= sum of R sine differences

$$+ \frac{\theta'}{225}\,(t+1)^{th}\text{ R sin difference}$$

$$+ \frac{1}{2}\,\frac{\theta'}{225}\left(\frac{\theta'}{225} - 1\right)$$ [(t+1) th R sin diff. - t th R sin diff.] . . . . (2)

(2) is equivalent to quadratic formula.

(3) Mādhava's formula - If t is a positive integer and $\theta < 225'$, then

$$R \sin (225' t + \theta) = \text{sum of t R sin - diff} + \frac{\theta \times [R \cos 225 (t + 1) + R \cos (225 t)]}{2R}$$

This has been quoted by Nīlakaṇṭha in his commentary on Āryabhaṭīya.

Verses 73-74 : Finding arc from any given jyā - We substract the greatest jyā piṇḍa, smaller than given jyā, from the given jyā. The difference of jyā is divided by difference of the jyā piṇḍa substracted and next bigger jyā piṇḍa. It is multiplied by 225' and result is added to the completed arc.

Notes - (1) This is again linear interpolation whose proof is similar to reverse process of finding sine of a required arc.

(2) There is a similar quadiratic formula for this process also.

(3) Geometrical proof of the formula (linear and quadratic)—

Figure 13 shows the graha of R sine with its arc (or angle) On OX axis x_0, x_1 x_2 are points indicating angles or arcs where value of sine is

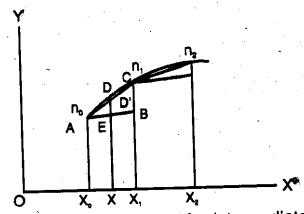



Figure 13 - Finding sine or arc for intermediate values

known. R sine at these points is no, n_1, n_2. We have to find value of R sine x = D on the graph when Ox is known in interval (1,2) Alternatively, if D is known, we have to find x point.

In linear interpolation, we assume that value of R sine changes along the straight line AC from no to n_1. For small intervals it is aimost same as curve AC; Dx is perpendicular on X axis cutting AC at D′ and AB at E. Thus D′ is a good approximation (linear) for D on the curve.

Now $D'E = D'x - Ax_0$ = increase in value of R sine for desired angle.

C B = increase in Rsine over interval (0,1)

Hence $\frac{D'E}{BC} = \frac{AE}{AB}$

When x is known, we know AE, AB = 225′ and BC is already known (value of R sines). Then fourth quantity D′E can be known. Required sine is $D'x = D'E + A\,x_o$ which gives the formula.

Similarly $x_o x = AE = \frac{D'E \times AB}{BC}$ can be known

and $O_x = Ox_o + xx_o$

Quadratic interpolation - n_1-n_0 is difference between the interval. (n_2-n_1) is difference in next interval.

Average rate of change is $\frac{(n_1 - n_0) + (n_2 - n_1)}{2}$

Increase in rate of change within the interval

$$= \frac{AE}{AB}\;\frac{(n_1 - n_0) - (n_2 - n_1)}{2}$$

Second term is the extra term for quadratic form of Brahmagupta.

Verse 75 : Relation between sine and cosine - From square of radius, deduct the square of jyā, take square root of the quantity. Result will be jyā of koṭi of the angle (3 rāśi - angle) or koṭijyā. Similarly, jyā can be calculated from koṭijyā.

Note - Cos $(90°-\theta)$ = Sin θ or sin $(90°-\theta)$ = Cos θ

$\sin^2\theta + \cos^2\theta = 1$

Hence $\sqrt{R^2 - R^2 \sin^2\theta} = R \cos\theta$

Verses 76 to 88 : True motion of mandocca by difference from parocca -

Mandocca of maṅgala, budha and śani and śīghrocca of budha move both forward and backwards as observed by me (author). To find their true motion another entity (devatā) called 'Parocca' has been assumed which affects these points (verse 76).

Parocca of maṅgala (its mandocca) is at 6 rāśi (180°) difference from madhyama sūrya. Parocca of budha mandocca is its śīghrocca. Likewise, mandocca of budha is parocca of budha's śīghrocca.

Parocca of śani's mandocca lies at 18° less from madhyama śani. Parocca also is an invisible form of Kāla like mandocca.

Similarly there are many small planets in the sky. Around planets, satelites move in circular orbits. Motion of these small planets and satellites can be seen only with instruments (telescope). They also move from west to east. They have not been described in Brahma and sūrya siddhānta, hence they are not been explained here.

Substract parocca from śighrocca and mandocca of budha and from mandocca of śani. R sine of the resulting angle is multiplied by 680 for budha and by 300 for śani. Product is divided by radius (3438). Result will be in liptā. If Para kendra (distance of manda or śīghra from Parocca) is between 0° to 180°, it is added in mandocca and for 180° to 360°, it is substracted. For sīghrocca of budha, its opposite procedure is followed (substraction for parakendra in 0° to 180° and addition other wise.

If śīghra kendra of maṅgala is in six rāśi's starting from makara (i.e. from 270° to 90°, then sphuṭa sanskāra is not needed for its mandocca. If śīghra kendra is in six rāśis beginning with karka (i.e. 90° to 270°) then mean mandocca will be corrected to find the true value. Mandocca of maṅgala for previous year (almost same for current year) is substracted from its parocca (mean sun + 180°). Result is its para-kendra. R sine of para kendra is multiplied by 450 and divided by 3438 = radius. Result is multiplied by koṭijyā of first sīghra kendra in kalā and then divided by kalā of 3 rāśi (5400). As before, result is added in Ist six rāśis of para kendra and substracted in other rāśis from mandocca of maṅgala.

For daily motion of mandocca - Substract mandocca from parocca of maṅgala (i.e. 180° + mean sun). Its koṭijyā is multiplied by 450 and divided by radius (3438). This will be koṭiphala for correction of mandocca speed. If parakendra is in 0° to 180°, koṭiphala is added to śīghra koṭijyā and substracted for parakendra between 180° to 360°. Result is multiplied by mean daily motion of sun

and divided by radius (3438). Result will be daily motion of maṅgala mandocca due to its parocca.

Similarly for śani, find kotijyā of its difference of parocca and mandocca. Multiply it by 300 and divide by radius 3438. Result is koṭiphala for correction. Koṭiphala is multiphed by mean daily motion of śani and divided by radius 3438. This will be daily motion of śani mandocca as corrected for its parocca effect.

For budha, Koṭijyā of its para kendra is multiplied by 680 and divided by radius 3438. Result will be Koṭiphala for correction. Multiply this Koṭiphala by daily mean motion of budha śīghrocca and divide by radius 3438. This will be daily motion of true budha śīghrocca. Correction in mandocca speed is addition for parocca kendra in Ist and 4th quodrant (270°-90°) and otherwise deducted. Correction in śīghrocca is in opposite manner.

Notes - (1) Author has not given reasons for such correction. His mention of observation of small planet by telescope indicates that these correction are based on some modern charts like Le - verrier's chart of 1850 or some nautical almanc available in his time. He has clearly mentioned that these have not been discussed in other siddhāntas which indicates his corrections are adopted from some almanc or results of telescopic observation.

(2) It is difficult to guess as to what correction was sought to be achieved by these methods. However, mathematical form of these formula will indicate the reasons of these corrections.

(a) Definition - Parocca' is a mathematical point in ecliptic from which deviation in mandocca of mangala, budha and śani and śīghrocca of budha can be calculated.

(b) Parakendra (P) = Sighrocca or mandocca - parocca.

Parocca of Budha śīghra = mandoca of budha

Parocca of budha mandocca = śīghrocca of budha

Parocca of maṇgala mandocca = madhyama ravi + 180°

Parocca of śani mandocca = madhyama śani - 18°

(c) Madhyama mandocca of all the planets is given in madhyamādhikāra. Śīghra of inner planets budha and śukra are the planets themselves. Sīghra of outer planets maṅgala, guru, śani is mean sun. Thus the tārā graha, affected by own orbit as well as earth orbit (or relative motion of sun) have sīghrocca as the planet of smaller orbit (and hence of faster rotation).

(d) Para kendra of Budha śīghrocca (B_1) i.e. PB_1, = $B_1 - B_2$ where B_2 is mandocca of Budha.

Para Kendra of B_2 (mandocca of Budha)

$PB_2 = B_2 - B_1$

Correction in mean B_1 = 680′ sin (B_1-B_2)

Correction in mean B_2 = 680′ sin (B_2-B_1)

Sin (B_1-B_2) is positive when $PB_1 = B_1$-B_2 is between 0° and 180° then it is negative correction For mandocca it is opposite. Hence for both budha mandocca and śīghrocca, correction is

680′ Sin $(B_2\text{-}B_1)$ - - - (1)

Parakendra of śani mandocca, Ps is

S_2 - (madhyam Śani - 18°), S_2 = mandocca of Śani

or Ps = S_2 + 18° - madhya śani

= S_2 + 18 - S say - - -

Correction to mandocca is 300′ Sin Ps - - (2)

Since it is to be added when Ps is between 0° to 180° i.e. Sin Ps is + ve, the formula indicates correct sign.

Śīghra kendra of maṅgala = mean sun - mean mangala. Skm = S_m - M

When S Km is between 270° to 90°, no correction is required. Correction to mandocca is done only when S Km is between 90° to 270° i.e. earth is on same side of sun as maṅgala.

Amount of correction -

Parakendra of maṅgala Pm = (Sm + 180°) – M_2

where M_2 is mandocca of maṅgala

(This is opposite substraction of the earlier process) Correction in maṅgala mandocca

$$= \frac{450' \sin Pm \times \cos S\,Km}{5400}$$

Sin Pm is + ve for Pm between 0° to 180° and it is added to mean mandocca.

Speed of mandocca is obtained by obtaining the differential coefficient of the corrections.

Position of true mandocca of mangala

$$= \frac{450' \sin Pm \times \cos S\,Km}{5400}$$

$$\text{Speed} = \frac{450' \cos Pm \times \cos S\,Km}{5400} * \frac{d}{dt} Pm$$

$$\frac{dPm}{dt} = \frac{dSm}{dt} = \text{speed of mean sun.}$$

Thus speed of mangala mandocca

$$= \text{mean speed of sun} \times \frac{450 \cos Pm \, . \cos S\,Km}{5400} \quad - (4)$$

Speed of Budha Sīghra or Mandocca - - -

$= 680' \text{ Cos } (B_2\text{-}B_1)$ - - - (5)

Speed of Śani mandocca = 300′ cos Ps (6)

(3) Reason and assumptions of these corrections -

(a) Śani mandocca motion - Motion of śani mandocca cannot be observed in a life time or even in a thousand years because it rotates only 39 times in a kalpa. Hence it is not oscillatory motion of mandocca which could be observed by the author. This appears to be correction due to effect of guru's attraction on Śani motion. At the time epoch of his observation after 1869 AD, guru was behind Śani for 5-6 years. Hence parocca of śani has been assumed to be slightly less (18°) than mean śani.

(b) Maṅgala is corrected, only when it is influenced by earth when both are on same side of sun. Hence this correction in mandocca is to account for influence of earth.

(c) Correction in śīghrocca and mandocca of Budha is to make correction of elliptic orbit of Budha śīghra (i.e. Budha itself) and its high inclination with sun's ecliptic; 7° as seen from sun.

These are reasonable assumptions of the origin. It needs further research and verification. But obviously, these corrections tallied with observations in author's time.

Verses 90-103 - Manda and śīghra Paridhi - For any planet, attraction by its mandocca is multiplied by kalā of a circle (21,600) and divided by radius (3438). Result is manda paridhi. Maximum mandaphala varies with manda paridhi. (90).

Śīghra paridhi of maṅgala, guru and śani is more in even quadrants compared to odd quadrants end. For budha and śukra it is opposite. (91)

According to Sūrya siddhānta - Manda paridhi of ravi is 14° in even quadrant and 13°40′ in odd end. Similarly, mandapraidhi of candra is 32° in even quadrant and 20′ less in odd quadrant i.e. 31°40′. (92)

According to Siddhānta Śiromaṇi of Bhāskarācārya, mandaparidhis of ravi and candra are constant and are 13°40′ and 31°16′ respectively. (93)

I (author) have calculated the values of manda paridhis of ravi and candra by observing conjunction of moon with stars and difference in rāśi of moon and sun (phases of moon) (94)

In odd quadrants, mandaparidhi of sun is 12°6′ and of candra is 31°30′ (95)

If manda kendra is at end of 4th quadrant, mandaparidhi of ravi is 11°30′. Now, method for finding manda paridhi at other places is being told (96)

Multiply koṭijyā of ravi manda kendra by 6 and divide by radius (3438). Result will be in kalā etc. If manda kendra is in Ist or 4th quadrant, substract this result from 18′ kalā. At other positions this will be added to 18′ kalā. (97)

Either of the results is multiplied by R cos of manda kendra (koṭijyā) and divided by radius (R=3438). Result in kalā etc. will be substracted from manda paridhi at end of odd quadrant (12°6′), when manda kendra is in 2nd or 4th quadrant. In other quadrants it is added (98)

This method is adopted for accurate calculation. For rough work, 1/9th part of previous result is added. This will give accurate results only on parva sandhi (Pūrṇimā or amāvāsyā).

Koṭijyā of candra manda kendra (R cosine) is multiplied by 30 and divided by radius (3438). Result in Kalā etc is added to the manda paridhi at end of odd quadrants (31°30′), when manda kendra is in six rāśis starting from karka. In other positions, it is substracted to find sphuṭa manda paridhi.

Manda paridhis of planets are - maṅgala 69° budha 27°, guru 34°30′ śukra 12° and śani 39°

Śīghra paridhis at end of even and odd quadrants are -

	End of even quadrant	End of odd quadrant
Maṅgala	238°	237°
Budha	139°	140°
Guru	70°	69°
Śukra	261°	262°
Śani	39°	38°

Notes (1) Mandaparidhi is correction method for elliptical orbit which have been assumed circular for first approximation.

(2) Sun and moon are directly in an ellipse around earth. But other planets take their position as a result of two orbits -- orbit of sun around earth (apparent) and orbit of planet round sun. Correction from mean position due to smaller of these orbits is done through śīghra paridhi.

(3) The method of correction by manda and śīghra parīdhis will be explained after calculation of these for tārā grahas.

Verses 104 to 112 - Sphuṭa manda and śīghra paridhis for tārā grahas - Difference (of 1°) between śīghra paridhis at the end of odd and even quadrants is multiplied by bhuja jyā of śīghra kendra and divided by radius (3438) i.e. multiplied by sine of śīghra kendra.

This result is added to the śīghra paridhi at end of previous quadrant, if it is rising in current quadrant. Otherwise, it is substracted.

For more accurate value of sphuṭa śīghra paridhi of maṅgala, we add 1/30 part of bhuja kalā of śīghra kendra (i.e. R sine of śīghra kendra is minutes).

Manda paridhi of maṅgala is 69° only at the end of quadrant. To find the intermediate values, we select the lesser part of manda kendra - amcng lapsed part and remaining part in the quadrant. R sine of that angle is multiplied by 8° when manda kendra is in six rāśis starting from karka, or by 4° when manda kendra is in 6 rāśis starting from makara. Result is divided by R sine of 1-1/2°rāśi

(45°) = 2431. Result is converted to degrees etc and added to 69° which gives sphuṭa manda paridhi of maṅgala (at any place).

When manda kendra of maṅgala is between 4868′ and (4868′ + 1590′) or between (15,142′) and (15,142′+1590′), its maṇḍaparidhi is taken as equal to its mandaphala of 3 rāśi i.e. 11°2′47″.

We find the lesser of lapsed and remaining parts in quadrant of Budha manda kendra. Its R sine (jyā) is divided by 9 and result is substracted from manda paridhi (27°). We get sphuṭa manda paridhi.

R sine of śukra manda kendra is multiplied by 2 and divided by radius R = 3438. Result in degrees is substracted from manda paridhi (12°) to find its sphuṭa value.

Notes - (1) As first approximation, planetary orbit is considered circle with earth at centre (for moon and sun).

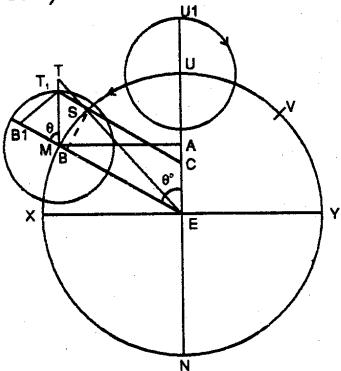
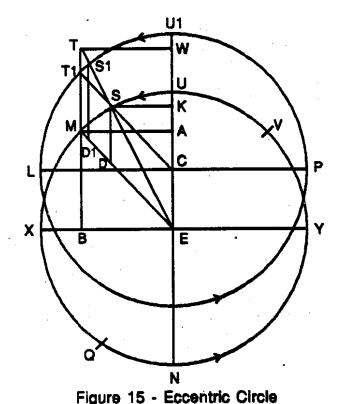



Figure 14 - Epicycle (Niocca vṛtta)

Figure 15 - Eccentric Circle

But orbit is elliptical with earth at focus which is away from centre at a distance of ae towards apogee (mandocca). Here a = semi major axis and e is eccentricity of ellipse.

At first step of approximation, we shift the circular orbit by distance ae in direction of major axis (Fig. 15). YU X N is orbit of mean planet whose apogee is U and V is the point of meṣa 0°, vernal equinox, from where longitude is measured. Another circle with centre at C in direction EU is drawn with same radius. In both the circles the planet rotates with same speed. Thus every point on the eccentric orbit (Prati vṛtta) PU_1 L is at a distance CE in direction of U. At apogee, mean planet is at U and true planet at U_1. When mean planet is at M, corresponding planet on prativṛtta is at T_1 where MT_1 = EC and both are parallel.

Same displacement can be done by assuming movement of spaṣṭa graha (true planet) on another

small circle whose centre is on madhya graha. The circle is called manda paridhi (epiciycle) which rotates fixed with radius vector of mean planet. However, movement of spaṣṭa graha on manda paridhi is in opposite direction to the motion of madhya graha but with equal angular speed. At apogee position in Fig 14, mean graha is at U and true graha is at U_1 in same direction, When mean planet moves to M in anti clockwise direction by angle $\theta = \angle UEM$ (manda kendra), the true planet moves by same angle $\theta = T_1BB_1$ in opposite direction. Thus T_1 is always in direction of mandocca i.e. MT_1 is parallel and equal to CE. Thus by construction of manda-paridhi also, all points of madhya graha orbit are shifted by distance EC in direction of EU towards ucca. Thus both the constructions are equivalent.

(2) Ellipse is symmetrical with respect to centre but not from focus which is centre of true orbit. Next step of approximation to make it toally equivalent to elliptical orbit is by changing the radius (or equivalently circumference) of manda paridhi at different places.

Let E be origin and EU direction of X axis, EX being direction of y axis. Radius of mean orbit (deferent) EM = R

Radius of manda paridhi for manda kendra θ $= m + n \cos \theta$

n has lowest value at 90° or 270° when $\cos \theta = 0$

m+n has highest value at apogee ($\theta = 0°$) or at $\theta = 180°$ (-ve) Coordinates of point T_1 are -

$x = EM, \cos \theta + MT_1$ - - in direction of EU

$= R \cos\theta + (m+n \cos\theta)$

or $x - m = (R+n) \cos\theta = a \cos\theta$

$y = R \sin\theta = b \sin\theta$

where $a = R+n$, $b = R$ are the semi major and semi minor axis. This is parametric equation of ellipse with centre at (m,o) i.e. at distance EC=m from centre of kakśā vṛtta towards mandocca.

From this,

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{(R+n)^2 - R^2}}{R+n} = \frac{\sqrt{2nR + n^2}}{R+n}$$

$$R = \frac{360^\circ}{2\pi},\quad n = \frac{20'}{2\pi} = \frac{1^\circ}{3 \times 2\pi} \text{ (sūrya siddhānta)}$$

Hence

$$e = \frac{\sqrt{\frac{2}{3} \cdot 360 + \frac{1}{3^2}}}{360 + 1/3} = \frac{\sqrt{6 \times 360 + 1}}{1081}$$

$= 0.043$ (real value for sun is 0.0167)

Geometric equivalent of the correction -

Without continuous varying the radius of mandaparidhi also, we can obtain the true position of planet.

In figure 14, join CT, which cuts deferent (orbit of mean planet) at S. Produce ES and MT_1 to meet at T. Then MT is the radius of true epicycle at M and T is true position of sun.

Similar construction can be made for prativṛtta (eccentric circle) also in figure 15. CT_1 cuts deferent at S and ES cuts MT_1 produced at T which is true position of planet.

(3) Bhujaphala is equal to equation of centre-

In figure 14, Sun's mean longitude = arc VUM

True longitude = arc VUS

Difference between the two, i.e. arc SM, is the sun's equation of centre

MA is perp. to EU and T_1B_1 and SB be perpendiculars to EM or EM produced. T_1B_1 is called bhuja phala or bāhuphala and B_1M is called koṭiphala.

Δ^5 B_1MT_1 and MAE are similar. Then

$$\frac{T_1 B_1}{T_1 M} = \frac{MA}{EM}$$

or T_1B_1, i.e. sun's bhujaphala

$$= \frac{T_1 M \times MA}{EM}$$

$$T_1M = \text{radius of epicycle (mean)} = \frac{14^o}{2\pi}$$

MA = R Sin θ, EM=R

Hence, Bāhuphala

$$= \frac{14^o}{2\pi} \text{Sin } \theta$$

$$= \frac{14 \times 60'}{2\pi} \text{Sin } \theta$$

= 133.'7 Sin θ = 0.388 sin θ radians.

With mean value of manda paridhi 11°48′, it is 0.327 sin θ which compares well with the modern value of 0.334 Sinθ. Plotemy had given 0.416 sin θ radiāns.

(4) In eccentric circle, geometric construction gives the method of successive approximation described later on while dealing with true speed.

(5) Heliocentric amomaly through Śīghra kendra. —Position of tārā grhaas depends on two orbits - apparent orbit of sun round earth and orbit

of planet around sun. Smaller of the orbits is called śīghra paridhi and madhya graha corresponding to average motion is bigger orbit.

Figure 16

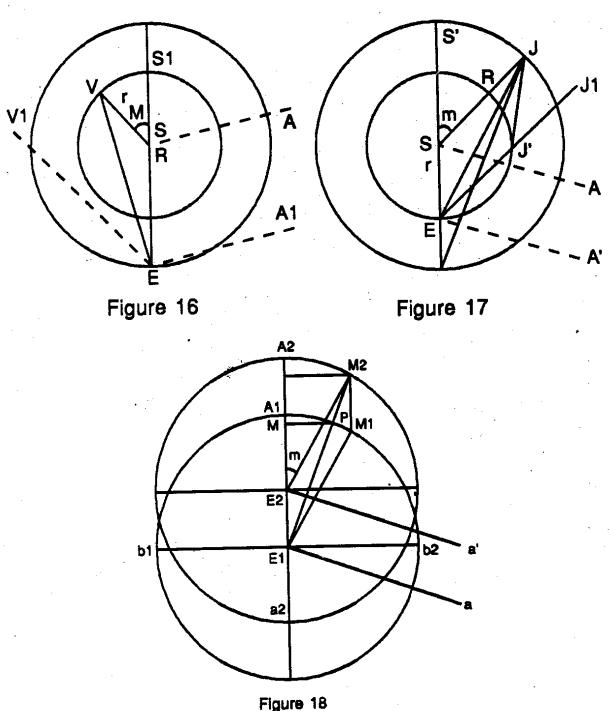



Figure 17

Figure 18

Like correction of elliptic orbit through manda paridhi, correction from heliocentric to geocentric position is done through śīghra paridhi. It can be an epicycle, or eccentric circle, as shown in figure 18.

It is to be proved that śīghra kendra is same as heliocentric anomady and śīghra phala is conversion from heliocentric to geocentric position. This will show correctness of śīghraparidhi method.

Figures 16 and 17 show the anomalies with sun as centre. First figure is for inner planets venus and mercury, marked as V. Second figure is for superior planets mars, jupiter and saturn, marked as J.

Figure 18 is for śīghra paridhi - both for inferior and superior planets.

Figure 16 and 17 - S = Sun, E = Earth, SA = direction to meṣa 0°, EA′ = direction to 0°. SV, EV′ are heliocentric and geocentric directions of śīghrocca. V is actual planet and V′ is imaginary point (śīghrocca of inferior planets) Draw EJ′ || SJ. Radius of inner and outer circles are r and R. K is radius vector to the planet (from earth).

Figure 18 - E_1 = Earth's centre, E_2 = Centre of eccentric circle (It will be proved as centre of sun) M_1, M_2 are mean planets in deferent (kakśā vṛtta) and eccentric (Prati Vṛtta). $E_1E_2 = r =$ antya phala jyā. R is radius of both circles. K is radius vector to planet known as śīghra karṇa.

Śīghra kendra (anomaly) = Longiude of śīghrocca - Longitude of planet (fig. 18)

$= \angle aE_1 A_1 - \angle a E_1M_1 = \angle a' E_2 A_1 - \angle a' E_2 M_2 = m$

In Fig 16, $\angle A' EV_1$ is longitude of śīghrocca, $\angle A'ES$ is longitude of sun treated as madhya graha of inferior planet.

śīghra anomaly. $m = \angle VSS' = \angle V'ES$

$= \angle A'EV' - \angle A'ES$

In fig. 17, $m = \angle S'SJ = \angle SEJ = \angle A'ES - \angle A'EJ'$

$= \angle A'ES - \angle ASJ$

= Longitude of Sun (treated as śīghrocca of superior planet) – longitude of planet from Sun known as mandasphuṭa graha.

= Śīghra kendra

Consider Δ^s ESV, JSE and $E_1M_1M_2$ of the three figures.

$\angle ESV = \angle JSE = \angle E_1 M_1M_2 = 180^\circ - m$

If value of śīghra paridhi is taken such that

$$\frac{SV}{SE} = \frac{SE}{SJ} = \frac{M_1 M_2}{E_1 M_1} \quad \text{- - - (1)}$$

all the three triangles will be similar.

Thus śīghra kendra is same as heliocentric anomaly and śīghraphala $\angle M_1E M_2 = \angle SEV = \angle SJE$.

$$K^2 = R^2+r^2 + 2 Rr \cos m \quad \text{- - - - (2)}$$

Comparison of values of orbit known in modern astronomy shows that value of śīghra paridhis have been chosen correctly, so that equation (1) holds -

i.e. $\frac{\text{Śighra paridhi}}{360^\circ} = \frac{\text{small orbit (radius or circum)}}{\text{Larger orbit}}$

Planet	Śīghra paridhi (average)	Deferent	Ratio	Value in modern astronomy earth = 1
Mercury	139.5	360	.3875	0.387
Venus	261.5	360	.726	0.723
Mars	237.5	360	1.519	1.52

Jupiter	69.5	360	5.18	5.20
Saturn	38.5	360	9.35	9.5

(6) Formula for sphuṭa paridhi - Difference between values at end of odd and even quadrants is 1°.

Hence if θ is angular difference from lowest position, addition will be Sin θ in degrees.

For further correction in maṅgala 1/30 part is added i.e. correction is $\frac{31 \sin\theta}{30} = 1.033^\circ \sin\theta$

Maṅgala manda paridhi is minimum in ends of quadrants (90° interval) and it is maximum in between (45° from ends). Difference from minimum 69° is 8° in 90° to 270° and 4° in other half. Hence correction in 90° to 270° is.

$+\frac{8^\circ \times \sin\theta}{R \sin 45^\circ}$ in other half it is $\frac{4^\circ \sin\theta}{R \sin 45^\circ}$

It is constant in two intervals 4868′ and 4868′ + 1590′ and (15,142 to 15,142′ + 1590′). Then mandaparidhi is 11°2′47′.

Budha manda kendra θ = lesser interval from ends of quadrant.

$$\text{Sphuṭa paridhi} = 27^\circ - \frac{R'}{9} \sin\theta$$

Śukra manda kendra = θ

Sphuṭa manda paridhi

$= 12^\circ - 2^\circ \sin\theta$

(7) For outer planets, earth is on same side of sun and closer to planet for śīghra kendra in even quadrants (closest at end). So its śīghra paridhi is more. For inner planets, it is opposite.

Minute chanes in śīghra paridhi are due to eccentricity of śīghra orbit also.

(8) Bhaskara II, has measured difference of mandocca - madhya graha in anti clockwise (position direction) and madhya - sīghrocca in opposite direction, Madhya is faster than mandocca but slower than śīghrocca. However, both measured same way make no difference.

Verses 113 to 120 - Bhuja and koṭi phala and karṇa

According to sūrya siddhānta, sphuṭa manda paridhi multiplied by R sin of bhuja of manda kendra and divided by 360° gives bhuja phala. When this paridhi is multiplied by koṭijyā (R cosine) of bhuja and divided by 360° it gives koṭiphala. (113)

When arc is smaller than 225′, it is same as its R sine (jyā). Then arc or R sine need not be converted to each other. (They are taken equal). Only when arc is more than 225′, its sine is to be calculated.

According to sūrya siddhānta, śīghra koṭiphala is added to trijyā (3438) when śīghra kendra is in six rāśis beginning with makara (i.e. 270° to 90°). For other śīghra kendras (i.e. 90° to 270°) it is substracted from trijyā.

This is koṭija bhujaphala, used for correction of radius and should not be considered an arc.

In sūrya siddhānta - Squares of bhuja and koṭi phala are added and square root of sum is taken. Then we get śīghra karṇa. Bhuja phala multiplied by trijyā (3438) and divided by śīghra karṇa gives śīghraphala in minutes of arc. Śīghra

phala is used for first and fourth corrections of five star planets starting from maṅgala. Sun and moon become spaṣṭa with only one correction with mandaphala. But in five tārā grahas, śīghra phala correction is done at first, then mandaphala correction is done twice. At fourth step, śīghra phala correction is done again. When śīghra kendra or manda kendra is less than 6 rāśi, sighra or manda phala is positive, hence always added for correction. When kendra is more than 6 rāśi, phala is substracted.

Notes : (1) List of given formulas

$$\frac{\text{Manda Paridhi}}{360^{\circ}} = \frac{\text{Mandatrijyā}}{\text{Trijyā}(3438)}$$

Hence manda trijyā r, bhuja of manda kendra θ give

$$\text{Bhujaphala} = \frac{r \, . \, R \sin \theta}{R} = r \sin \theta$$

In sighra paridhi, $R + r \cos \theta$ is calculated for known distance of true planet. Cos θ is positive from 270° to 90° hence it is added, otherwise substracted.

$$\text{Śīghra Karṇa} = \sqrt{\text{Bhuja Phala}^2 + \text{Koṭiphala}^2}$$

(both of śīghra paridhi)

i.e. $r = \sqrt{r^2 \sin^2 \theta + r^2 \cos^2 \theta}$ = radius of śīghra paridhi or śīghra karṇa, θ = bhuja of sighra kendra.

$$\text{Sighraphala} = r \sin \theta = \frac{r \, . \, R \sin \theta}{R}$$

Proofs are obvious when diagram of śīghra or mandaparidhi is seen. Sin θ is positive when θ is between 0° to 180° hence manda or sīghra phala is positive and is added.

Verses 121 - 123 - Correction in madhya tārā graha— Madhya graha corrected by half of śīghra phala (addition or substraction) gives first graha (corrected). Manda kendra is calculated for first graha and half of its mandaphala correction gives second graha. Manda kendra is again calculated for second graha. Its correction by mandaphala (full) gives third graha. For third graha, śighra kendra and śīghra phala is calculated. On correction of third by this śīghraphala, we get fourth graha which is the true position of planet (spaṣṭa graha).

Notes (1) - If madhya graha is Po, Ist and 4th sighraphala are S_1, and S_4, 2nd and 3rd mandaphala are M_2 and M_3, graha after Ist, 2nd, 3rd, 4th correction are P_1, P_2, P_3, P_4 then

$$P_1 = P_0 \pm \frac{S_1}{2}$$

$$P_2 = P_1 \pm \frac{M_2}{2}$$

$$P_3 = P_2 \pm \quad M3$$

$$P4 = P_3 \pm S4$$

Here S_1 and S_4 are calculated for P_0 and P_3, and M_2 and M_3 are calculated for P_1 and P_2.

Correction order can be indicated by

$$\frac{S}{2} + \frac{M}{2} + M + S$$

(2) **Āryabhaṭa method** For superior planets

$$\frac{S}{2} + \frac{M}{2} + M + S$$

For inferior planets (vernus and mercury)

$\frac{S}{2} + M + S$ (only 3 steps)

Hė has calculated śīghra kendra in opposite direction (Śīghrocca - planet), hence it is substracted for 0° to 180°.

Bhāskara I method $\frac{M}{2} + \frac{S}{2} + M + S$

For inferior planets $\frac{S}{2} + M + S$ (only 3 steps)

S/2 for inferior planets is corrected in reverse way and śịghra kendra is calculated from its mandocca.

Sūrya siddhānta method is the traditional and most popular method in country. It has been followed in siddhānta darpaṇa also.

(3) Further explaination of variations in manda and śīghra paridhi. (In continuation of note 2 after verse 112)

Eccentricity e of orbit = $\frac{m}{a}$

where m smallest value of manda paridhi

a = semi major axis = R+n

n = difference between maximum and minimum values of manda paridhi.

Thus e = $\frac{m}{R + n} = \frac{m}{R}$ approx, n very small.

It is also given by e = $\frac{\sqrt{2n\ R + n^2}}{R + n}$

= $\sqrt{\frac{2n}{R}}$ approx as n is very small.

Thus approximately, $e = \frac{m}{R} = \sqrt{\frac{2n}{R}}$

or $m = \sqrt{2nR}$

or $n = \frac{m^2}{2R}$, $m = eR$

This gives method of calculating maximum manda pardhi and its correction term For sun, max. paridhi is 14° (= $2\pi m$) and max correction is 20′ (= $2\pi n$)

$e = \sqrt{\frac{2n}{R}} = 0.043$

$e = \frac{m}{R} = 0.039$

This is similar by both method. Thus correction depends on value of max manda paridhi.

(4) Reasons for starting correction with śīghra phala - Mandaparidhi is measure of eccentricity of orbit (e=m/R) which is very small and less than 1/50. Shighra paridhi is ratio of smaller orbit to bigger orbit among the orbits of earth and planet round the sun. This varies from 1/9 to 3/4 approximately. Hence at first step manda correction can be neglected and only śīghra correction in done.

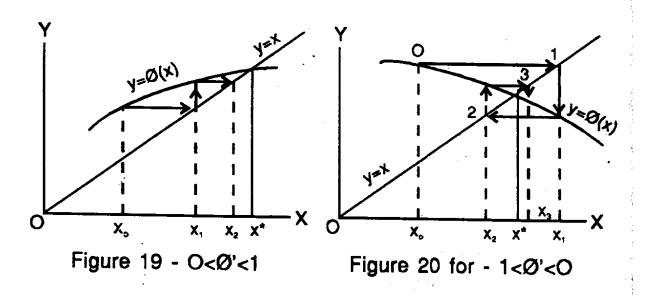
For inferior planets manda correction also is done in sun's orbit, not in the orbit of planets. Hence alternatively, manda correction can be done before śīghra as stated by Āryabhaṭa and BhāskaraI.

We do not calculate manda or śīghra kendra from true planet, but from mean planet which is an approximation. Hence only half corrections are done for śīghra and manda in beginning. Prob-

ability of negative or positive error will be both equal in half corrections and are likely to cancel each other. Then manda correction in full gives heliocentric anomaly of the planet - called manda spaṣṭa graha. Its last correction by śīghra phala has been explained in note (3) after verse 112.

Since śīghra and manda corrections are comparable, their half correction only is taken at a time. After 2nd correction error is reduced and after full manda correction, exact śīghra phala can be determined.

(5) This is a type of calculation based on probabilistic value of errors which is called 'Monte-Carlo method' in modern numerical analysis. Reduction of error at each step is similar to 'iteration method' for system of non-linear equations.

Figure 19 - O<Ø'<1 Figure 20 for - 1<Ø'<O

Method of iteration for numerical solution. Solution for y = $\Phi(x)$ is its point of intersection

with line y = x whose slope with x axis is 1. Figure 19 explains the approximations when slope is positive and figure 20 indicates negative slope of

$y = \Phi(x)$ Slope is $\Phi'(x)$ or $\frac{d\Phi}{dx}$, it is positive when

function in increasing, negative when it is decreasing. In both cases its numerical value is less than 1 i.e. slope of y= x. Only in such case successive approximations will reduce the errors at each step. For śīghra and manda corrections also, the corrections are much smaller than 1 as explained in previous para.

x_0 is the first approximation (like madhya graha). x_1, x_2, x_3 - - - are next approximations. When function (śīghra phala or manda phala) is increasing, i.e. correction is additive, all the approximations are on left side of, or less than true value x. When correction is negative, i.e. function is decreasing (Fig.10) x_1, x_2, x_3 - - - alternate on either side of the true value.

In both cases, diagram shows that errors decrease at each step, which was purpose of our corrections.

(6) Reasons of half corrections in first two steps - By full correction we may over correct and may not decrease the error which is required for iteration. Half correction will always reduce the error. Full investigation can be done only with Lyapunov's conditions of stability. However taking half of the approx value of correction, probability of positive and negative error both are same and it wil be approaching zero in end.

It is similar to methods used by computer which divide the line segment into two parts for

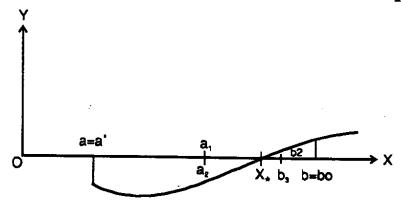



Figure 21

numerical approximation. In figure 21, solution of f (x) = 0 is its intersection with x axis. On one side of true x^*, there is a point a_0 for which f(x) is – ve and on other side $f(b_0)$ is + ve. We take midpoint c of interval (a_0, b_0). If f (c) is negative, we make it the new point in place of a, where it is negative. Thus we go on dividing the interval for better approximations.

Verses 124-131 - Special correction for maṅgala and budha - For mangala, 3rd and 4th phalas in kalā are muthplied and divided by 10. Result is substracted from last karṇa.
Then we get result in lipta etc. This result is added to 4th (sphuṭa) graha when manda kendra is 0° to 180°, otherwise substracted. If 3rd kendra (manda) of mangala is from 90° to 270°, 4th phala is substracted from 55, result is multiplied by manda koṭi phala of 3rd operation. This result is substracted from 4th karṇa. This is substracted from 5th graha, then we get 6th sphuṭa graha. If manda kendra of 3rd planet is 270° to 90° then this correction is unnecessary. (fifth graha will be true).

Madhyama budha is substracted from budha śīghrocca, already corrected for parocca. This śīghra kendra is used to find half of first śīghra phala,

which is kept in Ist place. In 2nd and 3rd places, we keep half mandaphala obtained from madhyama budha after 2nd operation (correction with half manda phala).

At 2nd place, this mandaphala is multiplied by half śīghra phala at Ist place and divided by half of the 4th śīghra phala. 1/3 of the result is substracted from mandaphala at 3rd place.

The new manda phala is used to make 3rd correction of madhyama budha. From that 4th śīghra phala is obtained and kept in 2 places. At 2nd place it is multiplied by 3rd koṭiphala divided by radius (3438)). Result is added or substracted at Ist place (addition is done when manda ḳendra is 90° to 270°). This śīghraphala is used for 4th correction. Then we get more correct result compared to Sūrya siddhānta.

Notes : The rules are lengthy and confusing when stated in words.

(1) Rules for maṇgala -

P_0, P_1, P_2, P_3, P_4 are the mean planet and the planets after Ist, 2nd, 3rd and 4th correction. S_1, S_4 are śīghra phala for Ist and 4th corrections, when śīghra kendra is calculated for P_0 and P_3. M_2, M_3 are mandaphala for 2nd and 3rd correction where mandaphala is calculated from manda kendra of P_1 and P_2.

Thus $P_1 = P_0 + S_1/2$ (S & M may be + ve or - ve)

$$P_2 = P_1 + \frac{M_2}{2}, \ P_3 == P_2 + M_3, \ P_4 = P_3 + S_4$$

$$\text{Thus } P_4 = P_0 + \frac{S_1}{2} + \frac{M_2}{2} + M_3 + S_4 = \text{True graha}$$

r_1, r_4 are śīghra radius for S_1, S_4 and r_2, r_3 manda radius for M_2, M_3. If θ is manda or Śīghra kendra (bhuja),

M or S = r Sin θ For mangala we obtain P_5 and P_6 and further corrections of true planet.

$$\text{Fifth correction } X_5 = \left(r_4 - \frac{M_3 \times S_4}{10} \text{ in liptās} \right)$$

$P_5 = P_4 + x_5$ when manda kendra of P_4 is between 0° to 180°

or = $P_4 - x_5$ when it is between 180° to 360°.

When manda kendra of P_3 is between 270° to 90° this P_5 is the last correction needed. If manda kendra of P_3 is between 90° to 270°,

sixth correction

$x_6 = r_4 - (55 - S_4)\, r_3 \cos \theta_3$

$P_6 = P_5 - x_6$

(2) Correction for Budha - S' is śīghra phala of budha corrected for parocca. From $\frac{\acute{S}_1}{2}$ we calculate M_2'. For third correction we do not calculate M_3' from 2nd planet.

$$\text{3rd correction} = M_2' \left(1 - \frac{S_1'}{3\,S_4} \right) = X_3$$

S_4 is calculated by general method.

The new śīghra phala after $P_2 + x_3 = P_3'$, is called S_4'.

Fourth correction $x_4 = S_4'\,(1 \pm r_3 \text{ Cos } \theta_3)$

Addition is done when θ_3 is 90° to 270°

$P_4' = P_3' \pm x_4$ (addition for 0° to 180°)

P_3' and P_4' are planets obtained by revised method.

Verses 132-138 - True speeds of sun and moon.

Now true speed of graha is considered. The speed changes every moment, but sphuṭa gati of a day is the difference between sphuṭa graha on two successive days. Strictly this will be average daily speed for that day.

Dainika gati of mandocca, substracted from dainika gati of mean graha, gives danika gati of manda kendra. Dainika gati of manda kendra multiplied by manda koṭiphala and divided by radius gives manda gatiphala for one day. This is added for manda kendra in 6 raśis from karka ; in madhya gati of graha. Other wise it is substracted from madhya gati. This result will be manda sphuṭa gati for one day i.e. from sunrise to the next sunrise.

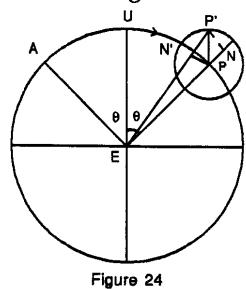
At sunrise, difference of true moon and true sun gives the balance part of current tithi. This (added to sunrise time) gives ending time of tithi. At that time true moon is again calculated and further correction of tithi end time is done. This accuracy in knowing beginning and end of tithi is needed only for ascertaining time of eclipse or of śrāddha (last rites). For normal works, the true position of sun and moon and their speeds at sunrise will be assumed constant for the day.

Notes : (1) List of all terms as revision and summary -

Mandocca-Madhya graha = manda kendra M.

Śīghrocca - madhya graha = Śighra kendra S

Manda kendra or Sighra Kendra = θ

Figure 24

True position of graha is P′ while mean graha is at P. Radius r of manda or śīghra paridhi is PP′.

EP′ is karṇa = K (Śīghra or manda)

EP = R, radius taken aș 3438′.

2 π r is expressed in degrees of manda or śīghra paridhi.

PN′ is perpendicular on Karṇa EP′, N′ is true position on Krānti Vṛtta. Thus PN′ is the correction in mean motion called śīghra or manda phala.

Mandaphala = PN′ = almost P′N.

It is slightly less than P′N, perpendicular from P′ to EP extentled.

P′N = Doh phala or Doh jyā

= r Sin θ

Mandaphala PN′ $= \dfrac{r \sin \theta \times R}{R + r \cos \theta}$

Koṭiphala PN is addition to the mean trijyā in that direction PN == r cos θ

Śīghra or manda karṇa K = EP′

$K^2 = (R + r \cos \theta)^2 + (r \sin \theta)^2$

$= R^2 + r^2 + 2 r R \cos \theta$

(2) Now the speed can be calculated with help of differential calculus. These results cannot come by any other method and are according to sūrya siddhānta.

In figure 24 in above para, Φ is angle of ucca point U with meṣa 0^0 at A. Then madhyagraha at $P = \Phi + \theta$, mandocca = ϕ.

$$\text{Thus } \frac{dP}{dt} = \frac{d\Phi}{dt} + \frac{d\theta}{dt} \text{ (t is time)}$$

or dainika gati of madhya graha = gati of mandocca + gati of manda kendra.

True graha is at $N' = P + r \sin \theta$ (negative correction)

$$\text{or } \frac{dN'}{dt} = \frac{dP}{dt} + r \cos \theta \frac{d\theta}{dt} \text{ as r is constant.}$$

Thus additional gati i.e. correction == $r \cos \theta, \frac{d\theta}{dt}$

= koṭiphala x gati of manda kendra.

Verses 139-142 - Śīghragati of tārā graha.

Śīghrocca gati —madhya graha gati = Śīghra kendra gati. Śighra phala is substracted from 90°, it is multiplied by daily motion of śīghra kendra and divided by śīghra karṇa. Result substracted from śīghrocca gati is śīghra sphuṭa gati. If it is more than śīghrocca gati, reverse substraction gives retrograde motion. In this way 5 tārā graha have two types of gati—manda sphuṭa and śīghra

sphuṭa. Ravi and candra have only manda sphuṭa gati.

Notes : (1) Like above, madhya graha (manda sphuṭa for śīghra gāti) is at P, and P′ is sphuṭa graha.

Sphuṭa kendra Φ−P is given by Φ where Φ is longitude of śīghrocca.

$$\frac{d\theta}{dt} = \frac{-dP}{dt} + \frac{d\Phi}{dt},$$ t is time measured in days

i.e. dainika gati of śīghra kendra = gati of śīghrocca - gati of madhya graha

P′ is sphuṭa, its component perp to radius is 'P + r sin θ

$$\text{Hence } \frac{dP'}{dt} = \frac{dP}{dt} + r\cos\theta \frac{d\theta}{dt}$$

$$= \frac{d\Phi}{dt} - \frac{d\theta}{dt} + r\cos\theta \cdot \frac{d\theta}{dt}$$

$$= \frac{d\Phi}{dt} - \frac{d\theta}{dt}(1 - r\cos\theta)$$

$$\frac{dP'}{dt} = \frac{d\Phi}{dt} - \frac{d\theta}{dt}\left(\frac{R - R\, r\, Cos\,\theta}{R}\right)$$

Thus negative śīghra gati phala is

$$\frac{R - r \,.\, R\cos\theta}{R}$$

$$= \frac{\sin 90^\circ - \text{Koṭiphala of śīghra}}{\text{śīghra Karṇa}} \text{ (approx)}$$

(2) Exact derivation assuming variation of karṇa also -

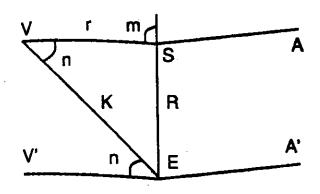



Figure 25
(Inferior planet like venus, mercury)

Figure 26
superior planet

E = Earth, S = Sun, V= inferior planet (Fig 25)

J = Superior planet (Fig 26)

SA, EA′ = direction of meṣa 0° from sun and earth

m == śīghra amomaly, n = sphuṭa kendra

R = bigger orbit radius = SE in Fig 25

or SJ in fig 26.

r = smaller orbit radius = S V in fig 25

or SE in fig 26.

K = Śighra karṇa i.e. distance from earth to planet (true) = EV or EJ.

True motion of planets δ (A′EV) or δ (A′EJ)

But δ (A′EV) = δ (A′EV′ - n), and δ (A′EJ) = δ (A′ES-n)

For inferior planet, δ (A′EV′) = δ (ASV) = sighrocca gati, δ n = sphuṭa kendra gati.

For superior planet, δ (A′ES) = Śīghrocca gati (sun is śīghrocca for superior planet)

δ n = sphuṭa kendragati as before

Thus in both cases,

Sphuṭa gati = śīghra gati – sphuṭa kendra gati- (1)

To find δ n, from figures

$$K \cos n - R \cos m = r \quad --- (2)$$

Differentiating (2), we have

$$-k \sin n\, \delta n + \cos n.\, \delta k + R \sin m\, \delta m = 0 \quad (3)$$

But $k^2 = R^2 + r^2 + 2 R r \cos m$

Differentiating, $2K\delta K = -2Rr \sin m\, \delta m$ - (4)

Eliminating δk between (3) and (4)

$$-k \sin n\, \delta n - \frac{R_r \sin m\, \delta m \times \cos n}{K} + R \sin m\, \delta m = 0$$

or $K \sin n\, \delta n = R \sin m\, \delta m\, (1 - \frac{r}{k} \cos n)$

$$= \frac{R \sin m\, \delta m}{K} (K - r \cos n)$$

But $K - r \cos n = R \cos E$

So $\delta n = \dfrac{R \sin m\ \delta m \times R \cos E}{K^2 \sin n}$

But $R \sin m = k \sin n$

so $\delta n = \dfrac{R \cos E\ \delta m}{K}$

(3) Proof of approximate method.

Mandagati phala $= \dfrac{\delta m \times R}{K}$

δm = mean motion, k = manda karṇa

Thus mandagati phala $= \dfrac{\delta m \times R}{\sqrt{R^2 + r^2 \pm 2R r \cos m}}$

$= \delta m\, (1 \pm \frac{2r}{R} \cos m)^{-1/2}$ neglecting square of $\frac{r}{R}$

$= \delta m\, (1 \pm \frac{r}{R} \cos m)$

$$= \delta m \pm \frac{r\,R\cos m}{R} \cdot \frac{\delta m}{R}$$

$$= \delta m \pm \frac{\text{Koṭiphala} \times \text{manda kendragati}}{R}$$

(4) Approximation of true distance and daily motion can be done by epicyclic or eccentric circles also by successive approximation. That can be seen in commentary on Mahābhāskarīya by Prof. K.S. Shukla published by Lucknow University. Geometric explaination of Lalla method can be seen in commentary on Śiṣyadhīvṛddida tantra by Smt. Bina chatterjee published by INSA, Delhi-2.

Verses 143-150 - Gati phala at four stages for tārā grahas– When śīghra sphuṭa gati is more than daily mean motion, then madhyama gati is substracted. When daily mean motion is more, śīghra sphuṭa is substracted from it.

When śīghra sphuṭa gati is vakra, it is added in daily mean motion. The result in either of three cases is called first gati phala whose half is taken.

When sphuṭa śīghra gati is more, it is added in daily mean motion. When sphuṭa śīghra gati is less or vakra, Ist gati phala is substracted (half only) from madhya gati. This is Ist corrected gati.

First gati is forward or reverse. It is multiplied by manda koṭiphala and divided by trijyā. Half of the result (2nd gati phala manda) is taken.

When manda kendra is in 6 rāśis starting from karka (90° to 270°), 2nd gatiphala half is added to Ist gati otherwise substracted from Ist gati. Result will be 2nd gati.

Again 2nd gati is multiplied by manda koṭiphala and divided by trijyā (3438) and full result

(3rd gatiphala) is used for correcting 2nd gati (addition or substraction). Result is 3rd gati.

Third gati is multiplied by śīghra koṭi phala and divided by śīghra karṇa. Result is 4th gati phala, which is used to correct 3rd gati. Then 4th gati is the true gati. If 4th gati phala is more than śīghrocca gati then motion is reverse (vakrī gati). This method is more correct then sūrya siddhānta.

Verses 151 to 158 - Special methods for true speed– When mandocca of maṅgala, budha or śani is moving forward, its speed is substracted from Ist and 2nd gati. If it is vakrī (in reverse motion), its speed is added. From these corrected, Ist and 2nd gatis, we find 2nd and 3rd gati. New 2nd and 3rd gati are substracted from sphuṭa śīghra gati for Ist and 4th gati.

Fourth gati phala of budha is kept in two places. At one place, it is multiplied by manda koṭiphala and divided by radius (3438). Result is added to 4th gati phala in second place, when manda kendra is in six rāśis starting from karka (90° to 270°). Then sphuṭa gati of budha will be more correct.

If Sighra gati half is vakra, it is added to negative mandagati half or difference is taken from positive half mandagati. Result will be vakra (reverse) gati. Maṇdagati (2nd or 3rd steps) phalas are added in six rāśis starting from karka and substracted otherwise.

Many methods of finding true planet from mean planet are coming to mind, but these are not given here (by author), as they are very complicated.

Daily true motion is used for finding transition time of graha from one rāśi to next or in conjunction (yuddha) of planets. For ravi and candra, method is different.

Notes : Method for śīghragati phala and mandagati phale has already been explained. Reasons of vakra gati will be explained when its starting or ending point are calculated.

Verses 159-160 - According to Sūrya siddhānta tārā graha becomes vakrī when its 4th sighra kendra has the given values -

Mangala	163°	Śukra 167°
Budha	146°	Śani 115°
Guru	126°	

At 4th sighra kendra obtained by substracting these values from 360°, the graha again becomes mārgī.

Notes (1) Derivation of 4th śīghra kendra for vakrī gati. We assume that heliocentric orbits of earth and Jupiter around sun are both circular and coplanar.

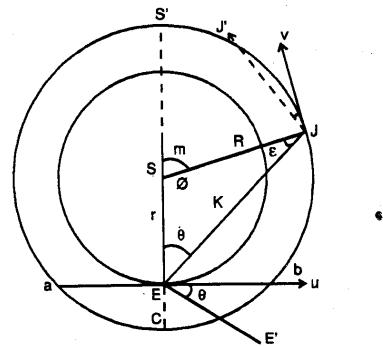



Figure 27 - Explaination of vakri gati and its position

Let S == Sun, E = Earth, J = Jupiter, u = earth's linear velocity. Śīghra karṇa EJ = k, v=velocity of jupitar.

r and R are orbital radii of earth and Jupiter.

EE′ and JJ′ are parpendiculars to EJ so that when relative velocity of Jupiter with respect to earth, i.e. perp to EJ is zero, Jupitér will appear stationary as seen from earth.

This means that $u \cos\theta + v \cos\varepsilon = 0$ - - - - (1)

or $\frac{u}{v} = \frac{-\cos\varepsilon}{\cos\theta}$ - - - - (2)

From ΔESJ, $R \cos\phi + k \cos\theta = r$ - - - (3)

$r \cos\phi + k \cos\varepsilon = R$ - - - (4)

From (3) and (4)), $\frac{\cos\varepsilon}{\cos\theta} = \frac{r \cos\phi - R}{R \cos\phi - r}$ ..(5)

Equating $\frac{\cos\varepsilon}{\cos\theta}$ from (2) and (5)

$-\frac{u}{v} = \frac{r \cos\phi - R}{R \cos\phi - r}$ so that

$\cos\phi = \frac{ru + Rv}{rv + Ru}$

If m is śīghra anomaly, then $m = 180° - \phi$

So, $\cos m = -\left(\frac{ru + Rv}{rv + Ru}\right)$ - - - (6)

This is equivalent to formula given by Bhāskara II

spaṣṭa gati = śīghra gati - $\frac{R \cos E \,.\, \delta m}{K}$

spaṣṭa gati = 0, if śīghra gati = $\frac{R \cos E \,.\, \delta m}{K}$ - - - (7)

i.e. Jupiter appears stationary as seen from earth, if

śīghra gati = $\frac{R \cos E \,.\, \delta m}{K}$

Angular velocity of earth and Jupiter are $\frac{u}{r}$ and $\frac{v}{R}$

so that sun's apparent velocity is also u/r

m == Kendra gati = sun's apparent velocity -- Jupiters heliocentric velocity.

$$= \frac{u}{r} - \frac{v}{R}$$

Substituting this in (7),

śīghra gati = $\frac{u}{r} = \frac{R \cos E}{K} \left(\frac{u}{r} - \frac{\nu}{P}\right)$

$$\therefore \frac{u}{r} \left(R \cos \frac{E}{k} - 1\right) = \frac{R \cos E}{K} \times \frac{v}{R}$$

Here $E = \varepsilon$, $R \cos E - K = - r \cos\theta$

So $\frac{u}{r} \times (- r \cos \theta) = \nu \cos E$

or $u \cos \theta + v \cos \varepsilon = 0$

which comes to equation (1)

Sighra kendra m is obtained from (6)

where r = radius of śīghra paridhi or antya phala jyā, R = 3438′, u = mean velocity of sun and v = mean velocity of the planet.

(2) Some observations on śīghra phala and śīghra gatiphala -

We have $M_2 + E_2 = S$ ....(1)

Where M_2 = Mandasphuṭa graha, E_2 = Śīghra phala, s=sphuṭa graha.

Differentiating this

$$\delta M_2 + \delta E_2 = \delta S \quad ---- (2)$$

i.e. Mandasphuṭa gati + śīghra gatiphala = spaṣṭagati

(a) Let E_2 be maximum so that $\delta E = 0$, then $\delta M_2 = \delta S$

This means that when sighra phala is maximum for sighra kendra 90° or 270° (Sin is maximum) manda sphuṭa gati is the spaṣṭa gati.

(b) Planets starts retrograde motion only after the spaṣṭa gati vanishes i.e. $\delta M_2 + \delta E_2 = 0$

Taking δM_2 to be almost a constant, since mandagati phala is small, the negative value of δE_2 must cancel δM_2. δE_2 becomes negative when sighra kendra is between 90° to 180° or 270° to 360° when value of sine decreases in value. From 180° to 270° it is negative, hence its net value increases. Thus the planet will have zero velocity at two points symmetric to 180° (S′ towards a and b)

Thus if retrorade motion starts at $180° - \theta$ it will stop at $180° + \theta = 360° - (180°-\theta)$, where its velocity becomes 0.

Keeping earth constant, an inferior planet goes anticlockwise whereas superior planet goes clockwise which is direction of sun's motion.

(c) Values of spaṣṭagati at S′ and C will be by putting R Cos E = R in formula)

$$\text{Spaṣṭa gati} = \text{Śīghragati} - \frac{R \cos E \, \delta m}{K}$$

Here, śīghra gati = U, $\delta m = U - V$

$K = R + r$ at S′

$R - r$ at c

Spaṣṭagati $= \dfrac{U - R(U - V)}{R + r}$ at S′ and

$\dfrac{U - R(U - V)}{R - r}$ at c

$= \dfrac{RV + rU}{R + r}$ or $\dfrac{RV - rU}{R - r}$ respectively.

$\dfrac{RV + rU}{R + r} > \dfrac{RV - rU}{R - r}$

if $R^2 V\text{-}2\, RV + Rr\, U - r^2 U > R^2V - Rr\, U + rRV - r^2U$

i.e. if rR (U-V) > Rr (V--U)

i.e. U-V is + ve and and equal to V-U.

Thus positive velocity at S′ of the planet will be equal to its negative or retrograde velocity at c. Thus velocities direct or retrograde will always be less then S or c at any point between them on either side.

Verse 161-164 : Udaya (rising) and asta (setting) of planets is of two types - practical is rough (sthūla) and drik siddha is sūkśma. (accurate)

The planets set in west when their śīghra kendras cross the following values -

Maṅgala	332°	Śukra	177°
Budha	159°	Śani	343°
Guru	346°		

For rising in the east, last śīghra kendra is

Maṅgala	28°	Śukra	183°
Budha	201°	Śani	17°
Guru	14°		

For setting in east, śīghra kendra of inferior planets are, Budha 310° Śukra 336°

For rising in west, śīghra kendra are
Budha 50° and Śukra 24°

Notes : (1) Rising of a planet means that it is above horizon of earth. But tārā graha are visible only during night time, so their rising is only seen at night.

Obstruction due to sunrays makes the tārā grahas invisible during day. When they are away from sun sufficiently, they can be seen. That is called heliacal rising or dṛk siddha udaya.

(2) Sun's velocity is greater than superior planets, so sun overtakes them so that they set in west and rise in the east. When these planets are situated within particular limits from the sun, they will be invisible in the rays of sun. Thus they will be invisible at conjunction with sun and within particulars limits from position of sun. The total difference from sun depneds not only on difference in longitudes, but also on difference in śara (north south distance.)

The limits of invisible distance from sun depends on their distance from sun and relative brilliance. The brilliance also depends on their phase, i.e. part of illuminated disc facing earth. Phase is $\frac{1 + \cos EPS}{2}$, $\angle EPS$ = sighraphala E_2,

$$\text{hence phase} = \frac{1 + \cos E_2}{2}$$

At conjunction $E_2 = 0$, entire planet will be iluminated but we cannot see them, because they

will be immersed in rays of sun. With increase in E_2, Cos E_2 will decrease and lesser part of disc will be illuminated. Since distance also will decline, luminousity will not be affected. (from S′ to a). In path acb, planet gains in illumination and distance also decreases. Thus superior planets appear more and more brilliant when they are retrograding, being most brilliant at c.

Spherical radius of jupiter, saturn and mars are in decreasing order, so that they will be visible at angular distances in increasing order. Inverse square law of reduction in brilliance with distance (karṇa) works but doesn't counter the effect of sizes. Thus sighra kendra of these planets are 14°, 17°, 28°. In udayāstādhikāra, Kālāṁśa is slightly less, because distance will be (śīghra kendra – śīghra phala.)

(3) Inferior planets rise heliacally in the east after inferior conjunction and then they are retrograde. They attain gradually the maximum elongation in the east, then direct motion starts. When elongation gradually decreases and after going ahead of sun, they set in east. Thereafter, they heliacally rise in the west. There again, their elongation attains a maximum value, after which they become retrograde. After crossing sun again they gradually set in west and rise in east. (Figure 18 may be seen).

When the śīghra anomaly of budha and śukra are 50° and 24°, their sighra phala will be 13° and 11°, so that they are the kalāmśa i.e. elongation from there mean sun. Then, they rise in west, being near superior conjunction. When their śīghra anomalies become 159° and 177°, same sighra phala

will arise, so that they set heliacally in the west. Then as śīghra kendra attains symmetrical values on other side of 180° i.e. (360°-159°) and (360°-177°) i.e. 201° and 183°, śīghraphala are same, they rise in the east. Again, when they obtain sighra kendra (360°-50°) and (36°-24°) i.e. 310° and 336°, they set in the east due to same śīghra phala or kālāmśa.

Verse 165 - Moon sets when it is 11° behind sun and rises again when it is 11° ahead of sun.

Note : This is not related to rising in east or west. It is visibility near sun, which starts after 11° distance from sun. 12° difference from sun makes 1 ṭithi (in 360° difference there are 30 tithis 15 in bright half and 15 in dark half). Thus in amāvāsyā, moon is not visible. It is again visible slightly before 2nd day of bright half (12° advance of sun). Thus start of 'dūjā' in muslim calender is counted from sighting of moon.

Verse 166 : To find mean planet knowing the true.

Assume the true planet to be the mean; compute the manda and śīghra phala and apply them inversely. We have approximation of the mean planet. Treating this as mean planet, again obtain manda and śīghra phala and apply them inversely. The process is repeated, till constant values are obtained.

Notes : This is method of successive approximation

Verses 167-187 - Use of tables for calculation of true planets.

Calculation of true planets is very long and difficult process and there are chances of error.

Hence I (author), am giving correct Khaṇḍaphalas in a chart for easy calculation (167.)

In appendix, there is chart of manda and śīghra phala, for parts of 0 to 24 (24 parts of a quadrant of 90° are 3°45′ each). This contains koṭiphala of all planets, gatiphala of tārā grahas, gatiphala of ravi and candra, śīghra of 48 parts (180°), difference of khaṇḍaphala, śīghra karṇa in liptā (minutes of arc), degrees for cakra entry, krānti, śīghra kendra for rising and setting etc (170).

From the values in the chart, manda kendra bhujaphala in degrees, minutes, seconds etc are separately multiplied by 8, vikalā (seconds) etc. are divided by 60, when they become degree, they are added to the degrees. Total degrees are divided by 30 to make rāśi. This will be past (gata) phala. (172)

For extra degrees, they are multiplied by difference for the khaṇḍaphala and divided by 60. Result is added to degrees obtained earlier. Remainder is multiplied by 2 and added to mandaphala khaṇḍa. This way, mandaphala of a graha is calculated, which is added or substracted according to rules earlier explained.

Manda kendra gati multiplied by difference of khaṇḍaphala and divided by 225′ (3°45′), is manda gati phala between two khaṇḍas. (173).

In appendix, parocca khaṇḍaphala of maṅgala, budha, śani also have been given. Khaṇḍaphala difference and ucca gati at end of khaṇḍa has also been written. From them parocca phala is calculated and is added or substracted from manda kendra

of maṅgala, budha, śani or budha śīghrocca, we get sphuṭa gati corrected for parocca. (174)

Śīghra khaṇḍa table also is prepared for 48 parts (khaṇḍa of 180° i.e. 1 part of 3°45′. Śīghra kendra is found by substracting manda sphuṭa graha from śīghrocca. Śīghra kendra in 6 rāśi's beginning with meṣa is caled gata and in 6 rāśis beginning with tulā it is called gamya. Rāśi, degrees etc. of kendra are multiplied by 8 and divided by 60 to get the khaṇḍa number (because there are 8 parts of 3°45′ each in 1 rāśi of 30°) as before. Khaṇḍa phala of completed parts is corrected for fraction parts by addition if khaṇḍa phala is increasing, or by substraction if it is decreasing. This is śīghra phala. (176)

If śīghra kendra is in first 6 rāśis, khaṇḍa phala is added (to manda sphuṭa graha), or in other six rāśis it is substracted. This way madhyamā graha is made sphuṭa by śīghra phala half, half mandaphala, full manda phala and full śīghra phala. (177)

For ravi, candra and maṅgala, manda paridhi is different for different quadrants. So their mandaphala also has been written for 48 parts of 180° like śīghra phala. For value between two khaṇḍas, we add fraction of khaṇḍa phala difference if khaṇḍa phala is increasing. It is substracted when khaṇḍa phala is decreasing. Manda phala is never retrograde. (178)

For manda phala of maṅgala, there is no need of calculation between 22nd and 28th khaṇḍas. For that interval khaṇḍa phala is constant 11°2′47″. (174)

Śīghra kendra gati is substracted from khaṇḍa phala and result is divided by 225′. Half of the result is added to madhya gati, if śīghra phala is increasing. It is substracted, if śīghra phala is decreasing. We get Ist corrected gati. (180)

First gati is multiplied by manda phala dfference between two khaṇḍas in which 2nd manda kendra lies and divided by 225′. Half of the result is added to first gati, if manda kendra is betwen 90° to 270°, otherwise it is substracted. We get second gati. (181)

2nd gati is multiplied by manda phala difference for 2nd graha and divided by 225. Result is added or substracted from second gati to get third gati śīghra. (182)

3rd gati substracted from śīghrocca gati gives fourth śīghra kendra gati. This is multiplied by khaṇḍa phala difference of 3rd graha (manda sphuṭa) and divided by 225′. Result is added to 3rd gati, if śīghra kendra is in 90° to 270°, otherwise substracted. We get spaṣṭa daily gati. If it is negative, graha is vakrī (retrograde). (183)

For maṅgala, budha and śani, vakra mandocca gati is added to Ist and 2nd gati and mārgī mandocca gati is substracted to find the kendragati from mandocca. Mandaphala of this manda kendra is found for second and 3rd gati. śīghra. (184)

If mārgī (forward) mandocca gati is more than first gati, then first gati is substracted. From remainder second gati will be calculated. Similar method is used for finding 3rd gati. Gatiphala is corrected in reverse manner i.e. substracted for

manda kendra between 90° to 270° and added for other values. (185)

This way we get second and third gati of the three planets maṅgala, budha and śani. (186)

If Ist gati of budha and śani is vakra and less than mandocca gati, then it is substracted to get second gati. If vakra gati is more, mandocca gati is substracted from it but mandagati phala is added or substracted in opposite order. (187)

Verse 188 - If in chart of khaṇḍa phala, some khaṇḍa phala is missing or unclear, then its khaṇḍa number is multiplied by 225′ and for kendra of that kalā, we find bhuja and bhuja koṭi.

Verses 189-191 - Difference from sphuṭa sūrya in degrees is given at which a graha sets due to sun rays

Vakrī Sukra 7°, Śukra (mārgī) 9°

Guru 10°, Chandra 11°

Budha 12°, Śani 14°, Maṅgala 16°

These values in degree are multiplied by 1800 and divided by rising time of the rāśi in which śāyana sun is situated. This will be kśetrāṁśa. If it is in west, then 6 rāśi is added to the result. Then kśetrānśa is substracted from (sāyana sun + 6 rāśi.)

When maṅgala, guru and śani are less than ravi by at least the kśehānśa, they rise in east before sunrise. When they are ahead of sun by kśetrāṁśa, they set in west after sun. (Thus they are visible only in night). When vakrī budha and śukrā are behind ravi by this kśetranśa, they set in east and when ahead of ravi, they rise in west.

(Just before sun rise, since sun is coming upon horizon, they go down being vakrī. During night, they are visible when sufficiently away). Similarly, they rise in west just after sun set when vakri).

Notes : (1) Rising times of rāśis is explained in Tripraśnādhikāra. Briefly, rāśis rise in different time because it is oblique with equator (23-1/2)°). At places farther from equator, obliquity rises and difference in rising time of rāśis increases. This calculation is done for sāyana surya, because sūrya goes on equator when sāyana surya is at 0° or 180°. Roughly the planets are assumed in same plane as sun, as their inclinations to ecliptic are very small. So rising time for their difference along ecliptic will be same as rising time of sayana sun for that rāśi. Since rising time is given for 1 rāśi of 1800 kala in asu, equivalent difference on ecliptic is given by multiplying given degrees (kālāṁśa) by 1800 and divided by rising time of rāśi.

This is almost same as kālāṁśa, being its projection on ecliptic.

(2) When planets are behind sun, they rise before sun in east, if difference is more than kālāmśa. Being behind, earth horizon in east meets them after wards. Vakrī budha and śukra have already been explained.

Verses 192-193 : Finding time of udaya or asta

From the kālāmśa given we can calculate the time in days since when graha has set or risen (heliācally). If their difference with sun is more than Kālāmśa, the planet has already risen or set. If it is less than kālāmśa, the time to reach kālāmśa

can be calculated, which will be days after which planet will rise or set.

(Difference of planet and sun - kālāmśa) is divided by difference in speeds of sun and the planet. The no. of days will be found since when planet is rising (or setting) or after which it will rise again.

Verse 194 : Start and end time of rising and setting of planets should be written in the practical calender, because it is very difficult to find it by dṛk karma.

Verse 195 : In appendix, khaṇḍaphala and their differences are given. Similarly differences of gati phala, and karṇa (in kalā) also should be calculated and written. Śīghrakarṇa, gati and sphuṭa positions etc will be found by values given for places just before the given position. Difference of phala is to be added or substracted when the value (phala) is increasing or decreasing.

Verse 196 : Frequency for finding true positions - Sun and moon should be made sphuṭa every day at sunrise time. At end of a pakśha, all graha should be made sphuṭa. Budha should be made sphuṭa in middle of pakṣa also (i.e. every week). When a planet becomes mārgī from vakrī or vice versa, or changing from one rāśi, nakśatra to another, or start of rising time or setting should be calculated more accurately by method of successive approximations.

Verse 197 - There are 200 kalā (minutes of arc) in a quarter of a nakśatra, 800 kalā in a nākśatra and 1800 kalās in a rāśi. To find the days since when the graha is in a particular rāśi, nakśatra or

quarter of a nakśatra, we take the difference of rāśi etc of graha and the rāśi etc of the beginnig of rāśi, naksatra or its quarter. The difference is divided by sphuṭa gati kalā. Result will be days etc since when the graha had entered that rāśi etc. When graha is less than rāśi of nakśatra etc, the reverse difference will be divided by sphuṭa gati. Result time in days etc. will give the period after which graha will enter that nakśatra etc. When graha is vakrī, opposite process will be done.

Notes : (1) Ecliptic of 360° has been divided into 12 rāśis and 27 nakśatra of equal interval. Hence

1 rāśi = 30° = 1800′ Kalā

1 nakśatra = 13°20′ = 800′ kalā

1 nakśatra quarter (1/4 or pāda) = 3°20′ = 200′ kalā

(2) Rāśi's starting from 0° of ecliptic are

(1) meṣa (2) vṛṣa (3) mithuna (4) karka (5) simha (6) kanyā (7) tulā (8) vṛścika (9) dhanu (10) makara (11) kumbha and (12) mīna

Nakśatras starting from 0° of ecliptic are

(1) aśvinī (2) bharaṇī (3) kṛttikā (4) rohiṇī (5) mṛgaśirā (6) ārdrā (7) punarvasu (8) puṣya (9) aśleṣā (10) maghā (11) pūrvā phālgunī (12) uttarā phālgunī (13) hasta (14) citrā (15) svātī (16) viśākhā (17) anurādhā (18) jyeṣthā (19) mūla (20) pūrva āṣāḍha (21) uttara āṣāḍha (22) śravaṇa (23) dhaniṣṭhā (24) śatabhiṣ (25) pūrva bhādrapada (26) uttara bhādrapada (27) revatī.

(3) Within a rāśi or nakśatra a graha can be assumed to have the same true motion hence the formula uses the relation–

Distance in kalā = days X speed per day in kalā.

(4) Candra moves faster and position of candra and sun are to be known accurately for start of day, tithi etc. Hence they are to be calculated each day. Other planets are not so important so they can be calculated each pakśa (fortnight). Budha moves faster, hence its calculation should be done twice in a fortnight.

(5) For change of vakrī or mārgī gati or rising or setting times, the speeds change within a day also. Hence calculation needs to be made accurate by method of successive approximation.

Verse 198 : Dainika spaṣṭa gati of a graha can be found roughly by taking difference of spaṣṭa graha at beginning and end of the pakśa (fortnight) and dividing it by number of days in it (round figure of 14 or 15 when days are counted from sunrise to sunrise) Difference between spaṣṭa graha on two successive days at sunrise is more accurate dainika gati which is useful for calculation. Both differ very little, so very little error is made if we take average daily speed for a pakśa. If the two are different, then method of successive approximation is used.

Verse 199 - Fourth śīghra kendra is calculated at the end of every pakśa. As already stated, śīghra kendra of graha for which it becomes mārgī or vakrī, its rising and setting has been given in appendix. To find the position of śīghra kendra at any time between pakśa ends, divide the difference beetween values at end with days of pakśa and add them proporitionately for the time passed.

Verse 200 : 21,600 kalā divided by 30, 27, 12, 27 and 60 gives measures of tithi, nakśatra, rāśi, yoga and karaṇa, i.e. 720, 800, 1800, 800 and 360 kalās.

Notes : (1) Tithi, nakśatra, yoga, karaṇa and vāra are five parts of a calender - hence it is called pañcāṅga. Vāra is successive counting for days starting from sunrise, hence no calculation is needed.

(2) Definitions - 'rāśi' is 30° part of the ecliptic where planets move. Rāśi of a planet means its completed rāśis from 0° of ecliptic as well as degrees, minutes, seconds, lapsed in the current rāśi. Though it is not part of pañcāṅga, it is used to calculate all other parts.

Nakśatra is found by dividing ecliptic into 27 equal divisions of 13°20′ each (total 360° = 27 X 13°20′) Each part is nakśatra. 'Nakśatra' mentioned in pañcāṅga means the nakśatra which is occupied by moon at a particular time.

Tithi is $\frac{1}{30}$th part of a lunar synodic month, i.e. the time when moon goes one circle more than sun. It is measured usually from the time when sun are moon are together, i.e. difference between their rāśi is 0°. That is start of first tithi called amāvāsyā, i.e. when sun and moon live (vāsa) together (amā = amity = closeness) Month can also be counted from time when sun and moon are in opposition (i.e. 180° away) Then full moon is seen, so that is end of pūrṇimā tithi. The two systems of lunar month are called amānta (ending with anāvāsyā) or pūrṇānta (ending with pūrṇimā).

Tithis are not counted serially from one to 30 in lunar month. They are counted from 1 in each half (Śukla = bright and kṛṣṇa = dark) In śukla pakśa last tithi is written 15 and in kṛṣṇa pakśa it is written 30 (denotin$_{\text{o}}$ end of month).

Since 360° difference between moon and sun causes 30 tithis, 1 tithis is result of 12° difference. Thus difference of 0° to 12° is Ist tithi in śukla pakśa after amāvāsyā, 12° to 24°, 2nd tithi etc. upto 180° the pakśa will be śukla pakśa with 15 tithis. Between 180° to 360° difference it will be kṛṣṇa paksa with 15 tithis. Thus the number of complted tithis

$$= \frac{\text{Moon} - \text{sun}}{12^0}$$

Fraction will give the part elapsed in the current tithi which is next after completed tithi.

When the quotient is more than 15, than 15 is substracted to know tithi of kṛṣṇa pakśa.

Karaṇa is half part of tithi, caused by 6° difference between moon and sun. Thus completed karana since amāvāsyā end

$$= \frac{\text{Moon} - \text{sun}}{6^\circ}$$

These are not counted from 1 to 60 in a month, but there is rotation of 7 karaṇas like 7 week days, 8 times in a month and 4 remaining karaṇas are given separate names fixed at both ends of a month. This is explained later in detail.

Karaṇa and tithi both indicate the phase of moon, i.e. the fraction of its disc which is illuminated. Nakśatra and rāśi of moon (or any

other planet) can also be physically seen. But yoga is not a physical quantity. It is only a mathematical function given by sum of rāśi etc of moon and sun (for tithi and karaṇa, their difference had been taken). However, one full revolution of moon + sun is not divided into 30 parts like a tithi, but in 27 parts only like a nakśatra. Thus for each increase in sum of moon and sun by 13°20′ one yoga passes. Thus number of completed yogas counted from time when sun of moon + sun was 360° or 0° is

$$\frac{\text{Moon + sun}}{13^\circ\ 20'}$$

List of yoga is given later.

(3) In a full circle there are 21,600 liptā or kalā. Hence measure of nakśatra etc is found by their total number in circle by which 21,600 is divided.

Verse 201-202 - Calculation of tithi -

Time lapsed (gata kāla) and remaining time (gamya kāla) of the current tithi is found by dividing difference of moon and sun in kalā by 720 kalās. Remainder is converted to vikalā (on multiplication by 60). This will give gata kāla. Dainika gati of ravi and candra is found by difference of current day and next day's position. Gata or gamya tithi is divided by difference of dainika gati of moon and sun. This will give value in daṇḍa etc. (when gata tithi was in vikalā). This is rough approximation, sufficient for normal work. In this we have used dainika gati for 1 sāvana dina in stead of gati in 1 tithi. If further accuracy

is needed, we find gati of a tithi from dainika gati and ravi, candra are further corrected.

Verse 203 - Lapsed or remaining time in rāśi or nakśatra -- Sphuṭa kendra is converted to kalās and divided by 800. Quotient will be number of past (gata) nakśatras counted from aśvinī. By adding 1, we get the number of current nakśatra. Remainder is the lapsed part (in kalā) of the current nakśatra. Substracting this from 800′ we get remaining part. It is multiplied by 60 to make vikalā and divided by dainika gati (in kalā). This will give lapsed (or remaining) time of nakśatra in daṇḍa etc.

Sphuṭa candra converted to kalā and divided by 1800 kalā in a rāśi gives number of completed rāśis. By adding 1 to quotient we get the number of current rāśi, counted from meṣa. Remainder will be lapsed part (in kalā) of the current rāśi). It is substracted from 1800′ to give remaining (gamya) part. Gata or gamya part is converted to vikalā by multiplying with 60 and dividing by spaṣṭa dainika gati of candra. We get gata or gamya kāla of the current rāśi in daṇḍa etc.

Note - Gata or gamya part (in kalā) == x = 60 x vikalā.

Dainika gati = Difference in position in 1 day = $\frac{\text{kalā}}{\text{day}}$

Hence $\frac{\text{gata part}}{\text{gati}} = \frac{\text{x kalā}}{\text{Kalā/day}} \times \text{day} = 60\text{x daṇḍa}$

Hence n is converted to vikalā before division by gati.

Verse 204 -- Calculation of yoga

Add the rāśi of sphuṭa candra and sūrya. If it is more then 12 rāśi's, substract 12 raśi from the sum. It is converted to kalā and divided by 800 =

no. of kalā in a yoga. Quotient will be number of completed yoga counted from viṣkuṁbha. Add 1 to it, we get number of current yoga. Remainder gives part of yoga lapsed in kalā. By substracting it from 800′, we get remaining part of current yoga. Gata or gamya kalā is multiplied by 60 to make it vikalā and divided by sum of dainika gati of sun and moon. We get gata (or gamya) time in daṇḍa etc.

Note : List of yogas -- (1) viṣkumbha (2) prīti (3) āyuṣmāna (4) saubhāgya (5) śobhana (6) atigaṇḍa (7) sukarmā (8) dhṛti (9) śūla (10) gaṇḍa (11) vṛddhi (12) dhruva (13) vyāghāta (14) harṣaṇa (15) vajra (16) siddhi (17) vyatīpāta (18) varīyāna (19) parigha (20) śiva (21) siddha (22) sādhya (23) śubha (24) śukla (25) brahma (26) aindra (27) vaidhṛti

Verse 205 - Calculation of karaṇa

Add 360 kalā to spaṣṭa ravi in kalā. Deduct the sum from sphuṭa candra in kalā. Divide the difference 360 i.e. no. of kalā in a karaṇa. Quotient is divided by 7. Remainder is number of completed karaṇa. By adding, we get the current karaṇa. Karaṇa starts from second half of śukla Ist day with 'Bava'. After end of seventh karaṇa, again first karaṇa 'bava' starts. In 30 tithis of cāndramāsa, there are 60 karaṇas. 7 Karaṇas are repeated 8 times. Remaining 4 karaṇas are fixed (sthira) which are śakuni, nāga, catuṣpada and kinstughna.

Note : (1) Moving karaṇas start after Ist half of Ist tithi (śukra Ist tithi) has already passed. Hence 360 kalā is added to ravi so that in difference from moon, 1 karaṇa is deducted.

(2) Seven moving karaṇas (chala karaṇa) are - (1) bava, (2) bālava (3) kaustubha (4) taitila (5) gara (6) vaṇija (7) viṣṭi or bhadrā. Last karaṇa is considered inauspicious for good work. Similarly sunday was not supposed a day for doing work out of seven week days.

(2) Sthira karaṇas śakuni, nāga, catuṣpada and kinstughna start from kṛṣṇa 14th second half, 30th (15th kṛṣṇa both halves) and śukla Ist tithi.

(3) In vedāṅga jyotiṣa, 11 karaṇa or half days were deducted from solar half year (equinox to next eqinnox in opposite direction) to make it equal to lunar month. 371 tithis in a solar year are divisible by 7, though 365 days are not divisible, hence fraction of weeks remain. Similarly in half year, karaṇas (half tithis equal to 371) are divisible by 7. Out of 11 karanas last 4 are fixed, as in a month also 4 remain after 7 X 8 cycles of 56 karaṇas.

Verse 206 - If at time of sunrise, the total gata and gamya kalā of tithi (720) is more than the difference in dainika gati of candra and ravi (i.e. difference is less then 720′ per day), then tithi is long (tithi vṛddhi) i.e. more than 60 dandas. Tithi vikalā 720 x 60 divided by difference of candra and ravi gati, we get duration of tithi in daṇḍa etc. If it is more than 60 daṇḍa then there is tithi vṛddhi, otherwise tithi kśaya occurs.

Verse 207-209 - Extra and ommitted candra months– When in a cāndra māsa, there is sūrya saṅkrānti (i.e. sūrya goes from one rāśi to another), then it is called śuddha cāndra māsa (i.e. normal month). When there is no sūrya saṅkrānti (i.e. sūrya remains in same rāśi), it is called extra month

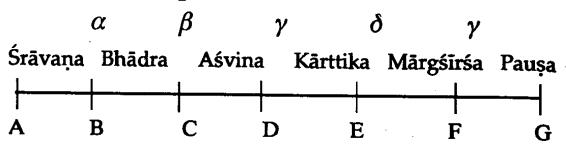
(mala or adhika māsa.) Next amānta month is called normal cāndra māsa. When there are two saṅkrāntis of sūrya in a cāndra masa, it is called kśaya māsa (lost month) - i.e. next, cāndra māsa is not counted. Before and after kśaya māsa, within 4 months there are one mala māsa each i.e. two mala māsa in that year. First mala māsa is called saṅsarpa, kśayamāsa is called aṁhaspati and later malamāsa is called mala. Both mala and kśaya māsa are prohibited for any auspicious work. (207)

In veda and smṛti, the works which are prescribed, monthly and annual śrāddha can be done in saṅsarpa or aṁhaspati, but not in the later malamāsa. Malamāsa is counted as a month for annual śrāddha of a dead man, when it comes within start and completion of a month. New work is not started in a malamāsa, but work started earlier can be continued. The following works can be done in a malamāsa–

Bath during eclipse, charity, observing rare yogas (auspicious times), sudden works, promised work, coronation, śānti, puṣti karma, functions related with child birth, śrāddha etc. (208)

A kśaya māsa is repeated after 141, 122 or 19 years. In current year (1869 when book was written) mandocca of sun was in mithuna, hence in 9 months from phālguna, a mala māsa is probable. 3rd months after kārttika may be kśaya māsa, Māgha month may be kśaya or adhika.

Notes : (1) A lunar synodic month is approximately 29.5 days long, where as sūrya remains in a rāśi of 30° for 30.4 days. Thus lunar month is completed earlier and after about 30

months extra days in solar month will amount one month and sun will not cross to next rāśi. Example of mala māsa is explained below -

ABCD - - -- are krānti of sun. Signs on upper part denote start of a lunar month. In Bhādra there is no saṅkranti so it is a mala māsa.

(2) Frequency of malamāsa - There are 1593336 malamāsa is 51840000 solar months of a yuga i.e. 66389 adhikamāsa in 2160000 solar months.

$$\frac{66389}{2160000} = \frac{1}{32}, \frac{1}{+1}, \frac{1}{+1}, \frac{1}{+8}, \frac{1}{+1}, \frac{1}{+1}, \frac{1}{+5}$$

Convergents are $\frac{1}{32}, \frac{1}{33}, \frac{2}{65}, \frac{13}{425}, \frac{15}{488}, \frac{25}{911}$

$\frac{1}{33}$ and $\frac{2}{65}$ are on either side of the true figure.

Hence adding numerator and denominator both, we get a better approximation. Thus 3/98 is ratio of adhika māsa i.e. 3 adhika māsa in 98 months (solar).

(3) Adhika māsa and year -- There are 1,593,300,00 adhika māsa in a kalpa of 4,320,000,000 years

i.e. 5311 adhika māsa in 14400 years

$$\frac{14400}{5311} = 2\frac{1}{1+}\frac{1}{2+}\frac{1}{2+}\frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{7+}\frac{1}{8+}\frac{1}{2+}$$

Successive Convergents are

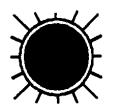
$\frac{2}{1}, \frac{1}{3}, \frac{3}{8}, \frac{19}{7}, \frac{122}{55}, \frac{141}{62}$

Thus there are approximately 7 adhika māsa in 19 solar years which was used in vedāṅga jyotiṣa (Ṛk veda). This was known in Romaka siddhānta and was called Metonic cycle in Greece.

Next approximations also indicate possibility of kśaya māsa in 19, 122, 141 years.

Verse 210 - Thus the rough pañcāṅga with its components like tithi and nakśatra is completed which may be accepted by the learned and they may perform every year the daily, occasional and conditional functions, fasting days, śrāddha, festivals etc. according to this pañcāṅga. This may do good of world as it is according to jyotiṣa samhitā and well thought of.

Verses 211-212 - For daily auspicious functions I am preparing this pañcāṅga with positions of sun and other planets. While doing the work I pray to lord Jagannatha who is on nīlācala shining like black soot (for eyes).

Thus the fifth chapter describing true planets with their khaṇḍa phalas is over in siddhānta darpaṇa written for education of children and calculation as per observation by Śrī Candraśekhara born in a famous royal family of Orissa.

Chapter - 6

CORRECTIONS TO MOON

Scope - Accurate pañjikā and further correction to motion of Moon

General Introduction

(1) Equation for elliptical orbit round earth.

Eccentricity of moon is 0.0548442 = e

So e^2 = 00.0030079, e^3 = .00016496

e^4 = .00000905

Higher powers e^5 etc are very small and can be neglected. Thus θ measured from mandanīca or perigee is given in terms of position m of mean planet as

$$\theta = m + (2e - \frac{1}{4}e^3 + \frac{5}{96}e^5) \sin m$$

$$+ (\frac{5}{4}e^2 - \frac{11}{24}e^4 + 17\frac{e^6}{192})\sin 2m$$

$$+ (\frac{13}{12}e^3 - \frac{43}{64}e^5) \sin 3m$$

$$+ (\frac{103}{96}e^4 - \frac{451}{480}e^6) \sin 4m + \frac{1097\ e^5}{960} \sin 5m \ldots$$

$= m + (0.1096884 -- 0.00004124) \text{ Sin } m$

$+ (0.0037599 - 0.00000415) \sin 2m$

$+ 0.0001787 \sin 3m + 0.0000097 \sin 4m$

$= m + 0.10964716 \sin m + 0.00375575 \sin 2m$

$+ 0.0001787 \sin 3m + 0.0000097 \sin 4m$

The sine ratios in radians are converted to kalā ($\frac{1}{60}$ degree) by multiplying with $\frac{180^{o}}{\pi} \times 60$ = 3437.75 kalā or 206265 vikalā. Then

θ = m + 376′56″. 4 sin m + 12′54″. 7 sin 2m + 36″. 9 sin 3 m + 2″.0 sin 4 m

Here m has been calculated from nīca or prigee. If it is calculated from apogee or mandocca, then

θ = m - 376′56.″ 4 sin m + 12′54″7 sin 2 m - 36″.9 sin 3 m + 2″. 0 sin 4 m

Here m on right side is manda kendra - i.e. distance of madhya graha from mandocca of moon. Remaining terms are mandaphala.

When θ = 90°, sin m = 1 and sin 2 m = 0

Then highest mandaphala depends only on its first term 377′ approximately or 6°17′. But our astronomers have taken highest mandaphala about 5° only (radius of mandaparidhi of 32°). However, on new moon or full moon day, when moon is 90° away from mandocca, then it is 1°20′ ahead of its calculated position. When moon is 270° ahead of mandocca or 90° from nica then it is 1°20′ behind its calculated position. Thus in both situations mandaphala correction is 6°16′56″.4-1°20′ = 4°56′.4 (correction is-ve for m = 90° and positive for m = 270°). Thus maximum mandaphala is about 5° only as observed.

However, in middle of a pakśa i.e. on 8th day, if this mandaphala correction for manda kendra 90° is taken as 5°, then observed moon is 3° behind calculated moon or 8° behind mean

moon. Thus cauculations in our siddhānta were true for pūrnimā or amāvāsyā when eclipse is to be calculated. One reason for such neglect is that accuracy is needed only for eclipse, other reason is that observations were done ony on pūrṇimā or amāvāsyā days or more accurately at time of eclipse. This is still followed by muslims and even now eclipses are studied for more accurate observation.

(2) Deviations in moon position due to effect of sun - Effect of sun is three types

(a) Attraction component of sun on moon in direction of earth moon radius, elongates the orbit in the direction of sun and away from it. It changes eccentricity of orbit and is called evection term. Since it changes eccentricity of orbit, called 'cyyti' it was called 'cyuti' sanskarā by Śrī Veṅkateśa Bāpūjī Ketakara in his Jyotirgaṇita. Since it changes angle from mandocca (or Tuṅga = top), it has been called 'Tuṅgāntara' saṅskāra in siddhānta darpaṇa.

(b) Component of sun's attraction on moon in direction of moon's motion advances it towards sun, which is maximum in middle of a pakśa and nil at its ends. This varied speed, hence it was called variation. Its frequency is in 1 pakśa, hence it is called pākśika sanskāra in siddhānta darpaṇa. Śrī Ketakara called it tithi saṅskāra because it depends on tithi of the pakśa.

(c) Due to difference of sun's distance from earth or moon depnding on its direction from earth, its attraction force on moon varies in a period of 1 year. This is called digaṁśa saṅskāra as it amounts to 1/10 of sun's equation. This is also called vārṣika sanskāra because its period of variation is one year.

Figure (1) (a), shows force of attraction G due

Fig 1 (a) Fig 1 (b) Fig 1 (c)

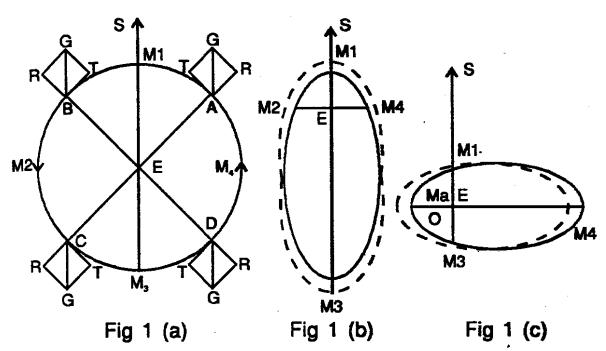



Figure 1 - Effect of sun's attraction on moon's orbit

to sun. In positions A and B which are near to sun compared to earth, extra attraction on moon is in direction of sun. In position C and D of moon, away from sun, the difference in force compared to earth is away from sun. Force of attraction G has two components, its component R is reducing the pull of earth on moon acting in opposite direction. Thus distance of moon increases from earth. This increase is maximum for positions M_1 and M_3 and nil for positions M_2 and M_4. Thus in Fig 1 (b), when major axis is in direction of sun, the axis will become longer and its eccentricity will increase.

In fig 1 (c), the distance perpendicular to major axis in sun's direction will increeease, due to which moon orbit will become round. Then eccentricity will decrease. Thus correction in man-

daphala due to eccentricity will increase for fig (1) (b) and decrease in position of figure (c).

Component T is maximum for position M_4 and increases the speed in middle of kṛṣṇa pakṣa. It decreases and becomes zero at M_1 as the force of attraction is totally in direction of EM_1, and other component is zero. It increases in value from M_1 to M_2 and again declines to zero at M_3. From M_1 to M_2 it is against the direction of motion.

It is in direction of motion between M_2, M_3 (decreasing) and again increasing upto M_4 but against the motion.

(3) Correction by different authorities - According to modern astronomy, principal terms of moon's motion are -

θ = m

+ (377′19″.06 sin m + 12′57″.11 sin 2 m.

· + 36″.9 sin 3 m + 2″.0 sin 4m) mandaphala correction.

+ 1°16′26″ sin [(2 (M--S) - m] - - Evection or Tuṅgāntara

+ 39′30″ sin 2 (M-S) - - variation or pākśika

+ 11′10″ sin (manda kendra of sun) - - - Annual or digaṁśa

The early astronomers of India recognised only the mandaphala correction, (equation of centre), but instead of its value to be 377′ sin m, they took its value 301′ sin m, by including effects of evection. By including 76′ part of mandaphala with evection we get - - -

301′ sin m + 76′ [(sin m + sin {(2 (M-S) – m}]+

= 301′ sin m + 152′ sin (M-S). Cos (S-α)

Here M = mean Moon, S = True sun, α = moon's perigee from which angle m has been measured. Thus m – (M-S) = S - (M – m) =s-α in cos term above.

Value of Ist correction to moon was the following according to different authors

Āryabhaṭīya 300′15″ Sin m

Khaṇḍa khādyaka 296′ sin m

Uttara khaṇḍa khādyaka 301′.7 sin m

Brahma sphuṭa siddhānta 293′31″ sin m

Greek value 300′15′ sin m

Siddhānta darpaṇa 300′49″.5 sin m

Sūrya siddhānta 302′23″.66 sin m

Bhāskara II, 301′46″.8 sin m

(a) Second correction term by Mañjula (932 AD)

In Laghumānasa, Ist mandaphala correction of moon has been given as

$$\frac{488' \sin m}{97' + \frac{488}{120} \cos m} \text{ degrees}$$

where m is mandakendra measured from apogee. Thus maximum value of mandaphala is for m = 90°,

$$\frac{488}{97} \text{ degrees} = 301'50''$$

Second correction has been given by

8°8′ cos (S-U) (True moon - 11) x 8°8′ sin (M-S)

where S,M,U are true sun, true moon and mandocca of moon.

For simplicity, daily motion of 790°35″ of moon is taken as true motion, then this becomes

8°8′ x 8°8′ x 2° 11′ cos (S-U) sin (M-S)

= 144°26′ cos (S-U) sin (M-S) - converted to minutes, 2nd correction - - - (1)

Thus Manjula's correction is sum of two correction -

(i) 76′ sin (M-U) - part of the mandaphala

(ii) 144′26″ cos (S-U) sin (M-S) - - - evection term which was not mentioned by previous astronomers.

Plotemy had given maximum value of 2nd correction as 159′ but didn't give any formula (150 A.D.)

Astronomer Yallaya gives credit of this discovery of these corrections to Vaṭeśvara (904 AD) but this has not been found in Vaṭeśvara siddhānta, the available book.

This appears in exactly the same form in karaṇa - kamala-mārtaṇḍa of Daśabala (1058). Subsequently it occurs in equivalent forms in siddhānta śekhara of Śrīpati (1039 A.D), Tantra Saṅgraha of Nīḷa Kaṇṭha (1500 A.D.), uparāga kriyā krama of Nārāyaṇa (1563 A.D.) Karaṇottama of Acyuta (1621 AD) and lastly in siddhānta darpaṇa of Candraśekhara (1869).

Equation (1) of Manjula is correct but constant is 8′, less.

Śrīpati's second correction amounts to the following correction term.

$$\frac{160' \cos m \sin (\text{mandaphala}) \times 1 - \cos (\text{mandaphala})}{\text{Mandakarṇa} - R}$$

where R = radius 3438′.

This is same as Manjula's equation except that the constant is now. 8′ more, instead of 8′ less earlier.

(b) Bhāskara II - Bhāskara II wrote a separate work called 'Bījopanaya' about corrections needed in true planets. Stanza 8 of the work starts with statement -

I have seen maximum difference between calculated and observed positions to be ± 112′

When moon is one quadrant ahead of mandocca and sun is half aquadrant ahead of moon, observed moon is 112′ behind calculated moon i.e. negative error.

When moon is 3 quadrants ahead of apogee and sun at half a quadrant behind her, the maximum positive discrpancy of + 112′ is seen

When eclipses of sun and moon take place and moon is at apogee and perigee, there is no error or bīja.

When eclipses take place at end of odd quadrants from apogee, error is negative equal to 34′

When moon is at the apogee, and sun is ahead or behind by half a quadrant, descrepancy is 34′

Same discrepancy is seen, when moon is at perigee and sun half quarter ahead or behind

His first equation of mandaphala was correct

$= - 301'46'' \sin m.$

But after Bījopanaya he gave the equation

$- 379'46''.8 \sin m + 34' \sin 2 (M-S)$

where m is manda kendra, M and S are true moon and sun. His new equation totally missed the evection term, and it became more incorrect at eclipses; though his observations about error were correct.

(c) Correction by Candraśekhara -

His first equation of apsis (mandaphala) is

$$\frac{(31^0 \; 30' - 30' \cos m) \; 3438 \; \sin m}{360^o}$$

$= - 300'49''.5 \sin m + 4'46''.5 \sin m \cos m$

$= - 300'49''.5 \sin m + 2'23''.25 \sin 2 m$

Though he has attempted to correct the second order of small quantities, his constant is too small (1/5th of the correct value).

(2) Tungāntara correction is of the form

$$\frac{160' \times 3438 \sin (\alpha - S - 90)}{3438} \times \frac{3438 \sin (M - S)}{3438}$$

(where α is apogee of moon)

$$\times \frac{\text{Moon's true deaily motion}}{\text{Daily mean motion}}$$

$$= - 160' \cos (S-\alpha) \sin (D-\theta) \times \frac{\text{Moon's apparent deaily motion}}{\text{Daily mean motion}}$$

(3) Pākśika equation or variation in Daily mean motion is $\frac{3438' \; \sin 2 \; (M - S)}{90} = 38'12'' \sin 2 (M-S)$

Here the constant is less by 1′18″ from modern value.

(4) Digansa sanskāra for annual variation is

$$\pm \frac{1}{10} \times \frac{12 \times 3438}{360} \sin S_m$$

(Sm = manda kendra of sun)

= ± 11′27″ sin Sm

Modern value of the constant is 11′10″. Tycho found it to be 4′30″. Horrocks′ (1639)) found it 11′51″. He has indicated in the text that new equations were to correct the discrepancies observed by Bhāskara II, in which he was brilliantly successful.

(4) Modern charts for calculating moon's position -

Constants of moon's motion at 1900 AD, 0.0 day epoch is

Mean longitude L = 294°.56984 + (1336 r) 307.8905722 T + 0.00918333 T^2 + 0.00000188 T^3

Mean anomaly M = 229°.97832 + (1325 r) 198°51′23″.5T + 44″.31T^2 + 0″.0518T^3

Mean longitude of node V = 259°12′35″.11 - 6962911″. 23 T + 7″. 48T^2 + 0.008T^3

For perturbations the constants are given by Hansen as-

A_0 = 69.80458 + (1148r) 55.37787761T + 0°.00881085T^2 + 0°.0000011374958T^3

B_0 = 352.81434 + (2473r) 254°.23441630T + °.000420645 T^2 + 0°.00000301393 T^3

C_0 = 204°.85020 + (99r) 359°.051667T + 0.0001988055T^3

D_0 = 190°.45443 + (1048r) 56°.32271091 T + 0°.007903044 T^2 + 0°.000011374958T^3

E_0 = 354°.45312 + (2373r) 255°.17924960 T + 0°004405255 T^2 + 0°.00000301393 T^3

F_0 = 341°.85083 + (1131r) 172°.20183595 T + 0°.00430092 T^2 + .000003347264 T^3

Components of perturbation effect are -

A = 4467″ Sin Ao = 1.24083° Sin Ao

B = 0.59583 Sin Bo

C = 658″ Sin Co = 0.18277 Sin Co

D = 0.55 Sin Do

Total effect of perturbation = G = A+B+C+D+E

Perturbation in latitude is

F = 0.1453 sin Fo

From the value of these constants equation of centre and latitude is calculated.

(5) Indian Charts -

In India also many charts were prepared from time to time. Makaranda sāraṇī was most famous. Candraśekhara has referred to tables of Kochannā of Āndhra pradesh. Then in south India, specially in Kerala, vākya karaṇa are very famous. Original Vākya karaṇa was written for moon--called candravākyāni by Vararuci, reputed to be in time of king Vikramāditya at start of Vikrama eera. Then Vākya karaṇa was prepared in 13th century. Its writer is not known, but Sundararāja commentary is available. These books calculated the days from kaliyuga beginning. The moiton was calculated for a convenient lump of days. For remaining number of days, the true position was calculated at about

200--300 positions. These were indicated by (vākya' for each of position to be read in Kaṭapayādi notation. This method could give correct position upto minute for 24 hour intervals. Mādhava of saṅgamagrāma in 1350 AD, prepared 'Sphuṭa candrāpti' to calculate true moon upto seconds of arc at 9 periods in a day. His method was to calculate position of moon at equal intervals of 24 hours from its mandocca position. Moon reaches from mandocca to mandocca in about 248 days, so 248 vākyas are used.

(6) Making of a calender -

One of the main aims of astronomy is to find suitable measurement of time. A time scale to indicate past time since an epoch is a calender.

Intervals of time which can be measured is one type of kāla and its measurement is called 'kalana' Thus 'calculate' means to count or to measure. In Arab, they were called 'kalamma' Work of 'kalana' is called chronology or calender.

The flux of time is apparently without beginning or end, but it is cut up periodically by several natural phenomena–

(i) by ever recurring alteration of day light and night

(ii) by the recurrance of moon's phases

(iii) by the recurrance of seasons

These have been used to define natural divisions of time-

Day - time of alteration of day and night

Month - Complete cycle of moon's changes of phase -

New moon to new moon (amānta month) or full moon to full moon (pūrṇimānta) months.

Year - Coming back of a season again and its smaller subdivision season.

Standards for day - Day for purpose of regular works was counted from sun rise to sunrise in India and from sunset to sunset in west asia (Babylonians and Jews). West Asia was called 'Asura' area and hence they were called niśācara (moving in night) because their day started from night time. Sunrise and sunset are convenient to see and day light only gives opportunity for doing works.

Sunrise time varies according to position of sun in south or north hemisphere of sun. Variation of day length is more in places away from equator, being nil at equator. Hence for calculation purposes day was counted from midnight to midnight.

Even midnight to midnight day varies, because during this time earth makes one rotation arounds its axis with respect to stars and has to move further to catch up with movement among stars. This second component varies with distance of sun which varies in an elliptical orbit. Thus revolution of earth with respect to stars is taken as a better standard called sidereal day. An average of solar day (midnight to midnight) is used and called mean solar day.

$$365\frac{1}{4} \text{ mean solar days} = 366\frac{1}{4} \text{ sidereal days}$$

$$1 \text{ hour} = \frac{1}{24} \text{ of mean solar day.}$$

Rotaton of earth = 23 h 56 m 4.100s mean solar time

Sidereal day = 23h 56m 4.091s mean solar time

Mean solar day = 24 h 3m 56.555s sidereal time Slight variation in rotation period of earth and sidereal day is due to obliquity of earth, rotation being counted in the ecliptic plane. Even earth's rotation period is not constant but fluctuates regularly and irregularly by amounts of the order of 10^6 seconds. Regular slowing down of rotation period is 14 seconds per century due to tidal friction caused by difference of attraction force on sea water in different parts of earth. It is mainly by moon and 1/4th by sun. Irregular variation is due to force exerted by wind movements or unequal rate of atmospheric rotation and sea currents, both of which are caused by heat of sun.

Month -

Period from new moon to new moon varies from 29.246 to 29.817 days due to eccentricity of moon's orbit and other causes like effect of sun. Period of mean lunation is given by

29.5305882-0.0000002 T days

where T = no of centuries after 1900 AD.

It may be noted that this is not the period of rotation of moon round earth. This is extra one round ahead of sun. When moon and sun are together, it is amāvāsyā (living together). Moon with its faster motion goes ahead in about 15 days by 180° when it is pūrṇimā (or full moon). After 29.5 days it is again with sun. This rotation is with

speed (moon-sun) and slower than moon's rotation in 27.3 days only.

Year and seasons -

1 year is one rotation of sun with respect to stars - it is called sidereal year. Seasons change according to position of sun with respect to earth in north south direction. It is perpendicular to equator twice in one year, while coming from south to north it is called vernal equinox and in opposite direction it is autumnal equinox. Equinox means equal day and night (nakta in sanskṛta = night) If axis of earth is fixed, tropical and solar years will be same. But it rotates in reverse direction in a conical manner, thus equinox points rotates west ward making a rotation in about 25000 years. Due to this precession of equinoxes occurs.

Tropical year = Sidereal year - speed of precession per year (crossing time by sun)

Present values are

Tropical year == 365.24219879 - 0.614 (t-1900)$\times 10^{-7}$ days, where t = Gregorian year

Thus it is 365.2421955 days = 365d 5h 48 m 45.7 sec.

Sideral year is 365.256362 days.

Only tropical year corresponds to the seasons

In addition to two equinoxes, we can take the points of longest day (in north hemisphere) where sun is northern most from equator i.e. summer solstice or the southern most position called winter solstice.

As the day is counted from midnight i.e. lowest position of sun in east west circle, year can

be counted from southern most winter solstice (which is lowest for northern hemesphere). This is like a grand day hence one tropical year is called a 'divya dina' (divine day). Since the grand day starts with winter solstice from vedic days, the first day 'christmas' is called 'badā (grand) dina'. Actually it is start of grand day. That month called mārgaśīrṣa has longest nights hence it is called Kṛṣṇa māsa (or black month). Thus Kṛṣṇa has compared himself with mārgśīrṣa month in gītā. This has become 'Christmas' (Kṛṣṇa māsa). 15 days before start of mārgāśīrṣa māsa will be beginning of great uṣā (Twilight before sun rise), hence it is called 'baḍa oṣā' in local languages (like in Orissa)

Problems in calender making -

Civil calender for use in human life has following difficulties

(a) Civil year and the month must have an integral numbers of days - perferably equal

(b) Starting day of the year, and of the month should be suitably defined. The dates must correspond to seasons.

(c) For the purpose of continuous dating, an era should be used and it should be properly defined.

(d) The civil day, as distinguished from the astronomical day, should be defined for use in the calender.

(e) If the lunar months have to be kept, there should be convenient devices for luni solar adjustments.

All the problems have not been solved till today. The errors in calculations also had to be corrected. Hence new calenders were started in different parts of the world by the intervention of dictators like Julius Caesar, Pope Gregory XIII or a founder of religion like Mohammad, or by monarchs like Melik Shah the Seljik or Akber.

Owing to historical order of development, calenders have been used for double purpose.

(i) of the adjustment of the civic and administrtive life of the nation.

(ii) of the regulation of the socio religious life of the people.

Divisions of day :

Present division of day is in 24 hours. Minute divisions of 60 each called minutes and second division again by 60 called seconds. Thus 1 mean solar day = 60 × 60 = 86,400 seconds. Division of time and angle measures by 60 was because of 30 days in a month and 12 lunar months in a year whose lowest common multiple is 60. A day has 365 but approximate multiple of 60 is 360. Hence a civil year was taken of 360 days and a circle was divided into 360°. Thus sun will move about 1° in 1 day. In India, day was divided into divisions of 60 at each step as degree is divided. Thus 1° movement is in 1 day, 1′ movement in 1 daṇḍa, 1″ movement in 1 pala and so on.

Time was measured by length and direction of shadow of a pillar called gnomon. For equal time intervals, specially during night time, water clock etc were used. Improvements were done through pendulum clocks by Galileo, spring clocks

using balance wheel. Most occurate are quartz clocks for normal use and ammonia clocks for scientific use.

For practical watches of duty or shifts of work, a day was divided into 6 parts (3 parts in day time and 3 in night). After each interval a bell was rung. In India there were 8 shifts in a day, hence the shift of 3 hours is called a 'prahara' i.e. when a bell is hit (prahāra). A watchman remains on continuous duty for a prahara, hence he is called praharī.

$\frac{1}{60}$ day = 1 ghaṭī is called so because water clock measured the time of its filling. Since it was shaped like a pitcher it is called 'ghaṭī' (i.e. water pot). Hence watches are called 'ghaḍi' in India. When water clock in turned a second time it is 2 ghati = 1 muhūrtta (repeated turning of water clock).

Watches observed in churches were

(1) Martins - last watch of night. Monk got up 2 hours before sunrise

(2) Prima - at sunrise

(3) Tetra - Half way between sunrise and noon - time of saying mass.

(4) Sext - at noon (hence the word siesta = midday rest)

(5) Nona - Mid afternoon - Hence the word noon.

(6) Vespers - An hour before sunset

(7) Compline - at sunset

In India mid day is 2 praharas after sunrise (i.e. 6 hours after), hence it is still called 'two pahars'.

Day was divided into 12 parts in Babylone of 30 gesh (4 minutes each). In each part approximately 1 sign of zodiac will rise, it is like 12 divisions of year. In India rāśi was divided into two parts (like day-night divisoin of day) called 'Horā' (short of 'ahorātra' i.e. day and night) Thus there are 24 horās is a day night or 12 in day and 12 in night. This 'hora' has become hour. This was also used in Egypt and continues till today.

Counting of days in a month :

The ancient Iranian calender gave 30 names for each of the days of a month. It was not very popular as the list is long and difficult to remember. Hence a week of seven days was popular through out the world. Origin of week days has been explained by Varāhamihira. Each hora (24 in a day is ruled by a planet. Planets are arranged in order of decreasing orbit or increasing speeds of rotation - Śani, guru, Maṅgala, sūrya (or earth), śukra, budha and Candra. In first horā of the day, lord of the day will rule. For example, Śani will rule Ist horā on śanivāra. On next day ruler will be 25th planet in the order given above. Deducting 3 cycles of 7 planets, 4th planet sūrya will be ruler of next day i.e. Ist horā on that day. So it is called ravivāra or sunday. Next day will be 4th from sūrya i.e. candra or moon called somavāra or monday.

Rulers of days are fixed for astrological purpose, hence it has astrological origin in India

and west. Ancient Egyptions had a ten days week (period in which sun covers 10° or 1/3rd of a rāśī called Dreṣkāṇa in astrology) Babylonians started a month with new moon and marked the Ist, 8th, 15th and 22nd days of the lunar month for religious festivals. This was a sort of weak of 7 days with one holiday. In Iranian calender in which 30 days had different names 8th, 15th and 23rd were called Diniparvana for religious practices. But last week in this system was of 9 or 10 days. In veda, ṣaḍāha has been mentioned, but this doesn't seem to indicate a six days week. It seems to be six extra days after 360 in a leap year called 'Gavām Ayana' every four years. The Jews reckon the days from saturday and indicate them by numbers i.e. Ist, 2nd - - - 7th day.

Seven days week was introduced to christian world by edict of Roman emperor Constantine in 323 AD, who changed the Sabath day (saturday for Jewish) to the Lord's day, sunday. In India it has been first mentioned in Atharva Jyotiṣa and by Āryabhaṭa. English names of week day have originated from Teutonic deities which are counter parts of Roman planetary deities.

Indian names	**Childean names**	**Teutonic daities**	**Roman daities**
Ravi	Shamesh	Sun	Sun
Soma	Sin	Moon	Moon
Maṅgala	Nergal	Tiu	Mars
Budha	Nabu	Woden	Mercury
Guru	Marduk	Thor	Jupiter
Sukra	Ishtan	Freyā	Venus
Śani	Ninib	Saturn	Saturn

It is note worthy that functions attributed to planets by Chaldeans are same as in Indian Astrology.

Ahargaṇa or heap of days -

Count of days is used all over the world from a standard epoch to calculate the mean posotion of any planet.

Mean position at required time

= Mean position at initial epoch + daily motion x ahargaṇa

To make a uniform standard, a French scholar, Joseph Scaliger introduced in 1582, a system known as 'Julian days' after his father Julius Scaliger. The Julian Period is

7980 years = 19 X 28 X 15

19 is length in years of the Metonic cycle

15 is length in years of the cycle of inclication

28 is length in years of the solar cycle

It was found by calculation that, these three cycles started together on Jan 1, 4713 B.C. Julian period and days are counted from that day and the day is completed at noon time. This is the standard for astronomical calculations now.

Julian days for some important epochs is given below

	Date	Julian day
Kaliyuga	17-2-3102 BC	5,88,465
Nabonassar	26-2-747 BC	14,48,638
Philippi	12-11-324 B.C	16,03,398
Śaka Era	15-3-78 AD	17,49,621
Diocletian	29-8-284 AD	18,25,030

Hejira	16-7-622 AD	19,48,440
Jezdegerd (Persian)	16-6-632 AD	19,52,063
Burmese era	21-3-638 AD	19,54,167
Newar Era	20-10-879 AD	20,42,405
Jalali Era (Iran)	15-3-1079 AD	21,15,236

In India, siddhānta jyotiśa uses ahargaṇa from creation after which 6 manus of 71 yuga each have passed, in current 7th manu 27 yuga have passed. In 28th yuga, Satyuga, Treta and divāpāra have passed. Present kali yuga started on 17-2-3102 B.C. Ujjain midnight. In this kali yuga is 4,32,000 years. Dvāpara, Tretā, Satya yuga are 2, 3, 4 times. A yuga is 10 times kali = 43,20,000 years. Before each manu there is a sandhyā of a satyayuga. Thus years from creation till beginning of kali yuga are 1, 97, 29, 44, 000 years. To find the ahargaṇa for calculation, we deduct the years spent in creation = 47,400 divya years x 360 solar years. After this period all planets started from zero position which is called epoch. Ahargaṇa at beginning of kaliyuga is

714, 402, 296, 628

Tantra granthas count the ahargaṇa from kali era. Each karaṇa book has used its own epoch. In present calculations Jan 1,1900 is important epoch. For this day Julian days are 2,415,021 and kali ahargaṇa are 1, 826,556.

(7) Solar calenders in History -

(a) Egyptian calender - This has 12 months of 30 days each, starting from Thoth on 29th August as per Julian calender. This was old religious calender, hence extra 5 days were attached

in the end which not part of any month. Since the year was short by 1/4 days from 365-1/4 days, the heliacal rising of Sirius star would re-appear at the beginning of year after 1460 years. This was called Sothic cycle as Sothis (Isis) was the goddess of sirius. In 22 B.C. the year started on 29th August the Pharoahs (kings) of Egypt tried to introduce leap year, but this never became popular. Ptolemy in 238 introduced a leap year, but old calender also continued side by side. Egyptians did not use any continuous era, but counted the number of years of each reign separately. For astronomical purposes, Nabonassar Era was used in Babylone. This was used as a reference by all countries for sumplicity

(b) The Iranian calendar - Around 520 B.C. Darius introduced a solar calender like Egyptian with 365 days each. It had 12 months of 30 days each and each day had a specific name. The names are similar to vedic names. 5 days extra were attached in the end. Adjustment of 1/4 extra day each year was done by adding a month of 30 days in a cycle of 120 years.

From 16-3-1079 A.D, Seljuc sultan Jelaluddin Malik Shah introduced a new calender Tarikh-e-Jalali, starting from 10th Ramadan of Hejira 471. It was 365 days year with 8 intercalary days in 33 years. The year started from vernal equinox day or next day. Its lenght was 365.242 42 days.

Riza Shah Pahlavi introduced a strictly solar year and restored the old Persian names of month; in use before Darius. The year started from 21 or 22 march. First 6 months were of 31 days each.

Last month was of 29 days or 30 days in a leap year.

Roman calendar (Christian Calendar) - The so called Christian calendar had nothing to do with christianity. It was originally the calender of semi savage tribes of Northern Europe, who started their year some time before the beginning of spring (March 1 to 25) and had only 10 months of 304 days, ending about the time of winter solstice (December 25). The remaining 61 days formed a period of hybernation when no work could be done due to on set of winter, and were not counted at all.

This calender was adopted by city state of Rome and some modifications were made. Second Roman king of legendary period Numa Pompilius added two months (51 days) to the year in about 673 B.C. making a total of 355 days. January (named after god Janus who faced both ways) and February were added in beginning and March became the 3rd month now. Number of days became now 29,28,31,29,31,29,31,29,29,31,29,29. Adjustment of the year to the proper season was done by intercalation of a thirteen month of 22 or 23 days (called Mercedonius) after two or three years between February and March, the extra month was actually 27 or 28 days but, the last 5 days of February due to be repeated after extra month, were not repeated. The correction at alternate year could have given 45 (22+23) days in 4 years or 11-1/4 days on average. Thus it made a year only one day longer than 365-1/4 days. But this was irregular and caused a lot of discrepancy from the seasons.

Julius caesar, on his conquest of Egypt in 44 B.C. was advised by Egyptism astronmer Sosigenes that mean length of year should be 365-1/4 days.Normal length should be 365 days and one extra day should be added every fourth year. Then the fifth month from March, Quintilis was changed to July (Julius) in 44 B.C. in honour of Julius Caeser and length of months were fixed at their present duration. Extra leap year was obtained by repeating the sixth day before kalends (first day) of March. In 8 B.C., sixth month after March, Sextilis .was changed to August in honour of Augustus, successor to Caesar. To correct the seasons, 90 days were added to 46 B.C. 23 days after February and 67 days between November and December. This year of 445 days was known as year of confusion. Caesar wanted to start the new year on 25th December, the winter solstice day. But people resisted, because new moon was due on January 1,45 B.C. Caesar had to accept the traditional landmark of the year.

Weekdays of 7 days week were introduced sometimes in Ist century AD on pattern of chaldean astronomers. Days of crucification of christ and his ascending to heaven was fixed arbitrarily on Friday and Sunday later on. New Testament only says that he was crucified on a day before Passover festival of Hebrews which was on full moon day of the month of Nissan.

The present christian era started at about 530 AD. When era beginning was fixed from the birth year of christ, birth day of christ was fixed on December 25, which was winter solstice day and ceremonial birth day of Persian god Mithra in Ist

century B.C. However, a Roman inscription at Ankara shows that king Herod of Bible who had ordered massacre of children after birth of christ, was dead for 4 years at 1 AD. Therefore, christ must have been born before 4 B.C.

The Julian year of 365.25 days was longer than the true year of 365.2422 days by 0.00788 days, so the winter solstice day which fell on 21 December in 323 AD, fell back by 10 days in 1582 AD. In 1572, Pope called a meeting to discuss the correction. In 1582 Pope Gregory XIII, published a bill instituting a revised calender. Friday, October 5 of that year was to be counted as Friday, October 15. The century years which were not divisible by 4 were not to count as leap years. Thus the number of leap years in 400 years was reduced from 100 to 97. length of years was 365.2425 days, the error being only one day in 3300 years. This was adopted immediately by the catholic states of Europe. But Britain adopted it in 1752, China in 1912, Russia in 1918, Greece in 1924 and Turkey in 1927. Revised rules for easter have not been adopted by the Greek Orthodox church.

World calender : To remove the working defects of Gregorian calender, a world calender was propsoed to UNO in Geneva meetign of ECOSOC in 1954. In this calender week days of every year are same. One extra week day in 365 days is kept after 30th December called W or world holiday. In leap year another world day was to be introduced after 30th June. Every year was same for counting of week days. Each quarters of 3 months was of 91 days, 13 weeks. First month of each quarter was 31 days and remaining of 30

days. So each quarter has same form of calender. Each year (each quarter also) begins on sunday. Each month has 26 working days, plus sundays.

(8) Luni Solar Calenders -

We need very accurate measurements and complicated procedure to tally lunar and solar calenders. Mean lunar synodic month = 29.530588 days

= 29^d 12h 44 m 2s.

with a variation of ± 7 hours

Mean sidereal period of moon = 27.321661 days

= 27d 7 h 43m 11.5s.

with a variation of ± $3\frac{1}{2}$ hours.

12 lunations (synodic) amount to 354.36706 days while tropical solar year is 365.24220 days. Length of lunar year is shorter by 10.87514 days, and there are 12.36827 lunar months in a solar year. Tropical solar year is varying very slowly and is becomig shorter by 8.6 seconds == .0001 days in 1600 years. Thus at kali beginning or in Sumerian times it was 365.2422 days.

All ancient nations had almost accurate knowledge of the mean synodic month. However, no rules could be fixed for tallying the lunar year with solar year. Hammurabi (1800 B.C.), law giver king of Babylonia, has a record saying that the thirteenth (extra) month was proclaimed by royal order throughout the empire on advice of priests. Practically the start of first month was adjusted with ripening of wheat.

Later Babylonians, called Chaldeans around 600 B.C. fixed some empirical relations in lunar and solar years for correction of calender in form

m lunar months = n solar years.

where m and n are integers

Some convenient periods were

Octaeteris - 8 tropical years = 2921.94 days

99 lunar months = 2923.53 days.

This gave 3 intercalary months in 8 years with error of only 1.59 days.

In about 500 BC (383 B.C. according to father Kugler) 19 year or Saros cycle was used with 7 intercalary months

19 solar years = 6939.60 days

235 lunar months 6939.69 days

This gives a discrepancy of 0.09 days in 19 years or of 1 day in 209 years.

Their 19 years cycle was of 6940 days with leap years on Ist, 4th, 7th, 9th, 12th and 15th year in the first month and in 18th year at 7th month.

First month started with 30 days, then other months were atternately 29 and 30 days. Thus a normal year was of 354 days, but in 5 years of 19 year era one extra day was added to last month, making the year of 355 days. After adding intercalary year, the year was of 354, 355, 383 or 384 days duration. Effect of this arrangement was that the first month Nisannu start was never more than 30 days away from vernal equinox. The Chaldeans used gnomon for ascertaining time of 2 equinoxes and 2 solstices which divide a solar year into 4 almost equal seasons.

Eras of Western world - Dated records of kings in Babylon beings from about 1700 B.C. (Kassite kings). In Egypt also regnal years were used. But in Babylon, months and dates were of lunar month is while they were solar in Egypt.

Hipparchus (140 BC) and Ptolemy (150 AD) of Greece used the records of systematic observations of Babylone from 747 B.C. since the time of one king Ṅabu Nazir. Though they counted the astronomical era from 26 Feb. 747 B.C. in that reign, they adopted Egyptian solar years of 365 days each for ease in calculation of dates.

Macedonian Greek had their own months, but after they settled in Babylon in 313 B.C, they adopted their months to Chaldean months, Ist month Dios starting with 7th month of Chaldeans at autumnal equinox.

Seleucus, a general of Alexander, a Macedonian Greek founded a big empire in west Asia and started his own era Seleucidean era. In official or Maccdonian reckoning it started from the lunar month of Dios near autumnal equinox in (-311 AD) or 312 B.C, with greak month names. In Babylonian reckoning, the months had Chaldean names starting from Nisan near vernal equniox. Parthian era was started in 248 B.C. when Persia again became independent empire.

Ancient Jewish calender was lunar and their month names are derived from Chaldean names or vice versa. The day began in evening and probably at sunset. Extra month was added when necessary by making two months of the last month Adar - original was named veadar followed by

Adar. Year beginning was changed from Nisan month to Tisri corresponding to Mecedonian month of Dios. Around 4th century A.D. rules were formed for intercalation. In a cycle of 19 years 3,6,8,11,14,17 and 19th years had extra month. Start of first months was adjusted, so that week days of important festivals do not change. Thus a common year could have 353, 354 or 355 days and a leap year of 383,384 or 385 days. 10 of the middle months had got fixed duration of 29 or 30 days. Extra month was of 30 days. The other two (Ist and 12th months) varied according to length of the year. Jewish era is called Anno Mundi or libriath olum or Era of Creation or Freedom.

According to mnemonic Beharād, this era is supposed to begin at the beginning of lunar cycle on the night between Sunday and Monday, Oct 7, 3761 B.C., at 11 hours 11-1/3 minutes PM. (Be = Beth i.e. 2nd day of week), ha (he = five, i.e. fifth hour after sunset) and Rad (Resh) delet i.e. 204 minims after the hour, 18 minim = 1 minute)

In Bible, eras have been mentioned from flood, exodus, the earthquake in the days of king Uzziah, the regnal years of monarchs and Babylonian exile. After exile, they counted years from Persian kings, and then from Seleucid era. Days have also been counted from fall of the second temple.

312 - Seleucidean era = Christian era B.C. (Jan to Sept)

Saleucidean era - 311 = Christian era AD (Jan to Sept)

Year 1 after destruction of second temple

= 3831 Anno Mundi

= 383 Seleucid = 71 A.D.

Islamic Calendar -

This is purely lunar calender now and has no connection with solar year. The year consists of 12 lunar months; beginning of each month is determined by Ist observation of crescent moon in the evening sky. The months have 29 or 30 days and the year 354 or 355 days. The new year day of Islamic calendar loses about 1 month in 3 years, and completes the retrograde cycle of seasons in $32\frac{1}{2}$ solar years.

Hejira (A.H.) was introduced by caliph Umar about 638-639 AD, stating from evening of 622 AD, July 15, Thursday (Since sunset Friday started in Islamic calender). Then crescent moon of the Ist month Muharram was first visible. This was the new year day preceding the emigration of Muhammad from Mecca (about Sept 20, 622 AD.). The months are alternately of 30 and 29 days from Ist month. Last month is 29 days in normal year and 30 days in a leap year. If Hejira year is divided by 30 and remainder is 2,5,7,10,13,16,18,21,24,26 or 29 then it is a leap year. Thus 11 leap years in 30 years, gives the cycle of 10,631 days which is 0.012 days less than the true value.

Dr. Hashim Amir Ali of Osmania University has showed that the mohamadan calender was originally luni-solar. Upto the last year of the life of Mohamad; i.e. upto AH 10 or 632 AD, a thirteenth month was intercalated when necessary. The family of astronomers, known as Qalamas

decided at hajj in last month, whether 13th month will be added or not. This should have been 3 times in 8 years or 7 times in 19 years, but use of discretion by eldest Qalama created confusion afterwards. Thus AH 11, a normal year started on 29th March 632 AD. after vernal equinox. Thus all the previous years with intercalation, started after sighting new moon after vernal equinox. Thus the initial epoch of Hejira era was at the evening of March 19,622 AD, Friday, the day following the vernal equinox.

Names of Lunar Months

Indian	Chaldean	Mecedonian	Jewish	Islamic
Caitra	Addaru	Xanthicos	—	—
Vaiśākha	Nisannu (30)	Artemesios	Nissan	Muharram
Jyeṣṭha	Airu (29)	Daisios	lyyar	Safar
Āṣāḍha	Sivannu (30)	Panemos	Sivan	Rabi-ul-awwal
Śrāvaṇa	Duzu (29)	Loios	Tammuz	Rabi-uls-sani
Bhādra	Abu (30)	Gorpiaios	Ab	Jamada alawwal
Āśvina	Ululu (29)	Hyperberetrios	Ellul	Jamada as sani
Kārttika	Tasritu (30)	Dios	Tisri	Rajab
Mārgaśīrṣa	Arah/Samnah(29)	Appelaios	Marheshvan	Shaban
	Kisilibu (30)	Audinaios	Kisilev	Ramadan
Pauṣa	Dhabitu (29)	Peritios	Tebeth	Shawal
Māgha	Shabat (30)	Dystros	Shebat	zil kada
Phālguna	Addaru (29)	Xanthicos	Adar and	Zil hijja
Caitra			Veadar	

(9) Old Indian Calendars :

A. Vedic Calender - Vedic calender was luni solar. Year was named in three manners - Solar year, civil year and lunar year (normal and intercalary).

Samā = Fixed year or constant. It is opposite to 'māsa' i.e. formal of 12 māsa of 30 days each. Thus it means **a year** of 360 civil days or 365 solar days (i.e. 365-1/4 days)

Lunar years are called vatsara - which are of 5 types– Saṁvatsara, anuvatsara, Parivatsara, Id-vatsara and Idāvatsara. Anuvatsara is also called Iduvatsara. When these indicate a sequence of 5 solar years of 366 days each, vatsara is a sixth year of 360 civil days or samā (as per yajuṣ jyotiṣa).

Names of thirteen months in Taittirīya Brāhmaṇa (3-10-1) are Aruṇa, Aruṇa rajas, Puṇḍarīka, Viśvajīta, Abhijit, Ārdra, Pinvamāna, Annavān, Rasavān, Irāvān, Sarvan ṣadha, Sambhar and Mahasvān, Mahasvān appears to be increased month (with extra days in a solar year).

6 seasons of two solar months each are as follows -

1. Vasanta - Madhu and Mādhava
2. Grīṣma - Śukra and Śuci
3. Varṣā - Nabhas and Nabhasya
4. Śarad - Īṣa and ūrja
5. Hemanta - Sahas and Sahasya
6. Śiśira - Tapas and Tapasya

Taittirīya Brāhmaṇa has given a list of 24 half months (1 fortnight), names of day times and night times in śukla and kṛṣṇa pakśas - 60 names, names of 15 muhūrtas in śukla pakśa day and night, Kṛṣṇa pakśa day and night - 60 names and 15 parts of each muhūrta / called prati muhūrta).

Name of lunar months were named after the nakśatras entered by moon on purṇimā day. Ṛk veda (1-15-1) tells that Indra drinks soma juice with seasonal ādityas on full moon day. Thus Indra is always at a point 180° away from sun.

Ādityā corresponding to different seasons are

(1) Mitra - śiśira (2) Aryamān - vasanta (3) Bhaga - grīṣma (4) Varuṇa - Varṣā (5) Dakśa or Dhātā - śarad (6) Aṁśa - Hemanta

Ṛk veda verse 10-72-4 by Śunahśepa gives method of deciding about inclusion of intercalary month -

Dakśa was born of Aditi and Aditi was Dakśa's child. The whole ecliptic was Aditi and its division were ādityas - 6 for each season, 12 for each month or 13th for extra month. First point of Dakśa division was the start of ecliptic zero degree. Year started with rise of this point on eastern horizon with sun. When the next rise was not before 13th full moon, 13th month was extra month otherwise it was month of next year. In śāntipāṭha also it is stated --

अदितिर्जातम् अदितिर्ज नित्वम्

In Vājasaneyī samhitā, two adhika māsa are named. Sansarpa is extra month before winter solstice. Another is malimluca. Kśayamāsa (lost month) was called Aṁhaspati. (Yajur-VS, 22-30)

In a solar year of 365-1/4 days, 5 or occasionally six days are extra after civil year of 360 days. These have been called atirātra (i.e. extra days after grand night). Taittirīya Samhitā (7--1-10) says that 4 atirātra make the year incomplete, while 6 atirātra give excess, so five are the best.

Aitareya Brāhmaṇa has defined Tithi as the time during which Moon sets and rises again (32-10). Thus like civil day from sunrise to sunrise, tithi is a moon day from moon rise to moon rise.

In śukla pakśa tithi was from moon set to moon set, and in other it was moon rise to moon rise.

Atharva Vedānga jyotiṣa has defined two karaṇa in each tithi - one from moon rise to moon set and second from moon set to moon rise. These tithi and karana were of unequal length. Later on they were made of equal length defined on basis of moon phase.

B. Vedāṅga Jyotiṣa (Ṛk veda) : This is described in only 36 verses in anuṣṭupa chanda including introduction and importance. This is one of the six parts of Ṛk veda. Though it is shortest, it gives a most comprehensive, luni solar calendar so far. It was written by 'Lagagha' whose place was 35° N latitude, nothern border of Kashmira, may be the present town of Almā-Atā of Kyrghiz. This place might have been first place of learning, hence first school is called alma-meter.

Efforts to explain its meaning on basis of 5 years cycle (yuga) were unsuccessful, by various anthors as B.G. Tilak, S. B. Dīkśita and T.S.K. Shastri, R. Shamshastrī etc. 'Pañca samvatsara mayam yugam' was interprated that a yuga has 5 years (meaning of samvatsara). But samvatsara is one of the 5 types of lunar years and its meaning should be - A yuga has 5 years of samvatsara type, remaining years of other 4 types. If calculations are made on that basis, a yuga has 19 years, with 5 types of vatsaras, out of which 5 are sanvatsaras. This also gives meaning of other types of years. This gives correspondance of solar and lunar years in terms of tithis, days and nakśatras also.

Time cycle : There are 360 tithis in a lunar year. Solar year is bigger by 10.89 days. With reasonable accuracy, 7 intercalary months (adhikamāsa) occur in a cycle of 19 years. Thus

228 solar months = 235 lunar months Additional 7 months form 7 × 30 = 210 tithis. Thus there is difference of $\frac{210}{19} = 11\frac{1}{19}$ tithis (10.89 days) between a solar and a lunar year. Thus a solar year consists of 371 $\frac{1}{19}$ tithis. If we assume a leap year in a cycle, we have 18 years with 371 tithis and one year (i.e leap year) with 372 tithis. This cycle of 19 years is called a yuga.

Calculation of Ṛtu śeṣa - A year has 12 months (lunar) each having two parts śukla pakśa (called, śudi - i.e. Sūkśma diwas - or śuddha diwas) and Kṛṣṇa pakśa (Badi i.e. bahula diwas, extra days). Thus 24 pakśa of a year have difference of 11 tithis from solar year. Difference in each pakśa is $\frac{11}{24}$= 0.458 = $\frac{1}{2}$ tithi approximately

Thus a lunar pakśa = 15 tithis

Solar pakśa = 15 $\frac{1}{2}$ tithis

For calculating extra tithis for each half of solar year, we have to add 11/2 tithis = 11 karaṇas. Thus we have to substract 1 karaṇa after half year from 12 karaṇas got after taking 1 karaṇa for each pakṣa approximately. One tithi is taken extra in a completed yuga of 19 years. When cumulative total of extra karaṇa after a semester is more than 60

karaṇas = 30 tithi = 1 month, one extra month is added in that semester.

Classification of years : If one karaṇa is not dropped in each semester, then Ṛtuśeṣa will be 12 tithis per year or 6 tithis per ayana (semester). Thus start of ayanas will be after 6 tithis each and after 5 ayanas (2-1/2 years) the cycle will be complete and one extra month will be added so that the month starts again with Māgha śukla 1 (Ist year of month). Thus years were classified according to range of tithis on which Ist day of year fell -

Saṁvatsara - Śukla 1 to 6th

Anuvatsara - Śukla 7th to 12th

Parivatsara - Śakla 13th to 18th (or badi 3rd)

Idvatsara - Badi 4th to 9th

Idā vatsara - Badi 10th to 15th

When we decide adhikamāsa for each lump of 60 karaṇa Ṛtuśeṣa, 5 years in 19 years yuga are of samvatsara type. 3 years are Idāvatsara, lagging behind most, hence the adhikamāsa occurs in Ist semester of those years (6th, 9th and 17th). which can be seen by calculation. Four years are of Idvatsara type lagging 18-24 tithis, hence the adhika māsa is added in 2nd semester of (3rd, 11th, 14th and 19th years)

Nakśatra calculation for sidereal lunar year -

In a lunar month, moon completes its circular journey of 27 nakśatras and travels about 2 nakśatras more. To be more accurate, it completes 13 revolutions in 12 lunar months. Thus in 19 solar

years, there are 254 sidereal months and in 19 lunar year 19X13 = 247 sidereal months of moon.

Thus difference in 2 cycles is 7 sidereal months = 7 x 27 = 189 nakśatras. Thus we have 10 nakśatras per year for 18 years and 9 extra naksatra in one year

1 solar year = 361 naksatra

Leap year = 360 nakśatras

1 lunar year == 351 nakśatras (= 13 x 27)

Solar semester - lunar semester = $\frac{10}{2}$ = 5 nakśatras.

Ṛtuśeṣ in terms of nakśatras is calculated by assuming a total of 190 nakśatra for 38 semester i.e. 5 for each. To be more accurate it is 5- $\frac{1}{38}$ nakśatra. Thus calculation is to be started from śraviṣṭhā (Śravaṇa + 38 parts of 1/38th i.e. complete dhaniṣṭhā). Complete nakśatra divisions will be counted from Śravaṇa at interval of 5 nakśatras each for every ayana (semester). Thus list of nakśatras is given at intervals of 5 nakśatras indicating only one letter from each nakśatra. This verse was deciphered brilliantly by Śrī S.B. Dīkśita.

According to moon nakśatra at start of year also years can be classified. Thus samvatsara can start from śravaṇa upto 5.4 naksatra (Aśvinī) - 2.7 difference for each semester. Anuvatsara is Aśvinī (5.4) to Ārdrā (10.8) - countings done from śravaṇa. Parivatsara can start up to 16.2 (uttarāphālgunī) Idvatsara upto (21.6) anurādhā and Idāvatsara in remaining nakśatras.

C. Viśvāmitra's astronomy - also indicates a 19 year yuga. His hymn in Ṛk veda III - 9-9 reads

त्रीणि शता त्रि सहस्राण्यग्निम्
त्रिशंत् च देवा नव चा सर्पयन

i.e. 3339 devas (dyues or parts of ecliptic = aditi) worshipped agni (sun or kṛttikā nakśatra) by rotations in the sky.

This is based on calculation of solar nakśatras for each parva (or pakśa). First solar year of 372 tithis is the basic year in which sun crosses the 27 nakśatras. Thus 1 solar nakśatra = 372/27 = 124/9 tithis. To avoid fractions, angular distance travelled by sun in a tithi is divided into 9 parts called 'Bha-amśas' or bhānśa. (Bh°)

1 Tithi = 9 Bh° of sun

1 Nakśatra = 124 Bh°

$$1 \text{ Bh}^\circ = \frac{13^\circ\ 20'}{124} = 6'27''.1 \text{ approx}$$

1 parva = 15 tithi = 135 Bh° = 1 Nakśatra + 11 Bh°

In 1 Ayana = 12 parva

= 12 (Nakśatra + 11 Bh°) = 12 Nakśatra + 132 Bh°

= 13 Nakśatra + 8 Bh°

Thus 8 Bh° arise in one parva (in addition to completed nakśatras).

In 371 tithis of a solar year there are 371x9 Bh° = 3339 Bhānśas, which are indicated by Viśvāmitra.

Saros cycle of Chaldea was of 18 years and 10.5 days after which eclipses are repeated.

3339 tithis = 111 synodic months + 9 tithis.

This is half of Saros cycle of 223 synodic months

$\frac{3339}{3}$ synodic years = $\left(\frac{3240}{3} = 1080\right)$ = 1080 sideral years are a yuga (1/3 of Viśvamitra's mahāyuga). This is the period of precession of equinoxes for 1 parva (15 tithi or 15° movement of sun in 15 x 72 = 1080 years).

A day is divided into 603 kalās = 30 muhūrta. Moon crosses 1 nakśatra in 610 Kalās.

D. Yajuṣ Jyotiṣa : This is part of Yajurveda whose commentary by Somākara was available. It has 44 verses out of which 30 are common with Ṛk jyotiṣa Due to that reason, scholars have tried to combine these two into one text of 50 verses and interprate both on basis of 5 years yuga. However, Ṛk has 19 years yuga and yajuṣ has 5 years yuga. This has been specified in verse 31 of this text -

(In a yuga) there are 61 sāvana months, 62 lunations and 67 sidereal months i.e. nakśatra māsas; 30 days make one sāvana month and 30-1/2 days make one solar month. This along with verse 4 tells that we have one adhika māsa after every 2-1/2 years - clearly specify a 5 year yuga. This is the different meaning of this text.

Adjustment of luni-solar years in this system can be done on basis of two statements in contemporary texts. Mahābhārata, śānti parva ch 301 tells.

क्षयं संवत्सराणां च मासानां च क्षयं तथा

(By way of leap) drop years as well as months.

Order of leap years is indicated by Taittirīya Brāhmaṇa (part of yajurveda) (3-10-4) which gives list of years.

(1) Saṁvatsara (2) Parivatsara (3) Iduvatsara which is same as anuvatsara according to Mādhava (4) Idvatsara (5) Idāvatsara and (6) Vatsara :

This extra sixth vatsara is a samā or sāvana year of 12 sāvana months (30 days each according to verse 31) and comes after each yuga of 5 years in a yuga in the above order. Thus each of the five years is of 366 days and sixth year of 360 days balance the extra days counted in the yuga. The year after samā or vatsara may not be samvatsara, it will be decided according to tithi or nakśatra at beginning of year as defined in Ṛk jyotiṣa. Thus the ommitted years can be thought as dropped years or kśaya years. Thus in a cycle of 5 yugas of 25 years we drop six years including 3 leap years whose adhikamasa also gets dropped in the process. Thus we get 19 year yuga as before. But we get a simpler 5 year yuga.

There is a difference of 4 hours 23 minutes between vedāṅga cycle of 19 years and 19 solar sideral years. It we add 8 years at the end of eight 19 years yugas, we get 160 years. The difference at that end reduces to 23 minutes only.

Time period - RVJ verse 5 tells that year started on full moon day in Māgha in winter season when sun was in vāsava nakshatra (Ist year of yuga)

YVJ verse 6 tells that sun in beginning of śraviṣṭhā indicated beginning of Māgha and sun

in mid point of aśleṣa started beginning of south solstice.

Assuming 1° precession in 72 years, this indicates Ṛk jyotiṣa in 2976 BC and yajuṣ jyotiṣa in 2352 BC. However verse 3 of RVJ indicates that the theory was coming since long when these verses were composed.

E. **Gavām Ayana** has been mentioned in Tattirīya samhitā, Śatapath brāhmaṇa, Gopatha brāhmaṇa and Baudhāyana śrauta sūtra. It indicates a 4 year yuga with 1 leap year according to Prof. R. Shāma Śastri (1908 - gavām ayana) Accumulation of 1/4th day of each of previous 3 years combined with 4th year to make one extra day like the Julian calender. Thus this is a cow with 4 legs or 3 parents of sun. Four years of this yuga were called kali, dwāpara, tretā and kṛta yuga. These are also called Ist, 2nd, 3rd and complete (kṛta). Kṛta is also called Satya or Ṛta i.e. which really came (as a full day) If year or yuga starts in evening, Ist year (kali) will end at midnight after 365-1/4 days (sleeping time). 2nd year dvāpara will end in morning (rising time) on 366th day. 3rd year Tretā will end on 366th day noon, when sun is at highest. 4th year kṛta or satya will end in evening when people are moving. Thus Aitareya Brāhmaṇa tells - sleeping is kali, rising is dvāpara, standing is tretā and moving is kṛta, so keep on moving (7-15). This is attributed to Manu. This was around 23,720 B.C. as Taittirīya saṁhitā indicates (7-4-8) vasanta at phālguna full moon. Ṛk veda indicates rains in Mṛgaśirā nakśatra indicating same time.

Thus 4 years yuga with 4th as leap year appears to be first system started around 24000 B.C. Then 19 years yuga with 7 leap years (lunar)

of vedānga jyotiṣa continued upto about 3000 B.C. i.e. kali beginning. With kali erā smaller 5 year yuga of yajurveda forming a 19 year yuga was started.

F. Jaina calendar - Sūrya prajñapti and Candra prajñapti are two principal texts written at the time of Mahavīra about 600 B.C. However Jain Tirthankars and their astronomical traditions might have started along with yajurveda or early brāhmaṇa texts.

There are five kinds of saṁvatsaras (years) -

(1) nakśatra samvatsara (2) yuga (cycle) samvatsara (3) Pramāṇa (standard) samvatsara (4) Lakśaṇa (symptomatic) samvatsara and (5) sanīcara (saturn) samvatsara

	Days in year	Months in year	Month of 5 year cycle
Nākśtrika	327-51/67 days	27-21/27	67
Lunar	354-12/62	29-32/62	62
Ṛtu	360	30	61
Solar	360	30-31/62	60
Abhivar-dhana	382-44/62	31-121/124	57-3/13

A five year yuga consisted of 5 lunar samvatsaras with 3rd and 5th years having extra months called abhivardhan samvatsara. This is almost like yajus jyotiṣa but simpler. Nakśatra samvatsara was named according to nakśatra occupied by Jupiter at the time of complation of samvatsara - these are same as present months names of India. Śanīcara samvatsara was time of śani in crossing 1 nakśatra out of 28 with mean motion.

(10) Indian Eras :

(A) The erās started after kali erā are based on concept of Mahāyuga of 43,20,000 years or a kalpa of 1000 mahāyuga. Yuga concept is attributed to Āryabhata in 499 AD and kalpa concept to Brahmagupta in 627 AD. However, Brahmagupta refers to Viṣṇudharmottara Purāṇa. Smṛtis also have been referred to. Āryabhaṭa himself has followed purāṇa tradition, except to treat four parts of yuga as equal. The five limbs of calendar, known as pañcāṅga have already been explained in the previous chapter. Their brief definitions are given again -

(1) Vāra - Runing weakday in a cycle of seven days.

(2) Nakśatra - Nakśatra occupied by moon.

This almost means one day and was most popular in mahābhārata era to indicate a day (in Vālmīki Rāmāyaṇa also).

(3) Tithi - Moon rise to moon rise system was changed. Vedanga jyotiṣa started equal division of tithis depending on phases of moon.

$$\text{Tithi} = \frac{\text{Moon} - \text{sun}}{12^\circ}$$

Quotients above 15 indicate kṛṣṇa pakśa and extra days beyond 15th are counted as tithi number. Hence Kṛṣṇa tithi is called' bahula divas' (extra day) or 'badī' in short. Śukla tithi is called sudi - śuddha divasa.

(4) Karaṇa is half of tithi. In veda it was moon rise to moon set or vice versa. From vedāṅga jyotiṣa it is exactly half of tithi.

$$\text{Karaṇa} = \frac{\text{Moon} - \text{sun}}{6^\circ}$$

Corresponding to 1 day = 2 karaṇas extra at beginning of 19 yeear vaidika yuga, one karaṇa at each end of amāvasyā are omitted from running cycle of seven karaṇas. These 4 are fixed karaṇas. Remaining 56 karaṇas start from śukla Ist tithi 2nd half in which seven karaṇas are repeated 8 times in a month.

(5) Yoga - It is only a mathematical concept. It means sum of longitudes of sun and moon and one cycle of 360° makes 27 yogas. Originally there were only 2 yogas. Vyātipāta was when krānti of sun and moon were equal but their longitude was equal and in opposite directions. When longitude is same but krānti is equal and opposite, it was called 'vaidhṛti'. Thus yoga was means to calculate these and subsequently others were included to make a complete cycle of 27 yogas like 27 nakśatras.

There is another kind of yoga which is combination of vāra, tithi or nakśatra for auspicious works.

B. Rules for calender -

(1) A lunar month stars from Śukla 1 (called anānta) or from Kṛṣṇa 1 tithi called Pūrṇimānta. Lunar month is named after the nakśatra approximately occupied by moon on pūrnimā of that month. Amānta is called mukhya and other gauṇa.

(2) A solar month starts with entry of madhya sūrya in a nirayana rāśi (i.e. fixed point of zodiac). The first day of month may start on same day,

next day or 3rd day according to occurrance of saṅkrānti in different parts of day or night.

(3) In luni-solar year a lunar year tallies with a particular saṅkrānti of sūrya every year. In a lunar month when there is no sankrānti the month is called adhika māsa. The month having two saṅknāntis, is called kśaya māsa, the month corresponding to 2nd sankranti of the month is dropped. Kśaya māsa is called amhaspati. The adhika māsa before it is called sansarpa (or in Ist ayana uttarāyaṇa of the year). The adhika māsa after kśaya masa or in 2nd ayana is called malimluca.

(4) Uttarāyaṇa starts when sun starts its northward journey after winter solstice or sāyana makara saṁkrānti (24ṭh or 25th December). Dakśiṇāyana starts when sun starts going south from summer solstice i.e. sāyana mithuna sankrānti (26 June). A year may start with start of uttarāyaṇa as in vedāṅga jyotiṣa or from equinox in uttarāyaṇa - vernal equinox which is middle point of uttarāyaṇa. Instead of exact equinox point of uttarāyaṇa, we count entry into fixed zodiac rāśi which follows 23 days later at present.

C. Rules for saṅkrānti

(1) In Orissa, solar month begins on same day as saṅkrānti of madhyama sūrya, irrespective of the part of day (sun rise to sun rise) it falls in.

(2) Tamil rule - If saṅkrānti takes place before sunset the solar month begins on same day, otherwise from next day.

(3) Malābāra rule - In parahita system of Kerala, if saṅkrānti takes place before lapse of 3/5th

of duration of day (i.e. about 18 ghati or 7h / 12m after sunrise - about 1-12 p.m.), month starts on the same day, otherwise from next day.

(4) Bengal rule - When saṅkrānti takes place before midnight, month starts from next day, if it is after midnight, then from third day (next to next day).

If sankrānti is within 1 ghaṭī of mid night, i.e. 24 minutes before or after, tithi at sunrise time is examined. If sankrānti is before lapse of that tithi then month starts on next day. If it is after tithi then from 3rd day. For karka and makara sankrānti this rule is not followed.

D. Erās started in India -

Eras of long period have been described in all old civilisations. They describe three great floods. After one great flood Brahma appeared and started the civilization. Then saptarṣis were born, rule of Daitya, Deva and Danava followed. That may be called Deva yuga.

Devayuga ended with another great flood in which rudiments of life were preserved by Manu (or Nuh). After re-settlement human erās began. These were formed into a cycle of 12,000 divya years, 1/10th was kaliyuga, 2, 3, 4 times were dvāpara, tretā and kṛtayuga (or satyayuga).

There was another erā called saptarṣi era which is equal to 2700 divya versa, assuming that they remain in one nakśatra for 100 such years. Another count in Vāyupurāṇa mentions saptarsi yuga as 3030 mānuṣa varṣa. Thus divya varṣa appears to be solar sidereal year of 365-1/4 days and mānuṣa varṣa is sidereal lunar year of 327.4

days. These values give the above ratio of values given in Vāyu purāṇa.

Each yuga was further divided into sub parts, like Vāyu purāṇa indicates 24 parts of tretā yuga and great personalities have been named in each part. Each part considered equal, parts of Treta were of 150 years each. Dvāpara had 28 parts = 2400 solar years = 85.7 years approx. It is more convenient to keep 4 parts as sandhi periods after treta and after dvāpara. Then each part is of 1 century, i.e. 1 nakśatra of saptarṣi. Some corroborating quotations for this time scale are -

(1) Megasthenese quoted by Pliny (Indika of Arian ch IX) -

From the days of Father Bacchus to Alexander the Great, their (Indian) kings are reckoned at 154 whose reigns extend over 6451 years and 3 months (Pliny}

Father Bacchus was the first who invaded India and was the first of all who triumphed over the vanquished Indians. From him to Alexander the Great, 6451 years 3 months.....reign by 153 kings in intermediate period (Solin)

From the time to Dionyson (or Bacchus) to Sandrokottos, the Indians counted 153 kings and a period of 6042 years. Among these a republic was thrice established, another for 300 and 120 years.

Note - Bacchus is mentioned in Bible, becomes Dionyson in Greek. This is derived from 'Danusūnu' (son of Danu third wife of Kaśyapa -- or dānava) or Vipracitti (Bacchus). Herodotus has stated that Bacchus was called Orotol in old Arabic. This is derived from Vipracitti.

(2) Among sons of Kaśyapa prajāpati, eldest were born from Diti called Daitya. Next were from Aditi called Āditya or deva. Daitya were earlier and first to rule over world, hence they were called 'pūrvadeva'. Last were born from youngest wife Danu called dānava. Daitya and dānava were called asura and they were anti to deva or Sura. Herodotus writes on basis of Egyptian priests (part 1, p.136)

The twelve gods were, they affirm, produced from the eight and of these twelve Hercules is one. Hercules belongs to the second class, which consists of twelve gods and Bacchus belongs to the gods of the third order (P.199).

Note - Hercules is derived from 'sura kuleśa' i.e. Visnu. One form of Viṣṇu was vāmana, youngest of the twelve ādityas, who conquered Bali.

Daitya Hiraṇya kaśipu = Zeus
|
Prahlāda = Epaphos = Libye
|
Virocana = Beor
|
Bali = Bala = Bel = Baalim
|
Bāṇa   Candramā = Cadmus

Greek names are according to Pedigree by Nounos (1-377), Bible Duternomy 23-4 tells - they hired against thee Balaam the son of Beor of Pethor of Mesopotamia. In Jesus 3/7 and 6/28,30, it is called Baalum and Baal.

Devayuga before kṛta yuga has been mentioned in Ramāyaṇa Bāla kaṇḍa 9/2 and Jaimini Brāhmaṇa 2/75, Mahābhārata Ādiparva 14/5 Sabhāparva 11/1, Vanaparva 92/7.

(3) Floods - Encyclopaedia of Religion and Ethics -

Article on ages - The cuneiform texts mention kings before the flood in opposition to kings after the flood. In times before the flood, there lived the heroes, who (Gilgames Epic) well in the under world, or like the Babylonian Noah, are removed into the heavenly world. At that time, there lived, too, the (seven) sages.

Berosus, priest of Marduk temple of Babylon under rule of Selucus writes - There were 86 kings after flood in first family who ruled for 34,090 years. Then 5 more families ruled one after the other.

It is note worthy that among the south Amercian Indians, it is generally held that the world has already been destroyed twice, once by fire and again by flood; as among the eastern Tupies and Aravaks of Guiana.

Saving of civilisation from flood in a great boat has been described in south America also - Tales of Cochiti Indians - Bureau of American Ethnology. Bulletin 98 page 2-3.

(4) Herodotus writes on basis of Egyptian calculations - (part 1 page 189)

Seventeen thousand years (from the birth of Hercules) passed before the reign of Amasis. And even from Bacchus, youngest of the three, they count fifteen thousand years.

Vāyupurāṇa tells 12 deva in Ist tretā yuga. For further material - Bhārata varṣa kā Bṛhat Itihāsa - by Bhagavaddatta, Praṇava Prakāśan, Delhi - 26 may be referred.

E. Eras since Kali (i) King Yudhiṣṭhira ascended throne after Mahābhārata war and the time since then is counted as Yudhiṣṭhira śaka. Varāhamihira writes in Bṛhatsamhitā that according to old Garga, Saptarṣi were in Maghā during the reign of Yudhisthira. 36 years after that kali era started with death of Kṛṣṇa

(2) Kali era - It started 36 years after Mahābhārata war on the day Kṛṣṇa died. After some months Yudhiṣṭhira relinquiished his throne. According to Alberuni (part 3 p.239), it started on 13th tithi of Āśvina. There are thousands of documents mentioning this era. All text books count the day from kali beginning. Accordingly, it starts on 17/18-2-3102 B.C. Ujjain midnight on Friday. Its months are both Caitrādi (luni solar) and meṣādi (solar). Year expired in kali era is obtained by adding 3179 to śaka year.

(3) Saptarṣi era - It was also called laukika kāla and Śāstra kāla in Rājataranginī where it has been followed as the standard. This era began on Caitra śukla Ist tithi in kali year 27. Saptarṣis remain for 100 years in one nakśatra and after each century in a nakśatra the years are counted afresh. Era is mentioned merely by the nakśatra name in which saptarṣi remain. Thus years are found by adding 46 to śaka era, neglecting the centuries.

(4) Old Śaka era of Varāhamihira - Bṛhat-samhitā 13/3 tells that Śaka starts 2526 years after

king Yudhiṣṭhira. This is 554 before Vikrama era. According to this śaka era, time of Varāhamihira was 427, i.e. 127 years before Vikrama era. (epoch of Pañcasiddhāntikā) He himself writes in kutūhala mañjarī that he was in beginning of Vikrama samvat. Traditionally he is reputed to be one of 9 wise men with Vikramāditya who started Vikrama samvat. Such a great astronomer was needed to start the new era since Vikrama, Sri S.B. Dīkśita also has stated in his history of Indian Astronomy vol II. p.2 that the siddhāntas mentioned in pañcasiddhāntika belong to 5th century before Śaka era (new). Thus his epoch of 427 old śaka is a convenient period about 100 years before Vikrama, not his time. The subject matter also is much older than Āryabhaṭa during whose time the older, theories were extinct.

(5) Śūdraka or Śrī Harṣa samvat - (2644 Kali) Śudraka was also called Śrī Harṣa who was a king of Āndhra Kula. Albiruni (chapter 49) has written that Śrī Harṣa was 400 years before Vikrama.

Āin - Akbari (description of Ujjainī) tells that difference between Ādītya Ponwāra (Sūdraka) and Vikramāditya of Vikrama era was 422 years.

Yalla in his Jyotiṣa Darpaṇa (Śaka new 1307) has written बाणाब्धिगुणदस्रोना (2345 or 2645) शूद्रकाब्दाः कलेर्गताः Taking 2645 as correct version, Śūdraka era started in kali 2645 or 399 years before Vikrama.

This Śūdraka has written 'Mṛcchakatikam' a famous drama. He ruled over Malwa, Kannauja, Kaśmīra etc. After 400 years, 2nd Vikram samvat became more popular and this era was forgotten.

This saṃvat was also called Kṛta samvat. King Samudragupta has written that (Kṛṣṇa Carita)

'His rule was rule of law and religious, Hence his era was caled Kṛta samvat.'

This was written as Mālava samvat because of its start in Mālavā.

Jain Ācārya Hemacandra also has mentioned that rule of Śūdraka was famous for righteousness (Kāvyānuśāsana - Bombay edition p.464).

(6) Pārad samvat - This is Indian name of Parthian era or Arsacid era starting in 246 B.C. in Iran. This was in use in West India.

(7) Vikrama Samvat (kali 3044) - This was also called Sāhasāṅka year. All Gupta kings used Vikram name, so this is connected with one of them. In old geneology, Samudragupta is considered 93 years after Vikramāditya of Avanti (or Ujjain). In north India, it is Caitrādi with purnimānta months. In Gujarat it is kārtikadi and months are amānta. Jain inscriptions have called it Gupta era also.

(8) Christian era started with British rule in India.

(9) Śālivāhana Śaka (78 A.D. - Alberuni has written (part-3) - One Śaka king ruled in areas around Sindha river and through his tyranny he tried to destroy the Hindu culture. He was either a śudra of north west border or from a western foreign country. In the end, one king from east came and expelled him. After killing Śaka king he was called Vikramāditya and another era started with him.

This Vikramāditya might have been Skandagupta according to purāṇa chronology.

This is most popular, among astronomers. Āmarāja Brahmagupta, Bhāskara II, have written that, 3179 years of kali had passed at the end of Śaka king.

Alberuni tells that Gupta - Ballabha samvat started 241 years after Śaka. Gupta empire lasted for 242 years. Thus Gupta empire and Śaka kāla started together.

Meṣādi solar years are followed in Tamil and Bengal and caitrādi lunar year is followed elsewhere. Lunar months are pūrṇimānta in north India and Amānta in south India.

(10) Kalchuri era or Cedi era : Kings of Bhoja kula ruled in Cedi (present Bundelkhaṇḍa)

Yallaya in Jyotiṣa darpaṇa has written that

Bhojarāja samvat = Śaka year + 50

According to this, it started in 28 AD or 85 years after Vikrama era.

Keelhorn assumes it to start in 255 AD. considering Narasinha deo of Kalinga and of Dahal (M.P.) as same person. This year started from 'Āśvina Śukla 1.

(11) Valabhī era - One Vallabha ended the rule of last Gupta king who was a tyrant and started ths era in Śaka 242. (This has already been mentioned in statement of Alberuni under para 9 above).

Vallabhi king Śīlāditya had dispute with a merchant Raṅka of his town. Raṅka invited Hindu king Hammīra of Gajani (Afganistan). In a night

raid Ballabhi was distroyed in Vikrama era 35 which was also referred.

(12) Hijrī Era - This started with Islamic rule in India.

(13) Kollam or Paraśurāma Era - This is known as Kollam (western) Āndu (year). This is used in Kerala and in Tirunelveli district. This is sidereal solar year starting from solar month of Kanyā in north Mahabar and simha month in south. This year runs in cycle of 1,000 years and present cycle is said to be fourth. It's 4th cycle started in Śaka 747 or 824 A.D. According to Mahābhārata, Paraśurām was in sandhi of tretā and dvāpara (i.e. 24th part). Thus Paraśurāma must have been above 5000 years before 824 AD, may be 6000 completed years.

(14) Nevāra year started in 878 AD, with Kārttikādi amānta months. It was used in Nepal upto 1768 AD.

(15) Cālukya Era - Cālukya king Tribhuvana Malla started this era in 997 AD.

(16) Siṁha Samvat - It was started in Gujrat in 1170 AD. Months are amānta and start with Āṣāḍha.

(17) Bangāli Fasalī etc -

Bangali san started in 593 AD. It is solar year and Ist month starting from meṣa saṁkrānti is called Vaiśākha (it is called caitra elsewhere). All month names are lunar.

Vilāyati san started previous year with Kanyā sankrānti i.e. 7 months before Bangali san. The year is solar with lunar month names. This was

used in Orissa. Difference in rules of saṅkrānti has already been explained. Amli Era also was used in Orissa with luni solar months. This year started from Bhādra śukla 12th (the month of kanyā sankrānti), which is supposed to be birthday of kings Indradyumna of Orissa, in purāṇa erā.

Fasali san was started by Akbar. This started with same year number as Hizri era but it was solar calender to tally with harvesting time. In north India, it started in 1556 AD with Hijrī year 963. In south India it started in 1636 AD when Hijri year had become 1046. Thus years in South India are 2 more. In north India, Fasali year started from Āśvina Kṛṣṇa 1 pūrṇimānta. Then it was luni-solar. In Madras, it started with karka saṅkrānti. The initial date was fixed by British on 13th July in 1800 AD and from Ist July in 1855 A.D.

(18) Lakśmaṇasena Era - It is current in Mithila region of north Bihar. This is Kārttikādi, amānta and started in 1118 A.D.

(19) Rāja Śaka - It started on Jyeṣṭha śukla 13th in Śaka 1596 with coronation of Śivājī.

(20) Ilāhī Era - This was started by Akbar and also called Akbar san. It started on 14-2 - 1956 A.D. with his coronation. Its years and months are solar. Month names were Persian starting with Farvardin and each day of month had a separate name as in Persia.

(11) Festivals and Yogas in India

A. Rules Festivals are generally based on tithis except saṅkrānti days. As a tithi generally covers a period of two days, a tithi may be counted on day when it is current on sunrise. But for religious purposes it may have to be celebrated on the previous day when it begins. Tithi for feast or fast

is observed on the day in which it covers the prescribed part.

For such purposes, a day is divided into 5 parts between sun rise and sun set -

(a) Prātah Kāla - 6 ghaṭikā from sunrise.

(b) Saṁjava - 6 to 12 ghāṭikā from sunrise.

(c) Madhyāhna - 12 to 18 ghaṭikā from sunrise.

(d) Aparāhṇa - 18 to 24 ghaṭikā from sunrise.

(e) Sāyāhna - 24 to 30 ghaṭikās from sunrise.

Relevant parts of night are -

(a) 4 ghaṭikās before sunrise are called aruṇodaya or uṣākāla

(b) 6 ghaṭikā after sunset are called pradoṣa

(c) 2 ghaṭikā in middle of the night are called niśītha - midnight

A tithi is pūrva viddha when it commences more than 4 ghaṭikās before sunset of one day and ends before sun set of the following day. A festival on such a tithi is calebrated on the first day of the tithi and not on the second.

Tithi dvayam - when 2 tithis meet between 18 and 24 ghaṭikās after sunrise, but a similar meeting does not take place on next day.

B. Festivals Connected with nakśatras as well as Tithis

In southern India, nakśatras are often linked with solar months to observe a festival. Śravisthā with Lunar śrāvaṇa makes upakarma. Tithi festivals are also connected with solar months. When a śukla pakśsa tithi falls twice in a solar month, the first is called a śūnya tithi and only the second is celebrated.

(1) Pratipadā (Tithi 1)

Caitra śukla pratipadā i.e. that which precedes the Meśa Saṅkrānti, is the beginning of Hindu Lunar year. New year's day (Lunar) falls on the day when pratipadā is current on sunrise. When there is an adhika caitra, that begins the year. This tithi is, therefore, called Vatsararambha. It is also Navarātrārambha.

There is another navavātra starting on Āśvina śukla pratipadā.

Kārttika śukla 1 is Balipratipadā or Balipūjā and is pūrva viddha as to time.

Bhādrapada bahula 1 is Mahālayārambha.

Phālguna bahula 1 is Vasantotsava

(2) Dvitīyā (Tithi 2)

Āśādha śukla 2 is Rathayātrā dvitīyā or Rāma rathotsava. Kārtika śukla 2 is yama dvitīyā or Bhrātṛ dvitiyā (sisters make presents to brothers in afternoon) Bahulā dvitiya in 'Aṣāḍha, Śravaṇa, Bhādrapada and Āśvina is called Aśunya śayana-vrata and fast is broken at moon rise.

(3) Tritīyā (Tithi III)

Caitra śukla 3 is gaurītritīyā, also Matsya jayantī (afternoon), also Manvādi (forenoon).

Vaiśākha śukla 3 is kalpādi (forenoon), Tretā yugādi (forenoon), Akśaya tritiya (special when combined with Wednesday and Rohini nakśatra, forenoon), also Paraśurāma jayanti.

Jyeṣṭha śukla 3 is Rambhā tritīyā, when Bhavānī is worshipped at pūrva viddha.

Srāvaṇa śukla 3 is madhu sravā in Gujrat.

Srāvaṇa bahulā 3 is kajjalī tritīyā

Bhādrapada śukla 3 is varāha-jayanti (afternoon); Haritālikā, when Pārvatī is worshipped, Manvādi (forenoon). It is also called śivā tithi.

Phālguna bahula 3 is kalpādi (forenoon)

(4) Caturthī (Tithi 4)

Śukla Caturthi in every month is called Gaṇesh caturthī on Vināyaka caturthī, the chief being Māgha Caturthī (Ganeśa jayantī). It is celebrated at midday. Tila caturthī is its another name; but is observed in evening. It is also called kunda caturthī.

Bhādrapada śukla caturthī is special when it falls on sunday or tuesday.

Similarly, bahulā caturthī in every month is Saṅkaṣṭa caturthī and is a fast day for people in difficulties. Fast is broken at moon rise. If it falls on tuesday, it called Aṅgāraka caturthī and continues till moon rise.

Srāvaṇa bahulā caturthi is the main Bahula caturthī, and cows are worshipped.

(5) Pancamī (Tithi 5)

Caitra śukla 5 is Śrī pañcamī. According to some, it is also kalpādi.

Śravaṇa śukla 5 is Nāga pañcamī, when snakes are worshipped. If the tithi starts within 6 ghaṭī after sunrise of one day and ends within 6 ghaṭī of sunrise on next day, the tithi is observed on the first day.

Bhādrapada śukla 5 is Ṛṣi pañcamī.

Srāvaṇa bahulā 3 is kajjalī tritīyā

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bhadrā. A śukla saptamī on a saṅkrānti is called Mahājayā which is superior to eclipse for making donations.

Vaiśākha śukla 7 is Gaṅgā saptamī or Gangotpatti (birth of Gaṅgā - midday).

Śrāvaṇa bahula 7 is Śītalā or Sitalā vrata, time pūrva viddha.

Bhādrapada śukla 7 is called Aparājitā

Aśvina śukla 7 - About this tithi Sarasvatī is worshipped under mūla nakśatra

Kārttika śukla 7 is kalpādi (forenoon)

Mārgaśira śukla is Sūrya vrata.

Māgha śukla 7 is Ratha saptamī or Mahā saptamī (time aruṇodaya), Manvādi (forenoon)

(8) Aṣṭamī (Tithi 8)

An aṣṭamī, falling on wednesday, is special and receives the name of Budhāṣṭamī. The Śukla-aṣṭamī in every month is sacred to Durgā or Annapūrṇā, Bahula-Aṣṭamī in every month called Kṛṣṇāṣṭamī, celebrated at purvaviddha, is sacred to Kṛṣṇa.

Caitra śukla 8 in 'Bhavānī utpatti'; when joined with Wednesday and punarvasu nakśatra, bathing on this tithi is special.

Śravaṇa bahula 8 - Janmāṣṭamī, Kṛṣṇaṣṭamī or Kṛṣṇa Jayantī (midnight) special when combined with Rohiṇī nakśatra; less so when joined on monday or wednesday. Manvādi (afternoon).

Bhādrapada śukla 8 - Jyeṣṭha Gaurī pūjana vrata; when combined with Jyeṣṭhā nakśatra.

Bhādrapada bahula 8 - Mahālakśmī vrata (pūrva viddha); Aṣṭāka śrāddha.

Āśvina śukla 8 - Mahāṣṭāmī, special when joined to tuesday.

Kārttika śukla 8 - Gopāṣṭamī - worship of cows.

Kārttika bahula 8 - Kṛṣṇāṣṭamī, Kāla bhairavāṣṭamī or kāla bhairava Jayanti.

Mārgaśira bahula 8 in Aṣṭaka śrāddha in afternoon, the same is case with bahula 8 in Pauśa, Māgha or Phālgana.

Pauśa śukla 8 in special when on Wednesday with bharaṇī nakśatra (Rohiṇi or Ārdrā according to some).

Māgha śukla 8 is Bhīṣmāṣṭamī at midday.

Māgha bahula 8 is birth of Sitā.

(9) Navamī (Tithi 9)

Bhādrapada śukla 9 - Adukha navamī.

Āśvina śukla 9 - Mahā navamī or Durgā navamī, Manvādi (forenoon)

Kārttika śukla 9 - Tretā yugādi (forenoon)

Mārgaśirṣa śukla 9 - Kalpādi (forenoon)

Māgha bahula 9 Rāmadāsa navamī

(10) Daṣamı (Tithi 10)

Jyeṣṭha śukla 10 - Dasa-harā (destruction of 10 sins) Gangā- vatāra.

Āṣādha śukla 10 - Manvādi (forenoon)

Āśvina śukla 10 - Vijayādaśamī (afternoon special with śravaṇa nakśatra, Buddha Jayantī.

(11) Ekādaśī (Tithi 11)

Every Ekādaśī is sacred and has a separate name. It is called Vijayā when combined with Punarvasu nakśatra.

	Month	Śukla	Bahulā
1.	Caitra	Kāmādi	Varūthinī
2.	Vaiśākha	Mohinī	Aparā
3.	Jyeṣṭha	Nirjalā	Yoginī
4.	Āṣāḍha	Viṣṇu śayanotsava Śayanī or Viṣṇu Sayanī (Viṣṇu going to sleep)	Kāmādi or Kāmikā
5.	Śravaṇa	Putrādi	Ajā
6.	Bhādrapada	Viṣṇu parivartanotsava or parivartiṇī (VIṣṇu turning on his side) called Viṣṇu Śṛṅkhalā when 11th and 12th tithis meet in Śravaṇa nakśatra	Indirā
7.	Āśvina	Pāpānkuśa or pāsāṅkuśā	Ramā
8.	Kārttika	Prabodhinī (Awakening of Viṣṇu), Bhīṣma pañcaka Vrata commences	Utpatti
9.	Mārgaśira	Moksadā	Saphalā
10.	Pauṣa	Putradā or Vaikuṇṭha ekādaśī, Manvādi (forenoon)	ṣaṭ-tilā
11.	Māgha	Jayā	Vijayā
12.	Phālguna	Āmalkī	Pāpa mocinī

12. Dvādaśī (Tithi 12)

This is called Mahādvādaśī in the following circumstances -

11th tithi current at sunrise on two successive days : the next dvādaśī is called Unmīlanī.

12th tithi current at sunrise on two successive days - first dvādaśī is called Vanjulī.

12th tithi followed by a full moon or a new moon tithi, current at two sunrises - Pakśa Vardhinī.

12th tithi with Puṣya nakśatra - Jayā

- do - Śravaṇa nakśatra- Vijayā

- do - Punarvasu nakśatra - Jayantī

- do - Rohiṇī nakśastra - Pāpanāśinī

- Vaiśākha śukla 12, with Hasta nakśastra, guru and maṅgala in simha, sūrya in meṣa - Vyatīpāta

Āṣāḍha śukla 12, commencement of caturmāsya vrata

Śravaṇa śukla 12, Viṣṇoh pavitrāropaṇam

Bhādrapada śukla 12 - Vāmana Jayantī (mid day); called Śravaṇa divādaśī when with śravaṇa nakśatra, specially on Wednesday.

Āsvina bahula 12 - Govatsa dvādaśī (evening)

Kārttika śukla 12 - (i) End of cāturmāsya vrata which began on same tithi in Āṣāḍha.

(ii) Prabodhotsava or Utthāna dvādaśī (preparation for waking Viṣṇu)

(iii) Tulasī vivāha (Marriage of Viṣṇu with Tulasī plant)

(iv) Manvādi (forenoon)

Māgha śukla 12 - Bhīṣma dvādaśī

Māgha bahula 12 - Tila dvādaśī or vijayā when with Śravaṇa nakśatra (when previous Māgha is adhika).

13. Trayodaśī (Tithi 13)

Caitra śukla 13 - Madana trayodaśī or Anaṅga pūjana Vrata (pūrva viddha)

Bhādrapada bahula 13 - Kali yugādi (afternoon)

(ii) Māgha trayodaśī - when with maghā nakśatra

(iii) Gaja chāyā - when with maghā nakśatra and sun in hasta.

Āśvina bahula 13 - Dhana trayodaśī

Māgha śukla 13 - Kalpādi (forenoon)

Phālguna bahula 13 (i) Vāruṇī when joined with Śatabhiṣaj

(ii) Mahāvāruṇī - do - + Saturday

(iii) Mahā-mahā-vāruṇī - when joined with Śatabhisaja nakśatra + saturday + śubha yoga

(14) Caturdaśī (Tithi 14)

Bahula Caturdaśī in every month is Śivarātri

Vaiśākha śukla 14 - Narasiṁha Jayantī (sunset) : special when joined with svātī nakśatra + saturday

Śravaṇa śukla 14 - Varāhalakśmī Vrata

Bhādrapada śukla 14 - Ananta caturdaśī

Āśvina bahula 14 - Naraka caturdasī (moon rise), Dīpāvalī may fall on this tithi if with svātī nakśatra (normally on Āśvina bahula 15)

Kārttika śukla 14 - Vaikuṇṭha caturdaśī (mid night)

Mārgaśirṣa śukla 14 - Pāṣāṇa Caturdaṣī

Māgha bahula 14 - Mahā Śivarātri (mid night when Śravaṇa nakśatra is current). Special when combined with sunday or tuesday and śiva yoga.

15. Śukla Pañcadaśī (Tithi 15) or Pūrṇimā

A śukla 15 or purṇimā is called somavatī when it falls on monday and is special for donations

It is called cūdāmaṇi - when further joined with a lunar eclipse. Special names are given below-

Caitra Pūrṇimā

(1) Manvādi (forenoon)

(2) Hanumāna Jayantī

(3) Special for bathing when combined with sunday, thursday or saturday.

Vaiśākha pūrṇimā - Kūrma Jayanti (late after noon)

Jyeṣṭha pūrṇimā (i) Manvādi (fore noon)

(ii) Vaṭa pūrṇimā or Vaṭa sāvitrī (pūrva viddha)

(iii) Mahā Jyeṣṭha when moon and jupiter are in Jyeṣṭha nakśatra and sun in Rohiṇī

Āṣāḍha Pūrṇimā (i) Manvādi (forenoon)

(ii) Śiva śayanotsava or Kokilā vrata or Vyāsa pūjā

Śrāvaṇa pūrnimā (i) Rk yajuh Śrāvaṇī (for followers of Ṛk and yajurveda

(ii) Rakśā bandhana (tying a string round the arm) or Rākhī pūrṇimā or nāralī pūrnimā (throwing coconuts into the sea)

(iii) Hayagrīva Jayantī

Bhādrapada pūrṇimā - (i) Kojāgarī pūrṇimā or kojāgara vrata (mid night) Laxmi and Indra worshipped; games of chance.

(ii) Navānna pūrṇimā - when new grain is cooked.

Kārttika pūrnimā (i) Manvādi (forenoon)

(ii) Caturmāsya vrata ends

(iii) Tripurī pūrṇimā or tripurotsava

(iv) Special when joined with Kṛttikā nakśatra

(v) Mahā Kārttikī, when joined to nakśatra Rohiṇī or when moon and jupiter both are in Kṛttikā nakśatra.

(vi) Padmaka yoga when moon in Kṛttikā and sun in viśākhā

Mārgaśirṣa pūrṇimā (i) Dattātreya or Datta Jayantī (evening)

(ii) Special for donation of salt when joined with Mṛgaśirā

Māgha pūrṇimā - māghī - when moon and jupiter both are in Maghā nakśatra

Phālguna pūrnimā (i) Manvādi (forenoon)

(ii) Holikā or Hutāśanī pūrṇimā (evening)

(16) Bahula Pañcadaśī (Tithi 15) - Amāvasyā

A solar eclipse on sunday is cūdāmaṇi and is special for donations.

Śrāvaṇa amāvasyā (at beginning of next month bhādrapada as in all cases) - Pithorī or Kuśotpāṭinī

Bhādrapada amāvasyā - Sarvapitri or Mahālayā amāvasyā, special when sun and moon both are in hasta

Āśvina amāvasyā - Dīpāvali, with previous or following tithis; that on svāti nakśatra is special.

Pauṣa amāvasyā - (i) Ardhodaya when joined with sunday in day time + Śravaṇa nakśatra + Vyātipāta yoga.

(This can happen only when some previous month is adhika)

(ii) Mahodayā - when any one of these special features is lacking.

Māgha amāvasyā (i) Dvāpara yugādi (afternoon)

(ii) special for śrāddha when joined with Śatabhiṣaja or dhaniṣṭhā nakśatra.

Phālguna amāvasyā - Manvādi (afternoon)

Notes - (i) Many festivals differ due to interpratation by different sects or regions.

(ii) The list is not exhaustive

(iii) There were 14 manus (7 yet to come). So 14 days are manvādi.

(iv) Birthdays also differ according to interpretation whether original reckoning was solar or lunar. Birthdays of other sains like Tulasīdāsa, Nānaka, Kabira or Raidāsa etc are also calebrated.

(v) There are many other local festivals.

C. **Methods for Citation** : (i) Normally the gata or expired years, amānta months and Caitrādi years are given. A lunar year begins only when the solar year begins with meṣa sankrānti. Thus it is same as counting meṣādi solar years. Compared to this, the current years in christian era are counted.

(ii) Sometimes Varttamana or current years, pūrnimānta months or kārttikādi lunar years are also given, peculiar to a system of samvat.

(iii) An era is called year, samvat, saṁvatsara, san (arabic or persian), śaka etc. Thus śaka is not only śaka era but year in any era is called śaka.

(iv) Saṁvatsara is named in 60 year cycle of guru varṣa or Jovian years. In south India it is merely a solar year with Jovian name. In north

India it is the Jovian year actually completed at the beginnig of a solar year or year at the moment.

Jovian years are also named in 12 years cycle, when it completes one revolution. The current rāśi of Jupiter is name of year. Alternatively, Jupiter years are named on lunar months, corresponding to solar rāśis in which Jupiter is present. Mahā is added before these lunar months to indicate that they are years (Jovian)

References : (i) Spherical astronomy by W.M. Smart, Longman Green, London or by R.V. Vaidya, Payal prakāśana, Nagpur etc can be referred. Godfray has written a book 'Moon' only about Kinematic theory and perturbations of moon.

(2) For finding correct positions, a Nautical Almanc can be referred.

(3) History of calenders can be referred to the relevant article in Encyclopadia Britanica by Fotherington. History of calendar has also been published by govt. of India, publications division, being part C of report by National commission on calendars under Dr. M.N. Saha.

(4) History of Astronomy by S.B. Diksita, Govt. of India, or by S.N. Sen and A.V. Subbarayappa, published by Indian National Science Academy, Delhi - 2.

(5) Bhārata Varṣa Kā Bṛhat Itihāsa by Bhagavaddatta, Praṇava Prakāśana, Delhi - 26.

(6) Indian chronology by L.D. Swami Kannu Pillai.

(7) Reference for deciding festivals is Nirṇaya Sindhu by Kamalakara Bhaṭṭa.

Translation of Text (Chapter 6)

Verses 1-3 - Scope - Now I (author) write accurate pañjikā for getting quick results in marriage, sacred thread ceremony, house construction, yajña and birth caremonies etc. With help of this, accurate position of sun and moon are known and krānti, śara, lunar and solar eclipse, conjunction of planet and nakśatra, rising and setting, mahāpāta, tithi, nakśatra, yoga and karaṇa etc can be calculated.

Old astronomers assumed maximum increase of 5 daṇḍa and decrease of 6 daṇḍa in a normal tithi of 6 daṇḍa. Due to this their pañjikā was inaccurate, because actual increase or decrease limit of a tithi is much more. After describing rough pañjikā according to old school in last chapter, now I an explaining the method for accurate pañjikā. Calculation as per these rules will give correct time for auspicious works and the tithis etc can be actually seen. When direct observation proves the accuracy, no further logic is necessary in support of these rules.

Mean sun and moon corrected only by mandaphala give correct position at amāvasyā and purnimā. This has been described in previous chapter.

Verses 4-6 - Need for further corrections

Pañjikā has five limbs - vāra, tithi, nakśatra, yoga and karaṇa - so it is called pañcāṅga. Except the first part vārā, all others depends on sun and moon. Hence these will be accurate if sun and moon are accurate. Manḍa paridhi of moon and sun is taken same for rough and accurate methods

both. Hence sun and moon corrected by mandaphala is called Ist (corrected) planet.

From this Ist graha and Ist ravi sphuṭa gati, we calculate diameter of planet, time upto parva sandhi (i.e. pūrṇimā or amāvasyā), bhuja, mandakarṇa and lambana correction of sun.

Motion of moon is very complicated. After long period of observation, I have thought it necessary to have 3 more correction in addition to mandaphala. These four corrections are - Manda, Tuṅgāntara, Pākśika and digamśa.

Verses 7-9 - Tuṅgāntara correction

Tuṅgāntara kendra = Candra Mandocca
- (sphuṭa ravi + 3 rāśi) - in Śuklapakśa
or = Candra mandocca - (sphuṭa ravi - 3 rāśi)
- in Kṛṣṇa pakśa

Find out bhuja jyā of tuṅgāntara kendra.

Tungāntara bhujaphala (its bhuja jyā) is multiplied by 16 and divided by radius (3438). Then it is multiplied by bhuja jyā of difference of sphuṭa ravi and sphuṭa candra and again divided by radius.

Result in kalā etc. is multiplied by Ist candra sphuṭa gati and divided by madhya candra gati (790′35″). Result in kalā etc will be tungāntara phala. When tungāntara kendra is 0° to 180°, this is added to Ist sphuṭa candra otherwise substracted. We get second sphuṭa candra.

Verses 10-12 - Pākśika phala

Pākśika Kendra = 2nd sphuṭa candra - sphuṭa ravi. Lapsed and remaining parts of the kendra in its quadrant are found. Lesser of the two is

converted to kalā and multiplied by 2. Jyā of the resulting angle is divided by the hāra, to be calculated as below, gives pākśika phala in kalā etc. When candra (2nd) is in Ist half of pakśa, pakśika phala is added to 2nd candra, otherwise it is deducted.

Pākśika hāra is found by substracting Ist sphuṭa sun separately from mandocca of sūrya and candra. Bhujajyā of the two remainders is calculated and multiplied together. Product is divided by 180 and to the quotient we add 90. Result is the hāra for pākśika phala.

Verse 13 Digamśa phala

Mandaphala calculated from sphuṭa sun is divided by 10; multiplied by sphuṭa candragati, and divided by madhya candra gati. Result is digamśa phala. The mandaphala being positive or negative, digamśa phala is added or substracted from 3rd sphuṭa candra. We get 4th sphuṭa candra which will be accurate position.

Verses 14-15 : Reasons for correction -

In plane surface snake moves in a wave like motion, but at the time of entering a hole, its motion becomes straight. Similarly, moon deviates from the mandocca resultant motion normally, but on pūrṇimā and amāvasyā, these deviations vanish and only mandocca effect remains.

When snake enters a hole, its wavy motion ceases under pressure from narrow sides, but its natural forward motion also is affected. Similarly on parvasandhi, moon is not affected by tungāntara and pākśika sanskāra, but digamśa phala is still effective (in addition to mandaphala).

Comments : Fortnightly veriations in orbit due to sun effect are not evident on parvasandhi (pūrṇimā or amāvasyā) because sun, moon and earth are in a straight line. Howevere, total attraction of sun, varies according to its own variation in distance, causing minor corrections of digamśa phala. Parvasandhis are like a hole for snake, hence the simili.

Verse 16 - Rough and accurate correction for sun - For correcting accurate sphuṭa of ravi, accurate chart should be used and for rough sphuṭa, rough chart is used. Use in reverse order will create errors.

Verses 17-21 - Accurate sphuṭa gati of candra -

.Accurate tuṇgāntara phala is multiplied by radius (3438) and divided by Jyā of difference between Ist sphuṭa candra and accurate sphuṭa ravi. Result is multiplied by koṭijyā of difference of (Ist sphuṭa ravi - candra) and divided by radius (3438). Result added to Ist sphuṭa candra gati phala gives 2nd gati phala. This is added when manda kendra is 90° to 270°, otherwise substracted.

Pākśika phala in kalā etc. is squared and deducted from the square of maximum pākśika phala. Square root of remainder is multiplied by difference of 2nd candragati and sphuṭa sūrya gati and divided by half radius (1719). Result is added to 2nd gati when candra is in Ist half of śukla pakśa or 2nd half of kṛṣṇa pakśa. Otherwise, it is deducted. This is third sphuṭa gati of moon. This `third gati will be accurate. Difference of sphuṭa candra on two successive days also gives sufficient-

ly accurate gaṭi for calculation of tithi and auspicious works.

Comments : Corrections have already been explained in the introduction of this chapter. This explains the change of speed due to two paksika variations due to sun's attraction. Last correction is due to change in distance of sun which is negligible within a day and correction is not needed for ralready small effect.

Tuṅgāntara phala = - 160′ cos $(\theta-\alpha)$ Sin (D-θ)

where D = moon corrected for mandaphala, θ= . Longitude of sun,

α = mandocca of moon.

For a short period, only D is variable which is position of moon.

Hence speed due to this correction is by differentieting Sin (D-θ)

= - 160′ cos $(\theta-\alpha)$ Cos (D-θ), δ(D-θ)

$$= \frac{\text{RxTungāntara phala}}{\text{Rsin}(D-\theta)}, \frac{R\cos(D-\theta)}{R}$$

Thus we get the formula

Pākśika phala = 38′12″ Sin 2 (D-θ), 38′12″ = max phala = P

where D′ = 2nd sphuṭa moon.

Gati due to pākśika phala is its differntial

= 38′12″ cos 2 (D-θ) x 2 d(D-θ)

$$= 38'12'' \sqrt{1-\sin^2 2\ (D-\theta)} \times 2\ (dD - d\theta)$$

$$= \frac{1}{R}\sqrt{P^2 - (\text{Pākśika phala})^2} \times (\text{2nd candra}$$

gati - ravigati) × 2

$$= \sqrt{P^2 - \text{Pākśika Phala})^2} \times \frac{\text{2nd candra gati} - \text{Ravi gati}}{R/2}$$

This is the formula given above

Verse 22 : Sthūla value is not entirely useless, it is good for daily use. But adverse moments like viṣṭi, to be strictly avoided, should be calculated only through accurate motion.

Verse 23 : Phases of moon

Sun deducted from moon gives the kendra, when this kendra is in ist 6 rāśi i.e. 0° to 180°, it is śukla pakśa. When moon is ahead of ravi by 180° to 360° it is kṛṣṇa pakśa.First 3 rāśis are Ist half of śuklapakśa, then upto 6 rāśis it is 2nd half of śukla pakśa. Similarly in kṛṣṇa pakśa, Ist half is from 6 to 9 rāśis and 2nd half is from 9 to 12 rāśis. One fourth of every pakśa (i.e. 45° difference) is called pakśa pāda (quarter).

Verses 24-26 - More correct motion of moon -

Now I am telling more accurate motion of moon which needs to be calculated for eclipse. For this gati, we find gata and gamya kāla (lapsed and remaining) times of parvānta (pūrṇimā or amāvasyā). From that, true moon is found out. From this sphuṭa gati, sparśa, mokśa, sthiti etc periods of eclipse are accurately known.

First gatiphala is kept in two places. At one place it is multiplied by parama tungāntara phala (160) and divided by paramạ mandaphala (300′50″), At second place it is multiplied by Ist sphuṭa gati and divided by madhyagati (790′35″). Result of both places are added. The sum is added to madhyama gati of candra when manda kendra is

in 6 rāśis starting from karka (90°-270°), otherwise it is deducted. This is true mean speed of moon.

This is again kept at two places. At one place, sphuṭa sūrya gati is substracted and divided by half of hāra. Here Hāra = bhuja of (candra mandocca - sūkśma sūrya) x Bhuja of (ravi mandocca - sūkśma ravi ÷ 180° + 90°. Quotient is added to the true mean motion of moon at second place. Sum is true motion of moon.

<u>Comments</u> - Ist gatiphala = $\frac{\text{Koṭiphala} \times \text{Kendragati}}{R}$

$= \frac{r.\cos m \times \delta m}{R}$

Thus the above formula is

$$\frac{\delta m}{R} r \cos m \left(\frac{160}{r} + 1 + \frac{r \cos m}{R}\right)$$

Gatiphala of Tuṅgantara is

160 cos m. cos $(D\text{-}\theta)$

m = D - α = mandakendra,

cos $(D\text{-}\theta)$ is 1 for parvānta as $D\text{-}\theta = 0°$ or 180°

Hence, tuṅgāntara gati phala = 160 cos $\frac{\delta M}{R}$

This is the first term of above formula

$$\frac{\delta M}{R} . r \cos m . \frac{160}{r} = \frac{160 \cos m}{R} \delta m$$

Remaining terms are second order corrections in mandaphala itself.

Second step of correction amounts to pākśika correction as stated earlier after verse 21.

Verse 27 - Accurate pañjika - Use of one rough panjikā for normal works and another accurate pañjikā for important works will not be appreciated by anybody. Hence only accurate panjikā should be used, even though it involves hard labour. It alway deserves more respect.

Verse 28 - Definition of true planet - We are on the surface of earth. The point of sky where line from earth's interior centre to our location point on surface meets is called 'svastika' (vertically upward point). When the planet is seen on the great circle from kadamba (pole of ecliptic) through svastika and calculated position of graha on ecliptic, graha is called spaṣṭa.

Note - This will be explained fully in chapter 7 and lambana saṁskāra for solar eclipse. (Chapter 9)

Verse 29-31 : Need for calculating true planet -

According to old teachers, all auspicious works are done only according to this true graha. For this purpose bhagaṇa and bīja corrections are done to the planet.

For daily and special works of vaidika and smārtta type, true position of all planets are needed. But correction to moon is needed more, because pañcānga is based on moon's position. Hence, accurate corrections like tungāntara, pākśika and digamśa etc. have been thought of.

Even after these three sanskāras, there is difference of 2-3 palas (upto 1 minute) in the calculated and observed position of moon. But it is preferable to error of upto 14 ghaṭī (about 6 hours) which will occur without these corrections.

Only Brahmā can know how to eliminate this small error.

Note : Though these corrections are great improvement, some error will always remain. Error withhin 1 minute is sufficient for day to clay work. More accurate position is needed for scientific works. Every formula will give some error, though it is about 1/10 seconds or less in modern methods.

Verse 32-33 : Lambana and śara

A planet will be seen in different position, when seen from earth's centre (which is calculated) and when seen from surface (where we are located).

The angular difference between two position is called lambana. (In vertical position, it is already in line from earth centre to surface, hence lambana will be nil).

Distance of the planet from krānti vṛtta along great circle from pole of ecliptic (kadamba) to centre of planetary disc (also passing through svastika - vertical up point) is called śara or vikśepa.

Calculation of lambana and śara is called dṛk-karma (change of axis). This is needed only for lunar and solar eclipse and conjunction of planets. In that context only, it will be calculated. It is not needed for calculation of tithi and nakśatra etc.

Verses 34-39 : Authorities on need of true planets - Vṛhatsaṁhitā (Varāhamihira) has stated - If grahaṇa occurs before calculated time, then damage to foetus or child in womb, or war with weapons occurs. If it occurs after calculated time,

then damage to crops, loss of flowers and fruits and fear for people occurs.

Garga saṁhitā states - The result of having eclipse before or after calculated time has already been stated. Persons knowing true planets, never have error in timing. If every (astronomical) event occurs according to calculated time, then enemies of kings are destroyed and troubles cease. People become happy, being free from fear and disease.

Vaṣiṣṭha states (not known in which text) - Tithi etc. should be decided according to that theory only which gives true position of planets.

Śākalya samhitā states - Corrections to calculated position of planets should be done after observing them through instruments like nalikā (tube or telescope) gola (mirrors or sphere), turiya (Fourth - compound telescope). Correction of observed error is called bīja sanskāra. After that correction only, all rules will be correct. Result of direct observation (pratyakśa) cannot be ignored.

What is use of the gold ornament which cuts the ears ? Similarly what is the use of that śāstra whose results are not actually seen ?

Verses 40-42 : No need of lambana for tithi calculation

The people who talk of lambana for calculation of tithi etc, do not know its meaning. Explaining them is like talking to a deaf.

Spaṣṭa graha is known from the point of intersection of line from earth centre to the planet, when the graha is seen at that place, it is called spaṣṭa (or true) planet.

If graha is seen from earth's surface, tithi will be different for different places due to separate lambana corrections. hence tithi needs to be calculated from earth centre, so that it is same all ove the world.

Verses 43-46 : Bīja sanakāra -

When graha is not same as per gaṇita (calculation) and dṛk (observatoin), it cannot be used for auspicious works. So I describe the bīja karma, i.e. corrections to calculated position to tally it with observed position.

Bhāskracārya (in Bījopanayana) has stated - After daily observations of moon, I have observed that moon is seen 112′ liptā east or west from its calculated position. These are the minimum or maximum values of Bīja.

In Sūrya siddhānta - Sun himself has stated in the end that he was explaining bīja for good of the world even though it was a secret; after praying to gods and vedas.

In Brahma-sphuṭa siddhānta - Graha gaṇita (calculation from planetary theory) as told by Brahmā himself was lost (became erroneous) after lapse of long time. So Brahma gupta, son of Jiṣṇu, seeks to correct it with bīja-sanskāra.

Verse 47-52 : Origin of Tungāntara correction -

The error in calculated position of moon upto 112′ liptā (stated by Bhāskara) is probably due to distance of moon from ecliptic, so it should be related to the maximum vikśepa (281′ liptā). Because, after adding 3 rāśis of sāyana moon, Jyā of its krānti is 1370′. This multiplied by maximum

vikśepa and divided by radius (3438) gives 112 kalā. This is the same amount which is found by calculating difference between calculated planet and observed planet.

Bhāskarcāyra has called it Āyaṇa dṛk--karma, there are different types of Āyana karma in other siddhāntas. So it should be called a bīja sanskāra. I have called it parama tuṅgāntara phala.

It appears from dṛk-karma of Bhāskaracārya that maximum value of tuṅgāntara sanskāra is continuing since long ago. According to ancient teachers, it fluctuates, so they have advised to correct moon with bīja sanskāra.

To know the change in maximum value of tuṅgāntara phala, moon will be corrected after one thousand years. The error from true moon will give the value of change.

Notes : Candraśekhara has not understood the theory or reasons behind this tuṅgantara correction, But from the nature of variations, he has correctly assumed the position of maximum deviation and hence has got the correct formula.

Verses 53-57 - Variations in duration of tithi -

A tithi which includes aparāhṇa of two consecutive days has beeen called sūkśma tithi in smṛtis. Thus such a tithi has more than 66 daṇḍas (as aparāhṇa period is 6 daṇḍa and a day is of 60 daṇḍas).

According to smṛti, if tithi just touches one evening and is over before first half of day, then the śrāddha of that day should be done on next day. It should be over by 'kutupa' of next day.

(Here 'Kutupa' means 8th muhūrtta of the day time out of 15 muhūrtta = 30 daṇḍas between sunrise to sunset). Thus tithi = sāyāhna 6 daṇḍa + night 30 danda + half day time 15 daṇḍa = 51 daṇḍa.

Gautama smṛti has stated - If in sāyāhna (evening) of caturdaśī (Kṛṣṇa pakśa), amāvāsyā starts and is over before midday, then śraddha should begin in kutupa muhūrtta (14 to 16 daṇḍas from sunrise) and should be over by rohaṇa ṁuhurtta (12-14 daṇḍa after sunrise) on next day. This is called amāvasyā śrāddha.

In śukla and kṛṣṇa pakśa, on 7th, 8th and 9th - the three middle tithis, maximum difference in tithi duration is less than 6 daṇḍa. So these tithis have only 5 types of classifications - the 6th category of above six daṇḍa difference from 60 daṇḍa doesn't exist. Other 12 tithis in both pakśa have 6 types of class. If smṛtis are interpreted in this manner, there is no error in dṛk siddha (true) calculations.

Ancient teachers, didn't observe the daily location of moon in constellations. With rough calculation, there is variations in middle tithis also upto 14 daṇḍas (i.e. I 7 daṇḍa from average). But they had strived for accuracy, only at the end of a pakśa when wrong eclipse time will cause insult to the astronomer.

Notes - Traditional view about variation of tithi is 'Bāṇa vṛddhi, rasakśayah' i.e. increase upto 5 daṇḍas and decrease upto 6 daṇḍa. This gives tithi limit from 54 to 65 dandas. But Candraśekhara

has found corroboration from smṛtis that it is actually from 51 to over 66 daṇḍas.

Verses 58-67 : Sūkśma nakśatra of unequal divisions - Now I explain the method to calculate sūkśma nakśatra (unqeual divisions) for use in journey, marriage and sacred thread ceremoney etc as decided by sages like Garga, Vaśiṣṭha.

Mean motion of moon in a day (790′35″) is the extent of sūkśma nakśatra. One and half times this value is the extent of these six nakśatras equal to (1185′52″18‴) - (4) Rohiṇī (7) Punarvasu (16) Anurādhā and three Uttarā nakśatras (12) Uttarā phālgunī (21) Uttarāṣāḍha (26) Uttara Bhādrapada.

Half extent (395′17″26‴) is of the six nakśatras (9) Aśleṣā (15) Svatī (18) Jyeṣṭhā and (24) Śatabhiṣā

Remaining 15 nakśatras have unit extent (790′35″). Deducting the total of these 27 nakśatras (21345′41″5‴) from kalās of full circle (21,600), remainder (254′18″35‴) is the extent of Abhijita which comes between (21) Uttarāṣāḍha and (22) Śravaṇa.

We substract the kalā of as many nakśatras from sphuṭa graha as it is possible. It is the number of completed nakśatras. Remainder (gata) kalā of the graha is the lapsed part of current nakśatra. This part deducted from full extent of current nakśatra gives remaining part (gamya or bhogya kalā). Gata and bhogya kalā, separately multiplied by 60 and divided by sphuṭa gati of graha, give the lapsed or remaining time of the graha in current nakśatra.

Each of the 28 nakśatras of unit, half, one half length or Abhijit being divided by 4 gives its one pāda (quarter).

As per rough rule, rāśi of 1800 kalā contains 9 nakśatra pāda (27 nakśatras excluding Abhijit have 27 X 4 = 108 pāda = 12 rāśi X 9). Thus 108 pada in 12 rāśis are according to mean equal values of nakśatras. With sūkśma rule, 1 rāśi doesn't have complete number of nakśatra padas.

For example, at the end of 4 rāśis 4 X 9 = 36 nakśatra pāda or 36 ÷ 4 = 9 nakśatra till Aśleṣā will be completed and 5th rāśi siṁha should start with 10th nakśatra. But according to sūkśma calculation maghā nakśatra starts 8° before siṁha raśi itself.

Notes : (1) There are three measures of a pañcāṅga for approximately one civil day. Week days are for fixing current routine of work and a day more than seven days ago or in future is not referred by the week day. Thus in modern university history books, even for modern eras, week days do not figure. This doesn't mean that use of week days is not common in modern days. Due to temporary nature of weekdays, and use for astrology only, they have not mentioned in histories of Rāmāyaṇa and Mahābhārata and in vedās. This should not mean that week days were not known in ancient India, as it is concluded by so called modern scholars. Another weakness of a week day is that it starts from local sunrise time in Indian system (local midnight in Gregorian or christian calender, local evening in Jewish and Islamic calender). Thus in all systems, it starts at different time at different places. Thus it cannot be made a world rference.

(2) Technically, tithi starts on same time all over the world. But for civil purposes, only the tithi current at sunrise is counted, hence it may

be useful for religious functions, but civil tithi will be different in different places. Another defect is that it is a mathematical calculation, even when moon has risen, only the approximate tithi can be known from its phase by rough eye estimate. However, nakśatra can be measured more accurately even with seeing moon's position among stars by naked eye. Even for calculation purpose, it doesn't suffer the errors in finding true position of moon, as it can be seen by direct observation. This is the reason that all the important events in Mahābhārata, Rāmāyana and Purāṇa are indicated by nakśatra of moon (instead of tithi) in addition of the lunar month. This is evident to the whole public and easily identifiable time in distant future.

(3) When nakśatra extent is made exactly equal to the mean motion of moon in one day, some part of the full circle will be left out as the moon takes more than 27 days for a siderial revolitions. Thus 27 nakśatra equivalent to 27 days motion of moon, doesn't cover the circle completely and a small 28th nakśatra abhijit is introduced equivalent to extra time beyond 27 days taken in moon's sidereal revolution.

Reason of unequal division is that, along the path of moon in sky, inclined at 5° angle with path of ecliptic, sufficient bright stars are not available for all nakśatra divisions. The three vacant places were identified with their preceding star groups causing division of 3 nakśatras in pūrva and uttara parts - Phālgunī, āṣāḍha and bhādrapada (or proṣṭha pada - old name). These three vacant places and 3 other star groups having lesser gaps - were given 1-1/2 times the length. Correspond-

ingly, the length of 6 nakśatras was reduce to half to compenisate the excess.

(4) Viśvāmitra made equal divisions for each of 27 nakśatras and further divided them into 124 parts each for accurate calculations of solar and lunar nakśatras at the end of day, pakśa or half year. Thus he created a different nakśatra system - which is proverbial creation of stars by him. This has been explained in introduction, while explaining vedāṅga jyotiṣa. Corresponding to 24 original nakśatras, there are 24 letters in a Gāyatrī chanda. But with 3 extra nakśatras by division of 3 into pūrva uttara parts, 3 extra letters (vyāhṛtis) were added to Gāyatrī mantra whose sage is Viśvāmitra, making 27 letters in it. This corroborates the view that number of verses in Ṛk veda and number of letter in its chandas are based on astronomical measurements at regular intervals. This unequal division will be more clear when longitudes of identifying stars are discussed.

Verses 68 - 71 - Saṅkrānti i.e. crossing from 1 division to another

Exact point of saṅkrānti of a rāśi is when centre point of a planet's disc reaches the last point of the rāśi. This sūkśma saṅkramaṇa is known by name of rāśi which is to be entered, not the past rāśi. Complete saṅkrānti period is the time taken by complete disc of graha from touching the border point to its complete crossing. To find the saṅkramaṇa kāla, drameter of graha biṁba (disc) in vikalā is divided by graha gati in kalā. Saṅkramaṇa kāla is obtained in daṇḍa etc. Within

saṅkrānti period, sūrya gives very favourable results.

Planets give mixed results of both rāśis during saṅkrānti period. Similarly while crossing over from one nakśatra to the next, as long as the border point is covered by biṁba (disc) of the planet, it gives results of both the nakśatras.

Candra biṃba (disc) in Vikalā, separately being divided by (1) difference of ravi and candra gati (2) candragati, (3) sum of candra and ravi gati, gives respectively the saṇdhi time of (i) tithi or karaṇa (2) nakśatra and (3) yoga.

Verses 72-74 - Different circles - Due to effects of ucca, krānti and pāta, many circular orbits are formed.

Orbit due to attraction of śīghra and manda ucca is called pratimaṇḍala (eccentric circle - explained in previous chapter). Path of krānti is called apamanḍala (to be explained in Tripraśnādhikāra).

Due to deviation of graha from krānti vṛtta due to pāta, another circle vimaṇḍala is formed which is path of pāta (apamaṇḍala and vimaṇḍala are great circles perpendicular to ecliptic and will be explaiend in next chapter.

Verses 75-91 : Precession of ecliptic and Ayanāmśa

Point of intersection of krānti vṛṭta (ecliptic plane of sun's orbit) and viṣuva vṛtta is called pāta which moves in the opposite direction to the normal motion of planets. Completed revolutions (bhagaṇa) of pāta in a kalpa are (6,40,170) as observed by the author.

This pāta is above all planets and circle of nakśatras (slowest rotation indicates farthest distance). This moves the nakśatra vṛtta from east to west in plane of krānti vṛtta.

When this pāta is in six rāśis beginning with meṣa (0° to 180°), it takes the nakśatra and planets etc 27° towards east. When it is in six rāśis starting from tūlā (180° to 360°), it takes the nakśatras etc 27° towards west.

Due to this pāta, planets like ravi and nakśatras starting with Aśvinī are seen towards north or south from ecliptic even on the position of 0° krānti.

To find out krānti pāta for desired day, kalpa revolutions of krānti pāta are multiplied by ahargaṇa and divided by sāvana dinas in a kalpa (i.e. 15,77, 91, 78, 28, 000). We get the complete revolution numbers and from remainder rāśi etc. of krānti pāta.

The result in rāśi etc is substracted from 12 raśi and remainder is converted to bhuja according to quadrant and then to kalā. Bhuja kalā divided by 200 is called calānśa. This is also called ayanāmśa.

Raminder after division by 200, is multiplied by 60 and divided by 200. We get kalā of ayanāmśa. Motion of ayanāmśa in one day is 9/28 parā etc. At the beginning of karaṇāba (1869 AD, meṣa sankrānti at Laṅkā), ayanāṁśa was 22°1′51″45‴42⁗ etc.

When krāntipāta is in six rāśis beginning with tulā, then ayanāmśa is negative and, when in six rāśis beginning with meṣa it is positive.

According to Sūrya siddhānta - Ayanāmśa corrected graha (or sāyana graha) only is used for calculating krānti, chāyā, carkhaṇḍa etc. Motion of krānti pāta can be seen at the time of viṣuva saṅkrānti (sāyana karka sankrānti in uttarāyaṇa and makara saṅkrānti in dakśiṇa ayana).

Saptarṣi, Agastya and Yama and the stars close to them have no motion due to krānti pāta (They are near north or south pole and very far from ecliptic). Their motion in nakśatra maṇḍala towards east indicates that nakśatra circle has moved west wards. Seeing west ward motion of Saptarṣi etc means that nakśatra circle has moved eastwards. With this concept, astronomers calculate the śara of nakśara, which is north or south deviation from krānti vṛtta along circle perpandicular to it.

Position of sun calculated from shadow (chāyā) is different from mathematical position of true sun. This difference is ayanāmśa. This ayanāmśa is also moving eastwards. If calculated true sun is more than sun found from shadow, then ayanāmśa is moving west wards.

At the time of karka and makara saṅkrānti, when krānti of sun is equal, the rāśi etc of sun at both points is added. Their half is ayanāmśa. When sāyana karka or makara saṅkrānti is seen before nirayana sankrāntis then ayanamśa will be added, otherwise it will be deducted.

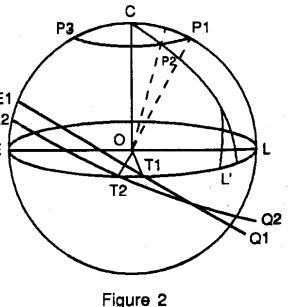
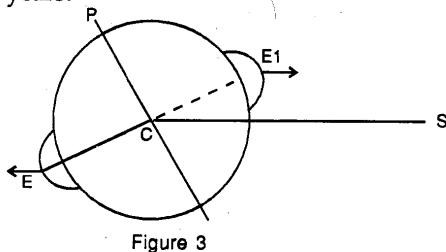



Figure 2

Note - (1) Newton's explaination : (Figure 2) C is pole of ecliptic EL'L. Let T_1 be mid point of E and L and thus the first point of meṣa for year 1. Then the celestial pole is at P_1 and celestial equator is E_1, T_1Q_1 Due to precession of equinoxes, the first point of meṣa is slowly moving in backward direction L T_2 E along the ecliptic. If T_1 shifts to T_2 in year 2, the celestial pole shifts to P_2 along a small circle $P_1P_2P_3$ - - - where CP is obliquity of the ecliptic. The celestial equator assumes a new position E_2 T_2 Q_2 in year 2. The celestial pole P_1 goes round the pole of ecliptic C and it makes a complete circle in a period of about 26000 years.

Figure 3

In Fig 3 - if earth is homogenous sphere, the force of attraction of sun will act, as if the mass is concentrated on its centre C., But it is an oblate spheriod, whose polar axis is shorter than equatorial axis by 43 Kms. The main pull due to sun is still along CS which keeps earth in orbit round the sun. But the bulge at equator EE_1 suffers additional pull. The nearer portion of bulge at E_1 is attracted more and E less. This extra forces at E_1 and E are equal and opposite in the direction of sun, but line EE_1 is inclined at an angle with CS. Hence it is a couple which tries to bring earth's

equator in plane of ecliptic. Due to this couple, precessional motion arises.

Overall reason of precession of solar orbits is that each planet influences the other and net effect is to bring angular momentum vector of all planets nearer to the direction of total angular momentum of the solar system. This mutual perturbation has a cycle of around 28,000 years. Due to motion of sun round the galactic centre also the angular momentum vector of solar system is turning in direction of galaxy's momentum. However this effect is very small and occurs in a period of about 250 million years.

Rigid Body Dynamics by A.G. Webster gives the following formula - Angle of precession P_1CP_2 = Ψ due to sun's attraction

$$\Psi = \frac{3\gamma m}{2\,\Omega\, r^3} \times \frac{C - A}{C} \cos\omega \left(t - \frac{\sin 2l}{2n}\right)$$

where = γ gravitational constant = 6.67×10^{-8} C.G.S. units

C = moment of inertia of earth round the polar axis

A = moment of inertia of earth round an equatorial axis

ω = Obliquity of ecliptic = 23°26′45″

m = mass of sun = 1.99×10^{33} gms

r = distance of earth from sun = 1.49×10^{13} cms

$\frac{\gamma m}{r^3}$ = tide raising term

l = longitude of the sun

n = angular velocity of earth in orbit

Ω = angular rotational speed of earth in radians

For a homogenous sphere, C=A and Ψ = 0. If polar radius C = a (1-ε), where ε is ellipticity of earth,

$\frac{C - A}{C} = \varepsilon = \frac{1}{297}$ if concentric layers of earth are assumed homogenous. But its real value has been found to be $\frac{1}{304}$. Putting the values in formula,

$\frac{d\Psi s}{dt}$ due to sun is

$\frac{3\gamma m}{\Omega r^3} \times \frac{C - A}{C} \cos\omega \ (1 - \cos 2I) = 2.46 \times 10^{-12}$ rad/sec.

It is multiplied by 2.063×10^5 = seconds in radian and 3.156×10^7 seconds in a year to get seconds of arc per year. Thus rate of solar precession = 16″.0 per year.

The tide raising force $\left(\frac{\rho m}{r^3}\right)$ for moon is more than double of the sun. Thus lunar precession = 34″.4 per year.

Moon's orbit is making an angle of 5°9′ average with sun's path (ecliptic) varying ± 10′. Point of interaction of moon's orbit travels on eecliptic in a period of 18.6 years (motion of rāhu). Figure 4 shows C, M as poles of ecliptic and of moon's orbit. P as celestial pole (earth north pole). Solar precession is vector along line PS. perpendicular to CP, lunar precession is represented by vector PR which goes up and down as M goes round C in a cycle of 18..6 years (Rāhu period) components of motion are

Along PS. = Ψms = $\Psi s + \Psi m$ CosM PC

Perp to PS. Ψn = Ψm Sin MPC

This causes certain irregulaties in precessional motion and also in the annual variation of obliquity - which is called nutation - with a period of 18.5 years

If t = no. of years after 1900 AD, then

Rate of precession = 50″.2564 + 0″.0002225 t

Angle between equator and ecliptic planes is 23°27′8″.26 - 0″.468 t

Correction in precession due to nutation is

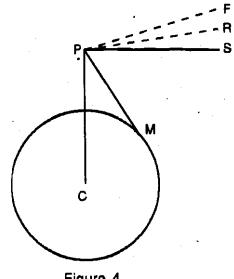



Figure 4

-17″.235 sin (sāyana rāhu) - 1″.27 sin (2 sāyana sun)

Correction in incline of equator is

+ 9″.21 cos (sāyana rāhu) + 0″.55 Cos (2 sāyana sun)

(2) Indian theories of precession : Correct theories : One theory states continuous backward motion which is correct as per modern theory. Other theory indicates-oscillatory motion whch is not correct either according to modern theory nor accordiing to references in Vedas or brahmaṇas.

Rates of Steady precession : Various quotations from puraṇas, brahmaṇas indicate different position of equinoxes.

Ṛgveda tells rains from Mṛga naksatra (I-161-13). Taittirīya saṁhitā (17-4-8) indicates vasanta at phālguna full moon. Both indicate a period of 23,720 B.C. when equinox was 352° behind present position.

Vālmīki Ramayāna indicates demon dynasty with Mūla nakśatra at vernal equinox. This should occur at 17000 B.C. which tallies with Egyptian countings mentioned by Herodotus. It also tells beginning of Ikśvāku dynasty with vernal eqninox at viśākhā at about 15080 B.C. This was the time of great deluge which is correct as per geology and sumerian records.

Mahābhārata indicates fall of pole star vega (Abhijit). At about 12,400 B.C. this was the pole star. Hence, around this star, a small extra nakśatra had been assumed.

Taittiriya Brāhmaṇa (I-5-2,6,) states kṛttikā to viśākhā are Deva naksatras which turn Sun from south. Anurādhā to Apābharaṇī are yama nakśatra which turn sun from north. This position of winter and summer solstice was in 8357 B.C. Varāhamihira tells that winter solistice was at Dhaniṣṭhā beginning at time of Vedānga jyotisa and at Makara beginning in his own time (about 100 B.C.). He has concluded backward motion of ayana.

Śatapatha Brāhmaṇa tells Kṛttikā at equator, present position being 36°9′ east and vikśepa 4°2′. This was about 67°56′ east of present position of vernal equinox. This was 2942 B.C.

Mañjula (932 AD) has indicated backward precession of vernal equinox 1,99,669 cycles in a kalpa i.e. 59″.86 per year. Bhāskara II has also accepted his authority. He has stated that ayana was non existant at time of Āryabhaṭa and negligible at time of Brahmāgupta and so they have not discussed. Even Bhāskara has mentioned it only in the context of constructing gola bandha (armillary sphere) Curiously Jagannātha Samrāta in his Siddhānta Saṁrāṭa has indicated 278 Śaka as year of zero Ayanāmśa and rate of precession per year as 51″. This value is accepted as per modern calculations Pṛthudaka (928 A.D.) has given 56.″82 per year.

Even Munjāla value is very accurate. In 932 A.D. yearly rate of precession was 50.2453-0.0002225 t (years from 1850 AD) = 50.041″. According to Indian practice, excess precession for tropical year is 9.76″, then correct precession should be 59.8″ per year which is very close to his value of 59″.86.

Liberation theory : A suspect passage occurs in Sūrya siddhānta, Tripraśnādhikara, (9-10) which states -

In a yuga, nakśatra cycle falls back eastward thirty scores (त्रिंशत् कृत्या 30 X 20). Number of days (ahargaṇa) is multiplied by this 600 and divided by number of days in a yuga to give the no. of revolutions and fraction rāśis etc. Its bhuja is multiplied by 3 and divided by 10, which will give ayana in amśa or ayanāmśa.

This gives an oscillatory motion of 27° east and west from equinox point.

This appears a defective and interpolated passage because-

(i) It occurs in Triprasnādhikāra and out of context just after discussing directions and shadow lengths.

(ii) No where else in this text kṛtya = 20 units has been used. 30 scores should have been written 6 hundreds or each digit should have been indicated separately through words as per general practice.

(iii) The verse indicates oscillation of nakśatra cycle around equinox. If it starts with east ward motion; in 5097 years since kali, it should be towards west from equinox. But the 0° of ecliptic is towards east from equinox point, as it has been clearly mentioned in next verse also. Thus the text should have stated oscillations of equinox point around 0° of ecliptic. Due to 600 speed, round number (540) had been calculated at the beginning of Kaliyūga.

(iv) Oscillations of equinox within 27° is not mentioned anywhere in ancient texts. They have mentioned the difference of upto 35° and values at different points of time indicates only backward motion.

(v) Bhāskara II has quoted Sūrya siddhānta differently. According to him, surya siddhānta tells 3 lakh backward rotations of Ayanāmśa in a kalpa. This means 300 backward rotation in a yuga. This can mean 3 backward + 300 forward = 600 oscillations in a yuga. But this interpratation has not been mentioned in own commentary or any other commentary. Thus he must have mentioned

some version of sūrya siddhānta prevalent in his time. This was lost due to the interpretation presently found.

(vi) Reasons of accepting this wrong version is that 0° position is same in both systems around 285 AD. and both indicate backward motion till 2298 AD. Due to approximately equal angular speed in both system, we get the same position of Ayanāṃśa. So no body has thought it necessary to refute this theory.

Reasons for oscillation theory and its value of constants-

(i) Bhāskara and Varāhamihira have commented that Ayanamśa was zero at the time of Āryabhaṭa 3600 years exactly after Kaliyuga. Now, it has been assumed that all the planetary positions were zero at beginnig of Kaliyuga and they started moving east wards since then. The same assumption was made for krāntipāta which was found west from 0° at the time of Āryabhaṭa. This means that, pāta started moving east wards with uniform speed like all madhya grahas, at mid point till time of Āryabhaṭa it started moving backward and reached zero position again. Thus half oscillation was completed within 3600 years. Remaining half oscillation will mean backward motion for 1800 years from Āryabhaṭa and again forward motion for 1800 years, so that it comes to zero position in east ward motion, as in Kali beginning. Thus 1 cycle is 7200 years, giving 600 cycles in a yuga. At about 600 years after Āryabhaṭa, if Ayanāmśa was 9° west, then maximum oscillation in 1800 years will be 27° on either side. Such measurement only can be basis of this limit.

In comparison, Hipparchus (100 BC) had found precession but did not give the value. Ptolemy had estimated it to be 36″ per year. Albatani of Arab in about 880 AD, found the speed as 55″.5 Then Nasiruddin of Iran calculated in 1250 AD as 51″ per year which was very accurate.

Siddānta Darpaṇa has assumed sūrya siddhānta theory of oscillation, but has slightly corrected the value to 6,40,170 oscillations in a kālpa instead of 6 lakhs for a kalpa according to Sūrya siddhānta.

These corrections are based on the following-

(i) Assumption of true 0° position which is with 1/2° error in eye estimates– This is according to position of identifying stars as given in Sūrya siddhānta. This will indicate the current value of ayanāṅaśa as to how much vernal equinox has shifted west from this 0°.

(ii) Assumption about the time of 0° ayanamśa– it is clear that sūrya siddhānta value is based on 0° ayanāmśa at the time of Āryabhaṭa in 3600 kali in which half oscillation was complete. Figure of 6,40,170 oscillations in a year by siddhānta darpaṇa indicates 0° ayanāmśa in 284 AD. At present it is assumed to be on meṣa saṅkrānti of 285 AD. So reasons of Candraśekhara must have been same as current reasons for accepting this figure.

It may be noted that both theories give same figure at present because, their speeds are almost same. 27°X4 = 108° oscillation in 7200 years means 1° in 66.6 years. Siddhanta Darpaṇa gives 1° in 61.4 years. Modern figure is 72.24 years per degree for 0 AD and 71.63 years at 1900 AD. Munjula

figure also is 1° in 61 years. This was accepted by Bhāskara and this figure only has been accepted by Candraśekhar though under different theory.

(3) Formulas explained :

$$\frac{\text{Revolutions of Ayana till desired day}}{\text{Revolution in a kalpa}} = \frac{\text{Ahargaṇa}}{\text{No. of days in a kalpa}}$$

In a full revolution of 360°, quadrants are of 90° each. In oscillatory motion the corresponding quadrants are

0° - 90°	0° to + 27°
90° -- 180°	+ 27° to 0° reverse motion
180° - 270°	0° to-27° reverse motion
270° to 360°	-27° to 0° forward motion
Thus 27°	Ayanamśa = 90° revolution

or Revolution X $\frac{27}{90}$ is Ayanāmśa.

Hence revolution is multiplied by $\frac{27}{90} = \frac{3}{10}$ or $\frac{60}{200}$

which has been mentioned here.

Verses 92-99 - Calculation of Krānti

Planetary orbits (ecliptic) and equator circle, both are in east west direction. Due to inclination, they cut each other which results in krānti (north south deviation). Thus, deviation of the planet, north or south from equator is measured along great circles passing through north pole and south pole (of earth projected in sky). This is also called 'apama' or 'apakrama'.

Note : Krānti (apama or apakrama) is north south deviation from equator as seen from earth.

Śara or vikśepa is north south deviation from ecliptic as seen from sun.

Both are measured along great circle perpendicular to reference circle (equator or ecliptic).

In celestial sphere, an imaginary circle of rotation of sun is called krānti vṛtta or mārga (ecliptic circle of path.) It is divided into 12 rāśis. Ayana correction is done in Ist and 7th rāśis (0° and 180° position). The corrected positions of these rāśis give the positions of intersection of ecliptic with equator circle. These points are called pāta. Since day and night are equal, they are called saṁpāta. Thus there will be two sampāta, vasanta and hemanta (vernal or autumnal equinox). At 3 rāśis from saṁpāta, krānti will be maximum (23°30′) in north or south directions.

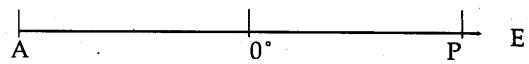
Jyā of maximum krānti (23°30′) is (1370′). Graha position corrected by ayanāmśa only is used for calculation of bhujaphala and jyā.

Ayanāmśa is added to spaṣṭa graha, sum (sāyana graha) is multiplied by Jyā of parama krānti (1370) and divided by radius (3438). This is equivalent to multiplication by 100 and division by 251. This will be krānti jyā of the spaṣṭa graha. This value converted to arc will give krānti in kalā.

Square of krānti jyā substracted from square of radius (1,18,19,844) and taking square root gives 'dyujyā' which is half diameter of ahorātra vṛtta (diurnal circle) - explained in Tripraśṇādhikāra.

Koṭijyā of sphuṭa graha (corrected with ayanāmśa) multiplied by 100 and divided by 251 and multiplied by daily motion of graha gives daily motion of krānti.

Ayana corrected graha moves northwards in Ist and last quadrants and south wards in 2nd and 3rd quadrants.

Notes : (1) Krānti from sāyana graha -

O is the O° of ecliptic. By definition, krānti at point A of intersection of equator will be zero, because it is at equator also. A is towards west from 0 due to backward motion. Planet P on ecliptic is counted in east direction from 0° of ecliptic. Thus krānti of planet P increases from A in the east direction, where it is zero.

Thus sāyana graha AP = OA (ayanāṁśa) + OP (distance from 0° of ecliptic i.e. true graha).

(2) As seen from equator, the pāta, A where kanti vṛtta appears moving north wards is the pāta taken as 0°.

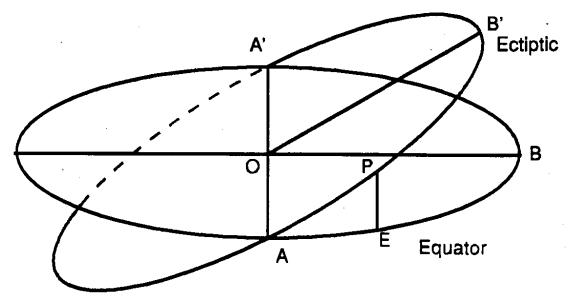



Figure 5

In figure 5, AE BA′ is equator and AB′ A′ is ecliptic which cut each other in line A O A′. OB and OB′ are radius perpendicular to AA′ at O

which is point of observation at centre of celestial sphere. This equator and ecliptic are inclined at an angle B′OB which is about 23-1/2°

Position of planet is at P on ecliptic whose distance from point A is the sāyana graha = AP. PE is arc of great circle perpendicular to equator, hence passing through pole of earth or equator. Thus length PE is the krānti, which can be determined from relations of right angled triangle APE on the sphere. Hence sāyana graha AP needs to be calculated to complete this triangle.

According to Napier's law for right angled spehrical triangles, sine of middle part = product of cosines of opposite parts

For middle part taken as PE, opposite parts are

$$\frac{\pi}{2} - PA \text{ and } \frac{\pi}{2} - \angle PAE$$

Hence sin (PE) = sin (PA) X Sin (PAE)

or Kranti jyā = R sin PE = $\dfrac{R \sin PA \times R \sin PAE}{R}$

Here PA = sāyana graha, ∠PAE = parama krānti

Hence, krantijyā

$$= \frac{\text{Jyā of sāyana graha} \times \text{Jyā of parama krānti}}{\text{Trijyā}}$$

Thus the position of highest krānti B′ is at 90°· from A of 0° krānti. Another point of highest krānti is opposite to B′ i.e. 90° from A′.

Since $\dfrac{\text{Jyā of parama krānti}}{\text{Trijyā}} = \dfrac{1370}{3438} = \dfrac{100}{251}$

a fixed quantity, hence alternate formula has been given

(3) Speed of krānti :

$$\text{Sin PE} = \text{Sin PA} \times \frac{100}{251}$$

Differentiating both, cos PE. d (PE) = cos PA. d (PA). $\frac{100}{251}$

For a single day, point A can be considered fixed and d (PA) = d (AO + OP) = d (OP) = speed of nirayana graha

as d (AO) = 0 for small period

PE is small and cos PE can be taken almost equal to 1.

So speed of krānti is d(PE)

$$= \frac{100}{251} \times \text{d (PA) cos PA}$$

$$= \frac{100}{251} \times \text{speed of graha} \times \text{cos of sāyana graha}$$

Verses 100--101 :

According to Bhāskarcārya, ayana doesn't move in west direction, hence he has asked to add ayanāmśa to the graha always. Still according to Brahma and sūrya siddhānta, I have assumed its motion in both direections. It will be clear by calculating ravi from chāyā (shadow of gnomon).

Verses 102-104 :

Day night values at a place-Krānti jyā multiplied by palabhā (shadow length of 12 length stick on equinox day) and divided by 12 gives kśitijyā. This, multiplied by trijyā (3438) and divided by dyujyā gives carajya. Its arc will be cara prāṇa.

Caraprāṇa added to $\frac{1}{4}$ of day night (15 daṇḍa) gives half day length when it is north krānti. On

substraction from 15 daṇḍas, half night length is obtained. When krānti is south, opposite procedure is followed - day half is obtained by 15 - caraprāṇa and night half = 15 + caraprāṇa. Multiplying them by 2 we get values of day and night (Quoted from sūrya siddhānta)

For finding day and night periods of nakśatras, moon and other planets, their śara is added to krānti, when they are in same direction, or difference is taken, when they are in opposite direction. From this spaṣṭa krānti, day or night time is found, (Day time is the period for which planet is above local horizon)

Notes (1) These topics have been discussed in Tripraśnādhikāra, but to understand the meaning of these formulas, it is necessary to explain the technical terms.

On equinox day, sun is perpendicular on equator, hence at local noon on an equator place it will be directly above, i.e. perpendicular to horizontal plane. Hence, a perpendicular to horizontal plane at other place with latitude ϕ, will be at an angle ϕ with sun's highest position at noon. Thus the length of a vertical pilllar's shadow at noon time on equinox day will give latitude of the place.

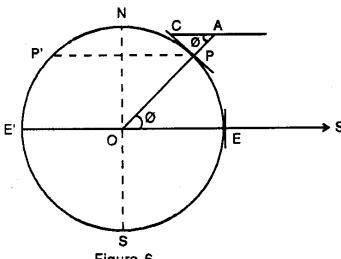



Figure 6

In Figure 6, S is perpendicular on equator passing through E, S being direction of sun. At place P, latitude = ϕ. Hence direction of sun in CA direction makes $\angle CAP = \phi$ with vertical direction of pole PA = 12 unit length.

$$\text{Tan } \theta = \frac{PC}{AC} = \frac{PC}{12} \text{ gives measure of latitude } \phi$$

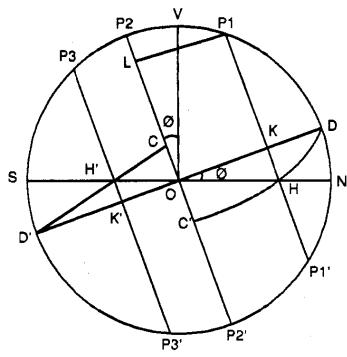



Figure 7 - Calculation of day time at a place

Figure 7, is a diagram for place O where day length of a planet i.e. period for which it is above horizon is to be found. NOS is horizontal line in north south direction at that place and DOD′ is the horizontal line for equator. D is celestial north pole (direction of earth's north pole in the sky) and D′ is south pole of earth. N D V S is the north south circle and V is the vertically upward point at O.

Due to daily rotation of earth, planets appear to move in circles parallel to equator. These circles are called ahorātra vṛtta (diurnal circle). For different positions of a planet or nakśatra, the

circles projected on vertical plane are P_1P_1', P_2P_2' and P_3P_3' all parallel to equator P_2P_2'. Sun on equinox day will appear moving on P_2P_2' - krānti for short time assumed constant.

When north krānti of a planet is arc P_2 P_1' then its diurnal circle is P_1P_1'. When south krānti is P_2 P_3, then the circle is P_3 P_3'. (diameter only is shown in projection). At equator, the horizontal line DOD′ cuts all the diurnal diameters in two equal parts. As long as the planet is above horizon or on V side of DOD′, it is seen or rising. Below it; it is set. Thus at equator, day and night are always equal.

However, for place O, the horizontal line is SON. Day portion of the planet is P_1H or P_3H'. It is bigger than 12 hours for north krānti.

OV is radius, P_1K =Dyujyā

(diameter of diurnal circle)

P_1P_2 arc or $\angle$ P_1O P_2 is krānti

Hence, krānti jyā = P_1L = OK

Versin of krānti = P_2L (versin θ = 1 – Cos θ)

Dyujyā = P_1K = OL = OP_2 - P_2L

= Trijyā - versine of krānti

= Krānti koṭi Jyā (1)

Kśitijyā = KH (extra motion on diurnal circle beyond half day).

Latiude ϕ = $\angle$HOK or $\angle$ VOP_2

In ΔKOH, $\tan \phi = \dfrac{HK}{OK}$

But $\tan \phi = \dfrac{\text{Palabhā}}{12}$

Hence Kśiti-jyā KH = $\frac{\text{Krānti jyā} \times \text{Palabhā}}{12}$ --- (2)

But DKO and DHC′ both are perpendicular on P_1P_1' and P_2P_2' (in the spherical triangle).

Hence $\frac{P_1 K}{KH} = \frac{P_2 O}{OC'}$

or $OC' = \frac{KH \times P_2 O}{P_1 K} = \frac{\text{Ksitijyā} \times \text{Trijyā}}{\text{Dyujyā}}$ --- (3)

This is value of OC′ = carajyā. Its angular kalā value is caraprāṇa, because earth takes 1 prāṇa to rotate kalā.

From the equations (1), (2), (3),

Carajyā

$= \frac{\text{Krāntijyā} \times \text{Palabhā}}{12} \times \frac{\text{Trijyā}}{\text{Krānti Koṭijyā}}$

$= \frac{\text{Krāntijyā}}{\text{Krāntikoṭijyā}} \times \frac{\text{Palabhā}}{12} \times \text{Trijyā}$

= Krānti sparśa jyā X Akśānśa sparśa X Trijyā --- (4)

In modern terms when Kranti is θ

Sin (cara) = tan ϕ tan θ --- (5)

Complete day is rising from horizon H to top position and then coming back to M again, after which it sets. Hence half day = $\frac{1}{4}$ day night + carajyā.

Verse 105 : Correction due to śara in day time

From sūrya siddhānta when krānti and śara are in one direction they are added to find spaṣṭa krānti of a planet (true declination from equator). When they are in opposite direction, their difference is taken for spaṣṭa krānti.

Notes : Krānti is inclination of planet from ecliptic. It is caused by two angles - Angle of

ecliptic with equator whch is called krānti (mean value). However, a planet deviates from ecliptic, whose angle is known as śara. Hence total inclination with equator is sum of these angles. This inclination only, decides their day and nights.

Verses 106-112 : Easy calculation of cara - Now a rough practical method is described to find out cara in pala.

(i) Find out the cara kalās at the end of 1,2 and 3 rāśis (corresponding to their krāntis)

(2) 3rd cara khaṇḍa = 3rd rāśi cara -cara of 2nd rāśi

2nd cara khaṇḍa = cara of 2nd rāśi - cara of Ist rāśi

Ist cara khaṇḍa = cara of Ist rāśi itself

These are the cara of meṣa, vṛṣa and mithuna rāśis in reverse order.

(3) Bhuja of sāyana planets is taken, its rāśi and degrees etc. are kept separately. If it is less than 1 rāśi, then degree and minute (kalā) are kept separately.

(4) Degree and kalā are multiplied separately by cara. Result at kalā place is divided by 60, quotient in degree added to degree place, remainder to be kept as kalā. Total degrees are divided by 30, remainder is kept there and quotient is added with rāśi.

If bhuja is more than 1 rāśi; but less than 2 rāśi, position is multiplied by Ist cara for meṣa rāśi, kalā and degrees are multiplied by cara khaṇḍa of 2nd rāśi. As before, excess kalā and degrees are added in higher places of degrees and rāśi.

If bhujā is more than 2 rāśi (it will be always less than 3 rāśi). then Ist and 2nd carakhaṇḍas are added at Ist place of rāśi. Degrees and minutes are multiplied by 3rd cara khaṇḍa. These are converted to rāśi, degree, kalā as before. Alternatively, cara of each rāśi of ravi is taken and accordingly, their fraction for each degree is calculated.

Notes : Rationale of method is obvious. It is linear interpolation which assumes that variation rate of cara within a rāśi (30° interval) is constant. This gives some error which can be ignored for practical purposes.

Verses 113-117 : Udayāntara pala from sāyana sūrya - Now method to find udayāntara saṅskāra is being explained. This is difference in pala between true sunrise time and madhyama sunrise time at Laṅkā. This is called time equation, arising out of inclination of ecliptic with equator. This rises steadily in first 3 half rāśis (i.e. 3 X 1/2 X 30° = 45°) and decreases till next 3 half rāśis.

From the first sampāta point, udayāntara (in pala) rises by, 12, 9, 4 pala for first 3 half rāśis, From 4th half rāśi to end of quadrant it declines by same amounts 4, 9, 12.

We find out the udayāntara palas for completed half rāśis. Fractional portion of lapsed degrees is multiplied by pala of that 15° part and is added to the result for completed half rāśis (if udayāntara pala is rising). It is substracted if udayāntara is declining.

When bhujānśa of surya is 45° (3 X 15° or 3rd half rāśi), its udayāntara pala is maximum 25 palas (12 + 9 + 4 pala). After that it starts declining. On equinox day or at 4th rāśis from that (0°,90°, 180° or 270°) udayāntara pala is zero.

Udayantara pala is multiplied by daily motion of graha and divided by no. of pala in a day (3600). Result is added to madhyana graha, if bhuja of sun is in even quadrant, otherewise it is substracted. Result will be the graha for sunrise time of Laṅkā.

Notes : This is approximate udayāntara palas at the end of each half rāśis. Its complete explaination will be given in Tripraśnādhikāra. (p/447)

Verses 118-120 : Rising time of planets -

We add ayanāmśa to graha, and from sāyana graha its udaya time in asu (prāna = 4 seconds) is found. Udaya asu is multiplied by daily motion of the planet and divided by no. of kalās in a rāśi (1800) Result is added to kalās in a circle (21,600), if the graha is margī (moving forward). If vakrī (retrograde), it is substracted from 21,600. The result will be day of the graha in asus i.e. after 1 rise, it will rise after that time again.

Sāvana dina for sun is roughly 60 daṇḍa. 59 liptā less from that is a nākśatra dinā. Method for finding sāvana dina of a planet has been told.

Notes : Udaya asu of a graha is its rising time, as its speed is seen from an inclined plane which will be less than its speed in the ecliptic. This will be less than its normal rising time. The corresponding apparant speed is found by dividing

the rising lime of that rāśi by 1800 kalā and multiplying it by gati of graha, this is movement in one day as seen from a latitude. If graha is moving ahead, this will be extra time taken by earth to reach its next rising place. Hence this time is added to 21,600 asu.

Verses 121-126 : Rising time of rāśis

The rāśī which rises (on eastern horizon) at a time is called lagna. At sunrise time, rāśi, aṁśa etc of sūrya itself is lagna. Sāvana day night (ahorātra) is found from daily motion of ravi as explained above. From true ravi at desired time, current rising rāśi (lagna is found).

In 1/12th part of krānti vṛtta (rāśi), there are 1800 kalā. Near equator, their inclination to equator is more. At the end of ayana (south or north), i.e. 90° east or west from equinox, krānti vṛtta (ecliptic) is paralled to equator. Hence, in diurnal circle (parallel to equator), different parts of ecliptic rise in unequal times.

To find out the rising times of rāśis at equator, the jyās of 1,2 and 3 rāśis and their krānti jyās are squared separately. For each rāśi, krānti jyā square is deducted from jyā square, and square root of the difference is taken. These are multiplied by trijyā and divided separately by jyā of 1,2,3 rāśis. Arc of the three results is calculated.

Rising time in asu for third rāśi is found by substracting arc of 2nd from 3rd rāśi. For rising of 2nd rāśi, arc of Ist is substracted from 2nd rāśi. Arc of Ist rāśi is its own rising time.

Thus we get rising times in asu (udayāsu) of (1) meṣa (2) vṛṣa and (3) mithuna rāśis. Udayāsu

of next 3 rāśis are in reverse order i.e. rising time of (4) karka is same as of 3rd rāśi, of 5th siṁha it is same as of 2nd rāśi and of 6th kanyā and Ist is same.

Rising times of tulā to mīna is in reverse order of the times for Ist to 6th rāśis.

Comments (1) Steps in calculation

Jyās of the rāśis (1,2,3) or 30°, 60°, 90° are 1719, 2978 and 3438

Their squares are (29,54,961), (88,68,484) and (118, 19,844)

Krānti jyā of 3 rāśis are (685), (1186), (1370)

Their squares are (4, 69, 225), (14, 06, 596) and (18, 76, 900)

Substracting krānti jyā squares from jyā squares, we get (24, 85, 736), (74, 61, 888) and (99, 42, 944).

Square roots of these results are

(1576/37), (2731/39) and (3153/15)

They are multiplied by trijyā (3438). Products are (54,20,408), (93, 91, 413), (108, 40, 873)

They are divided by respective dyujyās

(3369), (3227), (3153)

Results are (1609), (2190) and (3438)

Their arcs are (1675), (3471), (5400)

Rising time of meṣa = 1675

Rising time of vṛṣa = 3471-1675 = 1796

Rising time of mithuna = 5400-3471 = 1929

These are better approximations for modern values for 23°27′ inclination of equator. These are

based on 23°30′ declination and old siddhāntas assumed 24°. Comparison is given below

Sāyana rāśi	Rising time in asu in old siddhānta	Rising time in Siddhānta Darpaṇa	Modern values Asus	Modern values Minutes
Meśa	1670	1675	1675	111.7
Vṛṣa	1795	1796	1794	119.6
Mithuna	1935	1929	1931	128.7

Laṅkā rising time for all rāśis (Siddhānta darpaṇa)

Value	rāśis	rāśis	rāśis	rāśis
1675	(1) Meṣa	(6) Kanya	(7) Tulā	(12) Mīna
1796	(2) Vṛṣa	(5) Simha	(8) Vṛscika	(11) Kumbha
1929	(3) Mithuna	(4) Karka	(9) Dhanu	(10) Makara

(2) Derivation of rising time formula for 3 rāśis

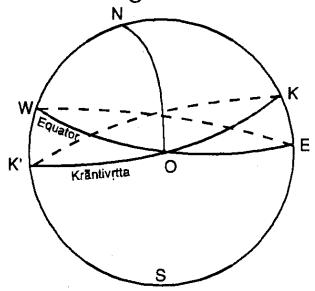



Figure 8 - Rising times of rāśis at equator

Figure 8 is horizon circle of equator in which E, N, W and S are the points in east, north, west and south.

WOE is equator circle

K′OK is ecliptic projection

O = Vasanta sampāta (or vernal equinox)

N is also direction of north pole of earth. Daily rotation of earth is along circle WOE, the time in which OE part of equator rises, is the time of rise of OK part of ecliptic also. But rising time of the whole equator circle 360° is 1 nākśatra dina (sidereal day) which is equal to 21,600 asus by definition. Hence rise of 1 kalā on equator will take 1 asu. Hence length of OE in kalā will give the rising time in asu which is rising time of OK part of ecliptic also.

OEK is a spherical triangle in which ∠OEK is right angle, ∠EOK is angle between equator and ecliptic which is maximum value of sun's krānti. EK is krānti of point K, arc OK is sāyana rāśi of point K measured from equinox point O. OE is its length measured on equator (viṣuvānśa).

Hence as per Napier's rule -

$$\text{Cos KOE} = \tan \text{OE} \times \cot \text{OK}$$

$$\text{or } \tan \text{OE} = \frac{\cos(\text{parama krānti})}{\cot(\text{sāyana rāśi})} \qquad \text{- - (1)}$$

For finding values in R sines (jyās), relations in spherical triangle NOK,

$$\frac{\text{SinNK}}{\sin \text{NOK}} = \frac{\text{SinOK}}{\sin \text{ONK}}$$

But ∠ ONK = arc OE

$$\text{Hence } \frac{\text{SinNK}}{\sin \text{NOK}} = \frac{\sin \text{OK}}{\text{SinOE}}$$

$$\text{or Sin OE} = \frac{\sin \text{OK} \times \sin \text{NOK}}{\sin \text{NK}}$$

Here OK = sāyana value of K

$\angle$ NOK = $\angle$NOE - $\angle$ KOE = 90°– Parama krānti of sun

Hence Sin NOK = cos (parama krānti) = Dyujyā of 3 rāśis

(because cos (krānti) = Dyujyā)

Sin NK = Sin (NE - KE) = Sin (90° - krānti of K)

= cos (krānti of K) = Dyujyā of K

Thus Sin OE

$$= \frac{\text{Sin (sāyanaK)} \times \text{Cos (Parama krānti)}}{\text{cos (krānti of K)}} \quad --- (2)$$

Allernatively it is, Sin OE

$$= \frac{\text{Sin (sāyanaK)} \times \text{Dyujyā of Parama krānti}}{\text{Dyujyā of K}} \quad --- (3)$$

Formula (3) has been given in the next verse.

In spherical triangle KOE

$$\frac{\text{sin KE}}{\text{sin KOE}} = \frac{\text{sin OK}}{\text{sin OEK}} = \text{sin OK (as sin OEK} = \text{sin } 90° = 1)$$

Thus in formula (2)

Sin OE

$$= \text{Sin OK} \times \frac{\sqrt{1 - \text{Sin}^2 \text{ KOE}}}{\text{D yu jyā of K}}$$

$$\frac{\text{sin OK} \sqrt{1 - \left(\frac{\text{sin Ke}}{\text{sin OK}}\right)^2}}{\text{K Dyujya}}$$

$$= \frac{\sqrt{\text{sin}^2 \text{ OK} - \text{sin}^2 \text{ KE}}}{\text{K dyujyā}}$$

$$\text{or R sin OE} = \frac{\sqrt{(\text{R sin Ok})^2 - (\text{R sin KE})^2}}{\text{Dyujyā of K}} \times \text{R}...(4)$$

This is the formula described in this verse.

(3) To prove that rising times of 4th to 6th rāśis are equal to those of 3rd to Ist rāśis in reverse order -

Equation (3) above tells

$$\text{Sin OE} = \frac{\text{sin OK} \times \text{cos (Parama krānti)}}{\text{cos (KE)}}$$

OE = rising time or length on equator in kalā.

OK = sāyana rāsi of K, KE = Krānti of K.

Sin θ = Sin $(180° - \theta)$

Hence Sin (180° - OE)

$$= \frac{\text{Sin } (180° - \text{OK}) \times \text{Cos (parama krānti)}}{\text{cos KE}}$$

Rising times of 90° at equator or ecliptic are same i.e. when OK = 90°, OE == 90°.

For rising time of mithuna (60°-90°), we substract the rising time of 60° from 90° time (6 hours = 15 daṇḍa = 5400 asu).

Rising time of 180° also is equal on both circles as it is equal for every 90°. Hence, rising time for karka (90° to 120°) is found by substracting the time of rising time of 3 rāśis from 120° time.

Now, when OK = 60°, OE is rising time (slightly less than OE)

When OK = 120° = 180°-60°, its rising time = 180° - OE

Hence rising time of Karka = (180°-OE) - 90° = 90° - OE = rising time of mithuna.

Similarly we can prove that rising times of simha, vṛṣa and kanyā, meṣa are equal.

The rising times of rāśis from meṣa to kanyā are equal to tulā to mīna in reverse order for all palces, not only on equator. So this result will be proved when rising time at other places is calculated. This is evident because both the ecliptic and equator circles bisect each other, hence other half 180° to 360° is similar to 180° to 0°.

Verses 127-128 : Alternative method for rising times at equator

Dyujyā of 3 rāśis (3153) is multiplied separately by jyā of 3,2, 1 rāśis (3438/2978/1719). Results (10,840,014), (93,89,634), (54,20,007) are divided by dyujyā of the rāśis (3153, 3227, 3369). Arc of the resulting ratios treated as jyā is found (5400, 3471 and 1675), which are rising times of 3, 2 and 1 rāśis.

Rising time of 2 rāśi is substracted from 3 to give time of 3rd rāśi. Time of 2nd rāśi is time of Ist rāśi deducted from rising of 2 rāśis. Rising time of Ist rāśis is already known.

Notes : This method has already been proved in previous verse.

Verses 129-130 : Rising times at different parts of sky.

Rising times of six rāśis in asu or prāṇa have already been stated as (1) 1675 (2) 1796 (3) 1929 (4) 1929 (5) 1796 and (6) 1675. (These have been calculated for rising on east horizon on equator). The rising time of rāśis for other points (on the east west vertical circle) are also the same. These points are udaya (east horizon), Asta (seetting point in west horizon), Daśama (Tenth house or vertically

upward point), Caturtha (fourth or vertically down wards).

Note : This is because all quadrants are same on both circles.

Veerses 131-142 : Lagna at any place -

To find .out rising times of rāśis at other places, we find out the cara khaṇḍa of first three rāśis as per formula described for that place. These cara khaṇḍas are deducted from first and last 3 rāśis in that order and are added to the three rāśi from karka and in reverse order to three rāśis from tulā. Addition and deductions of carakhaṇḍas is to the rising times of rāśis at Lankā (both in asu or prāṇa). These give the rising time at other place. for which carkhaṇḍa had been calculated.

According to rough calculation, whatever rāśi is rising in east horizon, its seventh rāśi (180° away) is setting in the west.

Rising times for each horā (1/2 rāśi = 15°), or dreṣkāṇa (1/3 rāśi = 10°) can also be calculated in same manner. For that krānti and dyujyā is calculated for each half or 1/3rd rāśi, hence it will be more accurate, than rising time for rāśis.

Ayanāmśa is added to sun at sunrise time position. Lapsed and remaining parts in the incomplete rāśi of sāyana sun is calculated. Remaining degrees of the rāśi are multiplied by rising time of full rāśi and divided by 30. This gives rising time of remaining part of that incomplete rāśi. This is substracted from desired time interval after sunrise (called iṣṭa kāla). From the remainder, rising times of next rāśis in successive order are deducted. Last remainder from which rising time

of next rāśi cannot be deducted - is multiplied by 30 and divided by rising time of the next rāśi. This result in degrees etc. is added to the completed rāśi which has risen. This gives sāyana lagna. Ayanāmśa is deducted from this to give the lagna for required time at desired place.

When fractional rising time of remaining rāśi of sāyana sun is more than iṣṭa kāla, the same sāyana lagna will continue to rise at iṣṭa kāla. This remaining rising time is multiplied by 30 and divided by rising time of sāyana sphuṭa ravi (or roughly by rising time of that rāśi). Result in degrees etc is added to sāyana sphuṭa sun and ayanāmśa is deducted to find sphuṭa lagna.

To find the moment when a particular lagna will rīse, ayanāmśa is added to it. Its lapsed part in incomplete rāśi is multiplied by rising time of that rāśi and divided by 30. This is lapsed rising time of the fractional rāśi. To this, we add the rising time of remaining fraction rāśi of sāyana sun at sun rise time, and the rising times of next completed rāśis upto the completed rāśi of sāyana lagna. The grand total will be iṣṭa kāla after sunrise, when the desired lagna will rise.

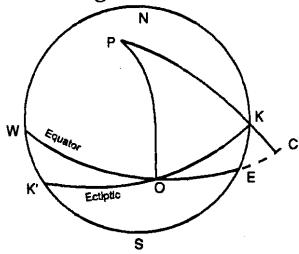



Figure 9 - Rising times of rāśis at places other than equator

Note : (1) Rising times for a place of latitude ϕ NESW is the horizontal circle at desired place of latitude $\phi°$ north. (fig. 9)

P is the north pole in sky :

O is vernal equinox point. WOE is equator circle, KOK′ is ecliptic circle. EC = Cara of K which is below horizon.

When point O is rising on east horizon, sāyana 0° of both ecliptic and equator are rising. When K point of ecliptic rises on horizon, E point on equator also rises.

Hence, rising time of OK in asu is same as that of OE. The polar circle PK passing through K meets OE extended at C which is below horizon. Thus OEC arc is the rising time at equator for point K. Hence rising time at 0° North is found by deducting EC from rising at equator. EC is the cara-kāla for point K.

Thus rising time = Equator rising time - carakāla.

Cara jyā = R tan $\phi \times$ tan θ

where θ is krānti of K. It has already been proved after verse 103

For meṣa rāśi, OK = 30°

Krānti of K is KC

Carajyā = R tan KC X tan ϕ = EC

Rising time OE = OC - EC

This holds good for meṣa to mithuna i.e. 0° to 90°. For karka rāśi, OK = 120°. Then krānti of K is same as of vṛṣa rāsi i.e. KC is same. Hence EC is also same as for 60° (vṛṣa).

Hence rising time of karka = OE = OC - CE

=(Rising time. of 3 rāśis + karka) - cara of Vṛṣa

= (Rising of 3 rāśis-cara of 3 rāśi) + karka + (cara of 3 rāśi - cara of vṛṣa)

= (rising time of 3 rāśi at ϕ lat) + karka + cara of mithuna

Hence extra rising time for karka = karka rising at equator + mithuna cara.

Similarly cara of vṛṣa is added to siṁha, and meṣa cara is added to kanyā rising time at equator.

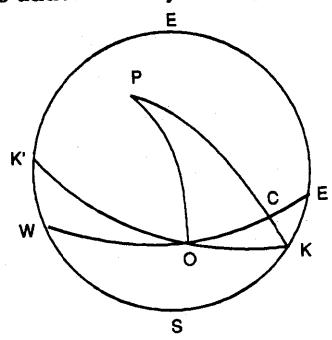



Figure 10 - Rising times for tulā to mīna

(2) Rising times for rāśis tulā to mīna - The figure 10 for 2nd half of ecliptic is same but the difference is that the two circles after crossing each other at autumn eqiunox O, have reversed their positions. K'O part of ecliptic which was above equator till 180° at O sāyana, goes below equator after O at OK i.e. after sāyana tulā. Hence, tulā to 3rd rāśi from it, cāra portion CE is to be added to the rising times at Laṅkā. Thus tulā rising time = tulā time at equator + cāra of Ist 30° (meṣa).

This is same as rising time of kanyā as proved in previous section.

Similarly times of vṛścika and siṁha are same and so on in that order.

(3) Calculation of lagna - OE is east horizon at sunrise time and OE′ is its position at iṣṭa kāla after sunrise. A_1, A_2, A_3 - - - A_7 are successive positions of starting of rāśis. (fig. 11)

At sun rise time, point E is on east horizon and lagna, and sun also is rising at E. Hence, at sunrise, rāśi of sun and lagna is same. The rising time of E′ is the sum of rising times of EA_2 (remaining part of fractional rāśi A_1A_2) then rising of complete rāśis A_2 A_3, A_3 A_4, A_5 A_6 and then lapsed part of fraction rāśi A_6 E′.

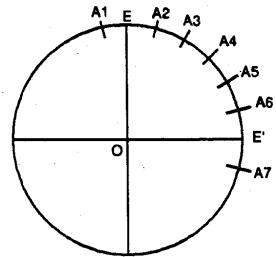



Figure 11 Calculation of Lagna

Within a rāśi the rising times can be considered as proportional to the parts, hence

$$\frac{\text{Rising time of E A2}}{\text{Rising time of } A_1\ A_2} = \frac{\text{degrees of EA}_2}{30^\circ \text{ of} A_1\ A_2}$$

Similarly rising time of completed part A_6 E′can be calculated as fraction of rising time of rāśi A_6A_7.

Since the rising times are not proportional to rāśi length (meṣa rising is much faster than vṛṣa for example) this calculation will be more accurate, if rising times of smaller parts like horā = 1/2 rāśi or dreṣkāṇa = 1/3 rāśi are calculated.

Verses 143-151 : Rule for finding dāśama lagna : Madhya lagna or tenth lagna (vertical top position) is found by rising time of rāśis at equator only for all places. (Because south north line bisects the diurnal circles at all places and corresponding times are same at any place and equator).

Before mid-day, the period for which sun will remain in east is called nata kāla which is desired time before mid day.

Nata kāla in east direction is expressed in asu. From this, we deduct the equator rising time of completed part of sāyaṇa sun rāśi. From remainder, the equator rising time of previous rāśis is substracted successively. Last remainder is divided by equator rising itme of next rāśi (whch cannot be deducted from remainder) and multiplied by 30. Result in degrees etc is substracted from 30°. This is added to the previous raśi which becomes sāyana madhya lagna. Spaṣṭa madhya lagna is found by deducting ayanāmśa.

When nata kāla is west, the time passed after mid day is nata kāla. From this we deduct the fractional rising time of sāyana sun at equator. Then rising times of next rāśis are deducted. Last remainder is divided by rising time of incomplete rāśi and multiplied by 30. Result in degree etc. is added to completed rāśis to give sāyana madhya lagna. From this, ayanāṁśa is to be deducted.

When in pūrva or paścima nata, naṭa kāla is less than the rising time of fractional rāśi (lapsed or remaining, then nata kāla is divided by rising time of the rāśi and multiplied by 30. Result in degrees etc is substracted from sāyana ravi for pūrva nata and added to it for paścima nata. Sphuṭa sāyana sūrya at midday is the ṃadhya lagna at that time.

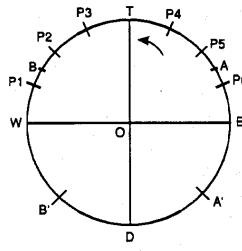



Figure 12

Notes : (1) Figure 12 is the vertical circle of any place O. E and W are east and west points. T is the top most or vertically upwards point. D is opposite to T and down ward point. Earth is rotating in clockwise direction, hence ecliptic appears moving in anti clockwise direction - shown by arrow.

Movement of ecliptic is not in this plane and only its projection is considered. At sunrise time, its projection will be at E which is not at 90° from D, it is upwards for north latitudes when krānti of sun is south. ETW is the day position and WDE is night portion of sun. DET is position of pūrva nata (mid night to mid day) and TWD is paścima nats. Vertical point T is same for all places, because

apparent rotation wil be paralel to equator. Hence rising times at equator are taken.

For a position of sun at A in pūrva nata, the tenth lagna is lagna at point T. P_1, P_2 - - - P_6 are the start of successive rāśis which will rise one after other in clockwise direction. Thus the rāśi of sun at A will reach T after travelling AT portion. Current rāśi at T is less than A. For point B or B′ at paścima nata, rāśi at B has aleady risen at T hence current rāśi is more than sun's rāśi at B. Position of sun at A or B indicate the time.

Thus, for pūrva nata of sun at A, tenth lagna at T is = $A - AP_5 - P_5P_4 - P_4T$

For paścima nata, Sun is at B, tenth lagna at T is = $BP_2 + P_2P_3 + P_3 T$.

For calculation of rising time of part rāśis, the rising time is considered proportional to degrees within the rāśi which is roughly correct.

Verses 152-153 - Rising time of Nirayana rāśis - when ayanāmśa is moving eastward, we take the difference of rising times of desired rāśi (sāyana value) and next rāśi, it is multiplied by ayanamśa and divided by 30. If rising time of next rāśi is more, then the result is added to rising time of sāyana rāśi to get the rising time of nirayana rāśi. If next rising time is smaller, it is substracted.

When ayanamśa is moving west wards (which is not the current position), we take the difference of rising times of the desired rāśi (sāyana value) and the rising time of previous rāśi, it is multiplied by ayanāmśa and divided by 30. If the rising time of previous rāśi is more, result is added to sāyana

rising time to get the rising time of nirayana rāśi. If previous rāśi time is more, it is added.

Notes : Method is obvious. Since ayana is moving east wards, sāyana rāśi is more. Hence, rising time of nirayana rāśis will be found by comparison with the next rāśi. Assuming ayanāmśa of 23°, nirayana meṣa 0° = sāyana meṣa 23°, nirayana meṣa 30° = sāyana meṣa 53°. Hence we have to find the rising times at 23° and 53° at sāyana value, and their difference is rising time for meṣa.

Verse 153 - When ayana (ecliptic) is moving west wards (from point of equinox) then rising times of cara rāśis (1, 4, 7, 10th rāśis) is same for sāyana and nirayana values at equator. At other places also rising times of meṣa and tulā will be same for sāyana or nirayana values (Their previous rāśis have same values).

When ayana (ecliptic) is moving east from equinox point, then dvisvabhāva rāśis (3, 6, 9, 12th) rāśis have same sāyana and nirayana rising times at equator. At other places, only the 6th and 12th rāśis have same rising times for sāyana and nirayana (as the rising times for next rāśis are same).

Verses 154-156 : Rising times for Orissa (22°N) and equator

Rising times for sāyana rāśi at middle of Utkala (22°N) in daṇḍa pala etc are as follows -

meṣa (3/56), vṛṣa (4/24), mithuna (5/7), karka (5/37), simha (5/34), kanyā (5/22). For second half circle starting from tulā, values are in reverse order.

Udaya times for 22° Ayanamśa - for nirayana rāśis are

meṣa (4/17), vṛṣa (4/56), mithuna (5/29), karka (5/35), siṁha (5/25) kanyā (5/22), tulā (5/31), vṛṣcika (5/36), dhanu (5/15), makara (4/35), kumbha (4/3), mīna (3/56).

Nirayana rising times at equator are meṣa (4/54), vṛṣa (5/16), mithuna (5/22) karka (5/35), simha (4/44), kanyā 4/39),

Values from tulā etc will be in reverse order. There values will change with change in ayanāmśa.

Veerses 157-160 - Values and Charts

Values of 28 nakśatras have been stated here according to sages like Garga and Vaśiṣṭha. Extent of rāśis (30°) is clear. Dhruva (constants) have been stated in liptā approximately. Motion of pāta of equator and ecliptic also have been given in appendix for 73 values of days. From them, ayanāmśa for any day can be calcualted.

Motion of pāta of equator and ecliptic in a day is liptās etc 0/0/31/32/51/35/6/53/28/23.

On first day of kali, viṣuva sampāta was 702 liptas from fixed meṣa 0° (towards east).

In Karaṇābda beginning, krānti pāta in rāśi etc. (for 1869 AD) was 3/16/33/47/27/40.

To find out the ayanāmśa since kali beginning easily, we find the years since kali beginning according to madhyama sūrya. (219/19) is deducted from it. Result is multiplied by 100 less (i.e. 2416) and divided by 2516. We get result in kalās. By

dividing with 60 we get degrees. Hāra of 54° is substracteed from it.

Notes : (1) Krānti pāta and ayanāmśa are considered same thing. But in terms used in the book, krānti pāta always moves in reverse direction, making complete circles of 360°. But ayanāmśa is 3/10 of its bhuja calculated according to quadrant of the krānti.

Kranti at kali beginning can be calculated by multiplying kalpa bhagaṇas by 1811 and dividing by 4000. (chapter 3, verse 51). Thus bhagaṇa at kali beginning

$$= 640170 \times \frac{1811}{4000} = 2{,}898{,}36.\ 9675 \text{ revolutions}$$

This is 0.0325 revolutions less than complete revolutions.

Since pāta in moving in backwards direction, it is 0.0325 east of 0°. Thus Kali position is 0.0325 revolutions = 0.0325 X 360° X 60 liptā = 702 liptās.

One revolution is in 432 crore years of kalpa divided by 640170 revolutions i.e. 6748.207507 years.

Hence pāta will come to 0° in reverse motion in

$$\frac{6748.207507}{360 \times 60} \times 702$$

$$= 219 \frac{19}{60} \text{ Years approx.}$$

Hence revolutions at kali beginning are counted from 219/19 yeears after kali (position of 0°). In Karaṇābda 4971 years had been completed since Kali. In about 3374 years half revolution will be complete in 3374+219 = 3593 kali year. Remain-

ing years are 1378 years in which less than 1 quadrant will be covered. Thus with reverse motion pāta crossed 4th and 3rd quadrants in half cycle and is now in 2nd quadrant at the end of which (i.e. at end of Ist quadrant in forward motion), krānti will be 90° correspondign to - 90° pāta or + 27° ayanāmśa.

By this method the Karaṇābda pāta is about 3 ♈ 16°34′ approx. Ayanāṁśa is less than 3rd rāśi position of 27° by 16°34′ × 3/10 i.e. 27°-4°58.2′ = 22°1′ approx, which is given at the end of verse 83.

(2) Ayanāmśa = (y-210/19) X 2416/2516 Kalā X 1/60 degrees.

219/19 years are deducted because, in that kali year krānti pāta and ayanāmśa were zero. As the ayanāmśa had become zero after 1/2 revolution of krānti 27°X2 = 54° movement of ayanāmśa in 2 quadrants, this amount is substracted, called hāra.

$$\text{Movement per year in kalā} = \frac{108 \times 60}{6748.207507}$$

= 0.960255 Kalā

0.960255 X 2500 = 2416.0016

$$\text{Hence annual movement of ayanāmśa} = \frac{2416}{2516} \text{ Kalās}$$

Thus we get the formula.

Verse 160 - Charts have been given in appendix for Jyā (R sine) for 24 khaṇḍas of 3 rāśis beginning from meṣa, krānti in kalā, semi diameter of diurnal circle, carkhaṇḍas in asu for Puruṣottama Kśetra (Purī), rising times of rāśi at equator in asu and udayāntara phala in asu for convenience of students. Intermediate values can be known by proportional increase.

Verses 161-162 - Prayer and end -

Supreme lord had directed Brahmā to create grahas for knowing the earned karma of previous births and fate in the present birth. Brahmā, in turn, regulates motion of planets through śīghra, manda, pravaha, pāta etc. The same supreme Lord Jagannātha has created Puruṣottama Kśetra for emancipation of beings. I pray that Lord Jagannātha living at Nīlācala.

Thus ends sixth chapter describing krānti, accurate sun and moon etc in Siddhānta Darpaṇa written by Sri Candraśekhara born in a bright royal family of Orissa, for purpose of educating students and for tally in calculation and observation.

Chapter - 7

THREE PROBLEMS OF DAILY MOTION

(Tripraśnādhikāra)

1. Scope - There are three problems regarding daily motion of earth, or rather it is used to find their answers -

(1) Place - Longitude or Latitude can be determined from daily motion. Both are needed to find the location of a place, specially in sea journey, when there is no other land mark for identification.

(2) Direction - North south direction can be measured roughly by a magnetic compass also, which gives other directions also. But this causes a lot of errors, because magnetic north pole is different from geographical north pole, which is on the axis of earth's rotation. In addition, there are local and general magnetic disturbances. Accurate method of finding the directions is only by astronomy, whether on land or on sea.

(3) Time - Measurment of time intervals are most accurate now with quartz watches for common use and most accurate laser and atomic watches for scientific use. However, that gives average standard time. True or apparent time can be found only by inclination of sun from vertical position. This is related to measurement of longitude also, as simultaneous measuring of time through sun at two places will be different, the difference depending on longitude. Thus time difference or lon-

gitudinal difference can be calculated from each other.

Siddhānta Darpana has treated this chapter in briefest manner and one of the vital use i.e. measurement of longitude has been left out. It has been explained roughly for purpose of making deśāntara correction in madhya graha in chapter 4. One reason of such neglect is that use of astronomy for navigation had ceased for Indians, who had lost the traditional excellence. This doesn't mean that astronomy is not needed for this purpose now. Even in modern astronomy, exactly the same methods are used for finding directions, place and time. With use of telescopes, their accuracy has increased, but formula is same.

Another reason for leaving some topics has been stated by the author that many more methods have been explained in detail by Bhāskarāçārya, whose book is most popular. Hence they need not be repeated. Before explaining individual methods, it will be useful to give a general idea of various right angled triangles used for calculations.

(2) Latitude triangles

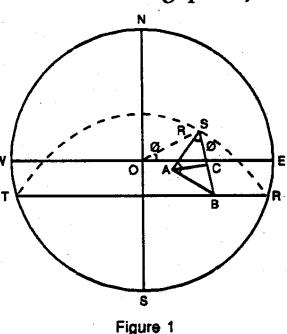
For calculation of 3 problems, some convenient right angled triangles are formed, whose one of the angles is latitude or akśāmśa . Hence they are all called latitude triangles or 'akśakśetra' in Indian astronomy. The other angles of such triangle are obviously $90^\circ - \phi$, and 90° as it is right angled triangle. $90^\circ - \phi$ is called colatitude or lambāṁśa. The side facing angle ϕ is called base (bhuja or bāhu), side. Facing $90^\circ - \phi$ is upright (koṭi) and the side facing right angle is hypotenus (karṇa).

The radius R of the celestial sphere is assumed to be 3438 or, more correctly 3437′44″ (which is value of one radian).

(1) Let S be the sun (or any other heavenly body) on the celestial sphere at any given time, SA be the perpendicular dropped from S on the plane of the celestial horizon, SB the perpendicular dropped from S on its rising setting line and AB the perpendicular from A on same line RT. (R is rising point on horizon and T is setting point).

OS = R, altitude of S is ∠ SOA = a. Hence height of S = SA = R sin a is the śaṅku. SB (hypotenus) is called 'Iṣṭahṛti'. It has been called 'dhṛti', 'svadhṛti' Iṣṭadhṛti', 'nijadhṛti' etc.

AB is called 'śaṅkuntala' or 'śaṅkvagra'

Figure 1

∠ AS B = ϕ, hence ΔASB is a latitude triangle. In this

Base	upright	Hypotenus
Śaṅkutala	Śanku or R sin a	Svadhṛti or Iṣṭadhṛti (1)

(2) When S is on prime vertical, SA is called sama - śaṅku' AB 'agrā' and SB 'samadhṛti' or tad-dhṛti'.

Base	Upright	Hypotenus
Agra	Samaśaṅku	taddhṛti - - - (2)

(3) When S is on prime vertical, then if a perpendicular AC is dropped from A on taddhṛti

SB, two more latitude triangles ACB and ACS are formed, AC = R sin δ where δ is declination. CB is called earth sine (kśitijyā), kujyā, Bhūjyā or mahājīvā etc.) SC = taddhṛti - kujyā

Base	Upright	Hypotenus
Earth sine	R sin δ	Agrā - - - (3)
R sin δ	taddhṛti	Samaśaṅku - - - (4)

(4) When Sun is on the equator and S its position on the celestial sphere at midday, SA is perpendicular on the plane of celestial horizon and O is centre of the celestial sphere, then SAO is again a latitude triangle. Then $\angle$ OSA = ϕ

Base	Upright	Hypotenus
R sin ϕ	R cos ϕ	R - - - (5)

(5) When Sun is on the equator, then at midday, the gnomon, (a vertical pillar of 12 unit length called śaṅku), its shadow (equinoctical midday shadow - palabhā, akśabhā, palacchāyā viṣuva chāyā etc.) and hypotenus of the mid day shadow(called palakarṇa, palaśravaṇa, akśkarṇa, akśaśruti etc.) also form a latitude triangle. This is called fundamental triangle and has been explained in previous chapter for calculation of lagna, day time etc.

Base	Upright	Hypotenus
Palabhā	gnomon or 12	palakarṇa - - - (6)

Then there are two altitude triangles for sun

Base	Upright	Hypotenus
(1) Śaṅku (R sin a)	dṛgjyā or natajyā (R sin Z)	R .... (7)
(2) Gnomon or 12	shadow	Hypotenus of shadow - - - (8)

(3) When the sun is on the meridian, śaṅku is called 'madhyaśaṅku' or madhyāhna śaṅku'. Shadow is called 'madhyāhna chāyā karṇa'.

(4) When the Sun is on the prime meridian (Samamaṇḍala), śaṇku is called sama śaṇku, shadow is samacchāyā and hypotenus of shadow is samacchāyā karṇa.

Translation of the text

Verse 1 - Scope - For happiness and benefit of the people, I begin this chapter named 'tripraśna' which will give knowledge of dig (direction), deśa (location) and kāla (time) in simple language.

Verses 2-5 : Finding the cardinal direction

To determine the directions, a place is made plain like a surface of water. It is cleaned and a circle of semidiameter 24 aṅgulas is drawn. At centre a Śaṅku of 12 aṅgula height is kept.

Shadow of śaṅku will touch the circle twice (when its length is 24 aṅgulas). Both points on circumference are joined by a line and with each point as centre, circles of 25 aṅgula radius is drawn.

Both the circles will intersect at two points and common parts of circles between them will form a fish shape. The points are like mouth and tail of the fish. The line joining them will be north south line which will be perpendicular on the line joining shadow position between first and second chāyā. Krānti movement is negligible and is ignored.

North south line will cut the circumference on two points called north and south points. A perpenducular on that line at the centre will cut the circle in east and west points. For finding

angular directions, arcs between east, north, west and south are bisected.

Notes (1) Types of śaṅku - Bhāskara I in his commentary on Āryabhaṭiya has described the following views -

Some astronomers prescribed a gnomon (śaṅku), whose one third in bottom is in shape of a prism on square base (caturasra), one third in middle in shape of cow's tail and one third in the top in shape of spear head.

Some other have prescribed a square prismoidal gnomon.

The followers of Āryabhaṭa I, used a broad (pṛthu), massive (guru) and large (dīrgha) cylinderical gnomon, made of excellent timber and free from any hole, scar or knot in the body.

For getting the shadow ends easily and correctly, the cylinderical gnomon was surmounted by a fine cylinderical iron or wooden nail fixed vertically at the centre of the upper end. The nail was taken to be longer than the radius of the gnomon, so that its shadow was always seen on the ground.

(a) Height of gnomon -

Gnomon could be of any length, but its height was divided into 12 units by convention. Smallest was gnomon of 12 aṅgula length, because it was portable and easy to handle. (about 9.8″ = 24 cm). Whatever may be length, it was called 12 aṅgula marked by 12 equal division which will be clearly seen in the shadow. Aṅgula also was divided into 60 pratyaṅgula for accurate measurement. It may be mentioned that accurate measurements were

based on very long gnomons. Viṣṇudhvaja or Kutubminar at Delhi was one such pillar. Since this indicated or marked (like a flag or dhvaja) the position of sun (Viṣṇu) it was called viṣṇu-dhvaja. Its arabic translation means the same thing, Kutub means north south direction (Kutub-numa=compass) mānar or minar is measurement or tower for that purpose.

(b) Testing the level of ground -

Test prescribed by Bhāskara I, Govinda Svāmī and Nīlakaṇṭha is -

When there is no wind, place a jar of water on a tripod on the ground which has been made plane by means of eye or thread, and bore a (fine) hole at the bottom of jar, so that water may have a continuos flow. Where the water falling on the ground spreads in a circle, there the ground is in perfect level. Where water accumulates, it is low. It doesn't reach at high level.

The same principle of 'water level' is used for modern levelling instrument. A long hollow glass cylinder is filled with water with a small air bubble in it, when the cylindrical rod, in kept on level ground, along the length touching the surface, bubble is at centre. The other side of length to be kept on ground is made flat so that it doesn't roll.

(c) Preparing the ground -

Ground should be plastered - so that it is not destroyed by pressure of walking, wind or rains. A prominently distinct circle was drawn with centre as centre of base of śaṅku. This line also had permanent or indelible marks by groove or permanent marks. Śaṅkaranārāyaṇa (869 AD) tells that lines were drawn with sandal paste. This may be because sandal was available in his area.

Verticality of śaṅku was tested by means of plumb lines (lambāka) on 4 sides.

It seems that fixed length compasses were used for drawing circles. This will be conventient for bigger circles and length of radius will not change in process of drawing. Hence, the radius has always been indicated as fixed. This is not necessary for finding perpendicular bisector.

(2) Cardinal directions :

Let ENWS (figure 2) be the circle drawn on the ground where gnomon is set. Let W_1 be the point where the shadow enters the circle (in the forenoon), and E_1 the point where the shadow passes out of the circle (in the afternoon). Join E_1 and W_1. Line E_1W_1 is directed from east to west.

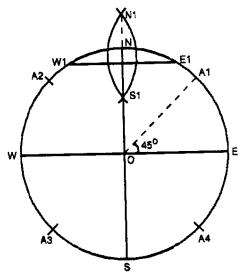



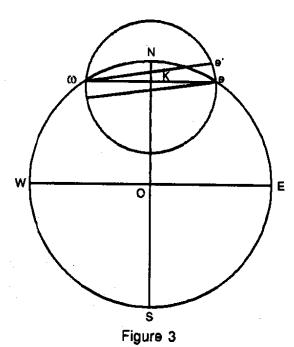
Figure 2 - Cardinal directions

Its perpendicular bisector is found by drawing two arcs of equal radius greater than 1/2 E_1W_1. This can be any length greater than this, 25 aṅgula radius prescribed here meets the condition. A fish figure is formed with N_1 and S_1 like mouth and tail point of the fish. Since E_1 W_1 was east west, its perpendicular bisector N_1S_1 will be in north south direction. N_1S_1 being bisector of chord E_1W_1, it will pass through the centre O and meet the

circle at points at N and S indicating north and south. Line EW parallel to E_1W_1 through centre O will mark east and west points E and W on the circle.

Angle points A_1, A_2, A_3 and A_4 between cardinal directions can be found by bisecting arcs EN, NW, WS and SE.

As the sun moves along the ecliptic, its declination (krānti) changes. By the time shadow moves from OW_1 to OE_1, the sun traverses some distance of the ecliptic, and its declination changes (though very small.) Hence, EW is not he true position of east west line. This minute correction was described first by Brahmagupta (628 AD), Bhāskara II (1150), Śrīpati (1039 AD) etc. As the correction is very small, this method is good for practical purposes.

Figure 3

(3) Correction for Krānti change -

From the east west line ew found as above, we make a circle with ew as diameter (Fig 3) Let d = correction in ew for change in krānti.

Φ = latitude of the place

δ = declination of sun when shadow tip enters the circle in forenoon at W

δ = Sun's declination when shadow tip leaves the circle in afternoon at point e.

K = chāyā karṇa

$$\text{Then } d = \frac{K\ (\sin\delta \approx \sin\delta)'}{\cos\phi}$$

To apply this correction, a circle with radius d is drawn with e as centre which cuts ew circle at e′ towards north when sun's ayana is towards north (as shown in the figure). e′ is south from e if sun's ayana is towards south. Now e′w is the correct east west line.

This figure is for situation when sun is having south krānti with respect to the place, so that shadow end is in north part. The south krānti will decline in north ward motion of north ayana, hence will be at lesser distance in north direction compared to w. Thus e′ is north from e.

Derivation of formula :

Assuming constant declination, w and e points have equal shadow lengths, hence their directions Ow and Oe are inclined at equal angles from ON direction.

It will be proved that K Sin δ/Cosθ is the agrā or the distance of shadow from the east west line passing through mid day equinox shadow end. Hence the change in north south position will be difference in the agrās at places e and w.

Hence this was named agrāntara correction by Caturvedācārya and then accepted by Śripati.

Unfortunately, derivation of this formula is not possible without use of spherical trigionometry in celestial triangles. Three dimensional diagrams are difficult to make on paper, they are approximate indications only.

Figure 4 (a)

Figure 4 (b)

Figure 4 (c)

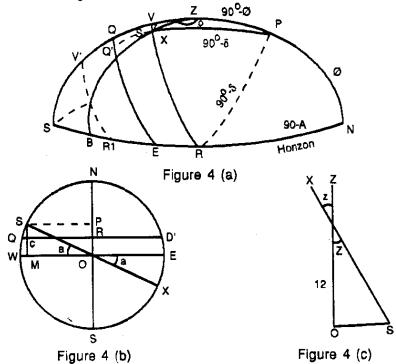



Fig 4 (a) is yāmyottara or meridian circle NPZS. (half circle over horizon), SEN is horizon showing south (S), East (E) and north (N) points. Z is vertical and P is pole of equator EQ. Hence ∠QES = NP = ϕ = latitude of the place. In north krānti, sun is moving in a diurnal circle R X V parallel to eequator towards pole P. In south krānti its position will be like R′V′. At a position X of sun, its krānti is distance Q′ from equator measured along great circle passig though P.

Hence PX = $90° - \delta$. Distance of sun from Z is measured along great circle Z X B = ZX = z.

Figure 4B is the direction circle with śanku at O in which WE and NS are direction lines. R is the palabhā position on equinox mid day. DD′ is east west line through it. At any instant OS is

shadow. Its distance from east west line WE is SM called agrā jyā. Thus agrā is the angle a between east horizon E and direction X of sun in a circle through vertical. Thus a = E X arc or ∠ EOX = ∠ SOM. In Fig 4(a) it is EB arc. on horizon circle (This direction along polar circle is krānti)

Bhuja of chāyā = SM = OP = OS Sin a

SC = Distance of shadow end from DD′, east west line on equinox day = Karṇa vṛttāgrā.

In ΔPZX,

$\text{Cos}\,(90^\circ - \delta) = \cos\,(90^\circ - \phi)\cos z + \sin\,(90^\circ - \phi).\sin z \quad \cos\,(90^\circ + a)$

where, ∠ PZX = 90° + a

or, $\text{Sin}\,\delta = \sin\phi \cos Z + \cos\phi.\ \text{Sin}\,Z.\ \text{Sin}\,a.$

Multiply both sides by $\frac{K}{\cos\Phi}$, where K is shadow length = $\sqrt{12^2 + s^2}$, 12 is śaṅku and S is shadow. Then

$$\frac{K \sin\delta}{\cos\theta} = K\ \text{Cos}\ z\ \tan\phi + K\ \text{Sin}\ z.\ \text{Sin}\ a \quad \text{- - - (1)}$$

But K Cos z = 12, K sin z = S - - - (2)

from figure 4 (c)

chāyā bhuja b = S Sin a already shown

Hence b = K sin z. Sin a - - - (3)

$$\text{Thus } \frac{K \sin\delta}{\cos\theta} = 12 \tan\phi + b$$

But 12 cos ϕ = palabhā = s = equinoctical mid day shadow (OR in fig 4b)

Hence $\dfrac{K \sin \delta}{\cos \theta} = s + b$ - - - (4)

When Sun is on horizon, ER is agrā A in fig 4(a).

In Δ PRN (∠PNR = 90°)

$\text{Cos}(90° - \delta) = \text{Cos}\,\phi\ \text{Cos}(90° - A)$

or $\text{Sin } A = \dfrac{\sin \delta}{\cos \phi}$ ....(5)

This agrā Jyā is in a circle of radius R. Reducing it to circle of radius K it is called Karṇāgra

$a = K \text{ Sin } A = \dfrac{K \sin \delta}{\cos \phi}$

Thus $a = s + b$ - - - (6)

In the figure 4(b) Karṇāgra is difference of s and b i.e. Karnagrā SC = PR = PO-RO = s - b

Sum or diffrence depends on opposite or same direction of shadow bhuja and palabhā.

Thus the formula $\dfrac{K\ (\sin \delta' - \sin \delta)}{\cos \phi}$ is difference of two shadows in north south directions by which they should be corrected to make its ends in true east west direction.

(4) Alternative methods :

Vaṭeśvara, Bhāskarā I and II, Lalla etc have given many other methods also, which deserve to be mentioned.

(a) Mark the points of extremities of two equal shadows, one before midday and one after that. Line joining them is east west line when due correction is made for change is sun's krānti.

This is same as the above method.

(b) When the sun enters the circle called prime vertical, shadow of a śaṅku is exactly in north-south direction, i.e. smallest shadow. It will be zero, when krānti of sun is same as akśāmśa of the place, and not useful.

(c) Bhuja and koṭi of a shadow (its distance from east west or north south line) is calculated. Two bamboo strips equal to bhuja and koṭi are taken. Koti strip is laid from centre towards west and bhuja strip is laid from shadow and towards south, so that their other ends meet. Then koṭi will be in east west direction and bhuja in north south.

(d) Any heavenly body with zero declination, rises exactly in east and sets exactly in west.

(e) The point where star Revati (ζ Piscium) or śravaṇa (Altair or α - Aquilae) rises is the east direction. Or it is that point which is midway between the points of rising of citrā and svātī.

Only those stars will rise in east which have zero krānti Observing citrā and svātī was used by people living in north of 30°N. Sudhākara Dvivedī has written in Digmīmāmsā, that śrāvaṇa, whose celestial latitude is about 30° N cannot rise in the east, as it will nevere have 0° krānti (minimum 30 $- 23\frac{1}{2} = 6\frac{1}{2}°$ North Krānti).

(e) The junction of two threads which pass through the two fish figures that are constructed with the extremities of three shadows (taken two at a time) as centre is in the south or north relative

to the foot of the gnomon, according as the sun is in the northern or southern hemisphere.

With the junction of the two threads as centre, draw a circle passing through extremities of the three shadows. The tip of shadow of a gnomon does not leave this circle in the same way as a lady born in a noble family does not discard the customs and traditons of the family.

Same views had beeen expressed by Lalla, Śripati and Bhāskara I (629 AD.) But this has been rightly criticised by Bhāskara II (1150 AD). As the sun is moving on a circle, locus of the line from sun to śaṅku top will be a cone with śaṅku top as apex. Its intersection by horizon plane will be always a conic section. The horizon plane is inclined at angle ($\delta + \phi$) with sun's direction which is not 90°, hence it cannot be a circle. As the shadows at sunrise and sunset time are of infinite length, they will be in general a hyperbola extending upto infinity. When ($\delta + \phi$) = 90° which is possible only within polar circle, its locus will be circle.

Siddhānta Darpaṇa has mentioned this view in verse 85 of this chapter and has criticised it there and in golādhyāya. However, this method will be approximately correct if the central position of hyperbola i.e. positions near mid day are taken.

Verse 6 : Relations between śaṅku and Chāyā - Add the squares of śaṅku and chāyā and take the square root of sum, which will he chhāyā karṇa. Square of śaṅku (144) is substracted from karṇa square and square root of the dif-

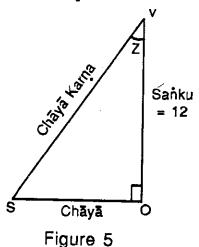



Figure 5

ference is chāyā which is base or pada. Square root of difference of squares of karṇa and chāyā is bhuja (śaṅku = 12) Line joining ends of bhuja (śaṅku) and koṭi (chāyā or pada) is called karṇa.

Note : Relation are obvious from figure 5.

OV = Śaṅku = 12 length = Bhuja for the angle z of Sun's direction from vertical (∠SVO = Z)

OS = chāyā = Koti or pada for angle z.

VS = Line from śaṅku tip to shadow tip = Karṇa of chāyā.

OS is in horizontal plane, OV is vertical, hence ∠VOS = 90°.

Thus $VS^2 = OS^2 + OV^2$

Verse 7 : Method to find square root.

Steps - (1) Given a number mark the even (sama) places and the odd (viṣama) places from right (unit place) by horizontal and virtical lines.

-1 -1 -1 - - - - - -

Example 11 97 16

(2) Substract the greatest possible square from the last odd place.

(3) Always divide the even place by twice the square root upto the preceding odd place

(4) Substract from the odd place (standing on the right) the square of the quotient

(5) Repeat the process as long as there are still digits on the right.

Notes : (1) This method was first given by Āryabhaṭa

Āryabhaṭa

```
               -1 -1-1
A     =        119716  (3
               3²
2X3   =      6) 29 (4
               24
               57
               4²
2X34  =     68) 411 (6
               408
               36
               -6²
               X
√A    =        346
```

New Method

3	$\overline{11}\ \overline{97}\ \overline{16}$ (346
3	9
64	297
4	256
686	4116
	4116
	X X

This is short version of same Āryabhaṭa. method.

(2) Proof of Āryabhaṭa method -

(1) Put $x_1 = [\sqrt{11}]$, $x_1 = 3$

$11 - x_1^2 = 2$

(ii) Divide 29 by $2x_1$ with quotient x_2, $x_2 = 4$

$29 = 2x_1x_2 + 5$

(iii) $57 - x^2 = 41$

(iv) Divide 411 by $2(10x_1 + x_2) = 2 \times 34$

$411 = 2(10x_1 + x_2)x_3 + 3$

(v) $36 - x_3^2 = 0$

Thus we have

$$\begin{aligned} 11 &= x_1^2+2 \\ 29 &= 2x_1x_2+5 \\ 57 &= x_2^2 + 41 \\ 411 &= 2x_3\,(10x_1 + x_2) + 3 \\ 36 &= x_3^2 \end{aligned}$$

Multiply these equations in order by 10^4, 10^3, 10^2, 10^1 and add. Corresponding terms are cancelled, as

$2 \times 10^4 = 20 \times 10^3$, $5 \times 10^3 = 50 \times 10^2$, $41 \times 10^2 = 410 \times 10$

we get

$11 \times 10^4 + 9 \times 10^3 + 7 \times 10^2 + 1 \times 10 + 6$

$= x_1^2 \times 10^4 + 2x_1\,x_2\,10^3 + x_2^2\,10^2 + 2\,x_1x_3\,10^2 + 2x_2\,x_3\,10 + x_3^2$

or $119716 = (x_1.10^2 + x_2.10 + x_3)^2$

$= (3.10^2 + 4.10 + 6)^2 = (346)^2$

or $\sqrt{119716} = 346.$

Some times we get smaller number at odd place then numbers which will be substracted from that. In previous place quotient is reduced by 1.

```
              1 - 1 - 1 -  1 - 1
              7 3 8 9 1 5 4 8 9 (2
              -2²
2 X 2 = 4    )33  (7     Here quotient should be 8
              28         as 4 X 8 = 32 is less than
              58         33. But at next stage, we will
              -7²        get 18 - 8² = negative Number.
2X27 = 54    )99  (1
              54
              451
              -1²
```

```
                              √7389l5489
2X271= 542 )4505  (8          = 27183
            4336              This adjustment is to be
            1694              done in short method also.
            -8²
2X2718=5436)16 308 ( 3
             16308
              09
              3²
              X
```

Verse 8 : Square root of sexagesimal numbers :

Some numbers are expressed in successive divisions of sixty like daṇḍa, kalā, vikalā which are called avayava or components. To find the square root of such numbers, steps are as follows-

(1) From the first component i.e. greatest division like daṇḍa, we substract the greatest square number. This gives first part of square root in daṇḍa (whose square has beeen deducted).

(2) If the remainder is less than the square root daṇḍa, then it is multiplied by 3. Then it is converted to next lower component (viz kalā) and number at that position in kalā is added. The sum is divided by square root in daṇḍa multiplied by 6 and added with 1. Result will be second i.e. kalā component of square root.

(3) If Ist remainder is equal or greater than daṇḍa root then it is multiplied by 2 and 1 is added. This is converted to 2nd component kalā (by multiplying with 60) and number at 2nd component is added. Total remaining kalās are divided by daṇḍa root X 4 + 3. Result will be kalā component of the square root.

Notes : (1) This is a very ingeneous method of finding square root, which I have not come across in any other text. This method of square root and cube root method in last chapter has not come across the modern world. The method is explained by examples for both cases.

Example 1.

7) 60° 20′ (7°
-7^2

11 → This is more than 7

11x2+1 = 23° Thus square root is 7° 45'

23x60′+20′ = Test

7x4+3=31)1400(45′

124

160

155

5

$$(7°\ 45')^2 = \left(\frac{31°}{4}\right)^2 = \frac{961°}{16} = 60\frac{1}{16}$$

Which is slightly less than the square no.

Example 2

7) 50° 20′ (7°
−7^2

1° This is less than 7°

1°X3 = 3° Thus square root is about

3°x 60′+20′ = 7° 4′.6

7X6+1=43) 200 (4.6 Its square is

172

280

258

22

$$\approx \left(7°\ \frac{4.5}{60}\right)^2 = \left(7\frac{3}{40}\right)^2$$

$$= \left(\frac{283}{40}\right)^2 = 50°3' \text{ approx.}$$

(2) Justification - This is an approximate method, hence an approximate proof or rather justification of method is given.

(i) Suppose $A^{\circ}B' = (a^{\circ}b')^2$

when $A\text{-}a^2 > a$ (Example 1)

Since $A < (a+1)^2$, $(a+1)^2 - a^2 > A\text{-}a^2 > a$

or $2a+1 > A\text{-}a^2 > a$

Hence, $A\text{-}a^2 \simeq \dfrac{(2a + 1) + a}{2}$ approx $= \dfrac{3a + 1}{2}$

This is multiplied by 2 and 1 is added, then it becomes

$(3a+1) + 1 = (3a+2)^{\circ} = (3a+2)60'$

Now $(a+1)^2 > A > a^2+a = a\,(a+1)$

or $A = (a+1)\,(a+\frac{1}{2}) = a^2 + \frac{3}{2}\,a + \frac{1}{2}$

$= \quad a^2 + 2\,\frac{3}{4}\,a + \frac{9}{16} = (a + \frac{3}{4})^2$

Hence b = 45′ approx (more than half degree)

B is betwen 1′ to 59′ = 30′ on average

Hence remainder is (3 a+2) 60′+30′

= 180 a + 150 approx

Dividing by b = 45′, $\dfrac{180a + 150}{45} = 4a + 3.3$ approx.

Hence the remainder is divided by (4a+3) to get the value of b.

(ii)) When $A\text{-}a^2 < a$

Since $A\text{-}a^2 > 0$ always, on average we can take

$$A\text{-}a^2 = a/2$$

$$A^\circ B' \simeq A + \frac{1^\circ}{2}{}^\circ \simeq a^2 + \frac{a}{2} + \frac{1}{2}$$

$$= (a^2 + \frac{2.a.1}{4} + \frac{1}{4^2}) + \frac{7}{16}$$

$$= (a + \frac{1}{4})^2 + \frac{7}{16} = (a1/4^\circ)^2 = (a^\circ\ 15')^2 \text{ approx}$$

Remainder is multiplied by 3 and converted to kalā then added to $B \simeq 30'$ becomes

$$\frac{a}{2} \text{ X } 3 \text{ X } 60 + 30 = 90a+30$$

On division by 15′, range of b it gives

$6a + 2$

Hence it is divided by 6a+1 to give approx value of b.

Verse 9 : When in astrology, we calculate proportionate life term from value of nakśatra, difference of 1 kalā will give age difference of 72 days. Hence component quantity roots should be found carefully. This is a rough method involving some error. Hence it should be checked by squaring.

Verse 10 : Multiplication of component numbers - A multiplication of two quantities with 3 components each will be in 9 places. First number is written at the top with three components at 3 places. 2nd number with 3 components is written below, by its first component we multiply the first line's components at 3 places. The multiplication by smaller component is written below it, drifted 1 place towards right. 3rd multiplication by next

smaller component is shifted 1 more place towards right. Thus total is in 5 places. First place from left is unit (rupa), 2nd place is liptā (1/60 part), 3rd is viliptā (1/60 liptā) and so on. Only 3 places are taken. Their square root can be found out by method of verse 9. Otherwise, for accurate calculation, they will be converted to vikalā whose square root will be in kalā.

Notes : This method is called go-mūtrikā in Indian arithmetic. Like urination by cows at separate spots, multiplication is done at different lines. Proceduce is as follows -

	a° b′ c″ X			
	d° e′ f″			
ad°	db′	dc″		
	ea′	eb″	ec‴	
		fa″	fb‴	fc″″
ad°	db′+	dc″+	+ed‴	fc″″
	ea′	eb″ +	+fb‴	
= A°	=B′	fa″		
= A°	=B′	=C″		

Only A°B ′C″ is kept which is sufficient for accuracy. (60 A°+B′) X 60 + C″ = Vikalā

Vikalā = Kalā X Kalā (e′xb′ = eb″ vikalā as above)

$$\text{as } \frac{1}{60} \times \frac{1}{60} = \frac{1}{3600}$$

Hence square root of vikalā will be in kalā.

Verses 11-12 : Setting of śaṅku

Circular base of śaṅku should be plane and from top to bottom, face should be plain and straight (i.e. smooth conical surface). Height of

cone and circumference of base will be equal). Shape of śaṅku may be any type, but 1/12th part of its height will be called 1 aṅgula.

For finding out time, our own body also can be considered a śaṅku and the distance of shadow is measured from middle point of the feet.

Convenient śaṅku is of eye level height made of soil or wooden pole. Its centre will be at centre of circle. Radius of base is measured already. Distance of shadow end is measured from base of śanku and radius of base is added to give shadow length.

Verse 13 - The shadow meant here is produced by centre of sun. But other parts of sun are not dark and they also contribute to the shadow. Hence the length of the shadow is increased by 1/211 to find the shadow length due to sun's centre.

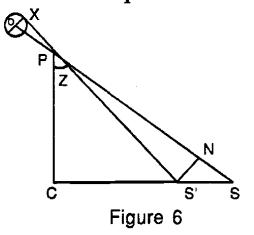



Figure 6

Notes : In figure 6, shadow of Saṅku CP due to centre O of sun is CS. Elevation of sun is $\angle CPS = z$. Due to upper most part X of sun, end portion SS′ is also lighted. Hence, only shadow CS′ is seen. To find correct shadow, length SS′ is added to it. Now PS = CP sec z, CS = CP tan z

S′N is perpendicular on SP.

Since S′N is very small compared to SP,

$$\frac{S'N}{SP} = \frac{\text{sun's radius}}{\text{sun's distance}} = \frac{1}{219}$$ (a known constant average value)

$$\text{or } S'N = \frac{SP}{219}$$

$S'S = SN \sec z$ (in right angled triangle S'SN)

$$= \frac{SP \sec z}{219} = \frac{CS}{\sin z} \frac{\sec z}{219}$$

$$\text{or } SS' = \frac{1}{219 \sin z \cdot \cos z} \cdot (cs' + ss')$$

$$\text{or } ss'\left(1 - \frac{1}{109 \sin 2z}\right) = \frac{CS'}{109 \sin 2z}$$

$$\text{or } \frac{SS'}{CS'} = \frac{1}{109 \sin 2z - 1}$$

Thus the correction will be for less than 1/2 the distance of SS', because shadow is not dark due to dispersion of light in atmosphere. Logic given here is that correection is equal to sun's radius; distance it is not correct.

Verses 14-23 : Definitions

(Text asks to explain the terms through spherical model constructed of bamboo to imagine the measures correctly. Diagram is a crudle substitue, but without it is impossible to describe).

Śaṅku is called nara or koṭi also.

Chāyā is called prabhā and bhuja also.

Square of bhuja and koṭi added are square of karṇa.

This koṭi, bhuja and karṇa form fundamental triangle.

The great circle (straight line for a spherical surface) passing through east west points and zenith (khasvastika) is called east west circle (pūrvāpara vṛtta).

Earth's equator extended into sky is called celestial equator (Ākāśa viṣuva). Its akśāmśa is considered zero. Great circle passing through poles and east, west points is called samamaṇḍala.

Ahorātra vṛtta becomes successively smaller as we procced from equator to meru (pole)

A sphere of bamboo or wood should be formed to show celestial equator, ecliptic, eccentric circle of planets and other circles.

On any day, if the midday shadow of śaṅku is north from śaṅku, then its difference from equinox midday shadow is called agrā (more correctly karṇa vṛttāgrā).

If shadow is south from gnomon (śaṅku) base, then sum of equinox shadow (north for north latitude only) and this shadow is called karṇa vṛttāgrā.

On equinox day sun makes day and night equal while on equator (perpendicular to equator on that day). Thus the distance of sun on this day from svastika of a place is akśāmśa or palāmśa (angular distance from equator) of the place.

Palāṁśa is the nati (angular distance from zenith or svastika) on equinox midday. Its angular height from horizon is unnātaṁśa equal to lambāmśa (complementary to akśāmśa - distance from north pole).

12 aṅgula śaṅku and palābhā multiplied by radius (3438) and divided by pala karṇa give respectively lambajyā and akśajyā.

Notes (1) Figure 7 is as per commentary by Paṇḍita Bāpūdeva Śāstrī on sūrya siddhānta.

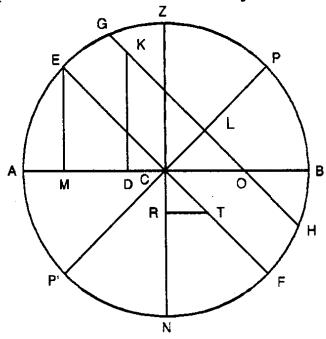



Figure 7 - Definitions in spherical triangles

ZANB is yāmyottara maṇḍala (meridian) passing through two poles P, P′, and zenith Z. All the other circles have been projected on this plane for diagram purpose. Samamaṇḍala is great circle through Z, N and east west points.

Kśitija (horizon) is circle passing through north south east west points. ACB is its diameter in the figure which is in north south line.

Nāḍī maṇḍala is celestial equator. Its diameter is ECF. P and P′ are dhruvas (north and south poles) of earth. PCP′ is a diameter of unmaṇḍala perpendicular on diameter of nāḍīmaṇḍala (or its diameter).

GH is diameter of ahorātra vṛtta (diurnal circle) of sun (or any planet or star). This meets

PCP at L (bisected there) and kśitija at O. Let EM be perpendicular to AB.

Then EZ is aksāmśa and CM is its sine or akśajyā. AE is lambāṁśa and EM is its sine or lambajyā. CE is trijyā, Thus EMC is a latitude triangle with lambajyā, akśajyā and trijyā as its sides. It is called W.

CL is distance between nādī maṇḍala and ahorātra vṛtta - and is equal to krāntijyā. L is point of intersection of ahorātra vṛtta and unmaṇḍala and LO is perpendicular on line of intersection of ahorātra vṛtta and Kśitija (this line is perpendicular to the plane of paper i.e. diagram). This LO is kujyā.

CO lying on kśitija is the distance between pūrvāpara and udayāsta sutra and is agrā (both the lines perp. to paper plane). Thus CLO is another latitude triangle with sides as krāntijyā, kujyā and agrā - called X.

Let the sun be at K. Perpendicular KD to kśitija is also called śaṅku (or mahāśaṅku). DO is śaṅkutala and KO, iṣṭahṛti. OKD is another latitude triangle called Y.

Midday śaṅku is called madhyāhna śaṅku.

Suppose sun is at E, the equinoctical point, let CR be śaṅku of 12 aṅgulas. RT is its shadow perpendicular to it meeting ECF in T. RT is called palabhā, and CT is pala karṇa. CRT is the basic latitude triangle called Z.

Verses 24-27 : Krānti from Palabhā

Now I tell the method of finding current declination (angular distance from equator - krānti) of sun forn palabhā (midday shadow)

Midday shadow on north south line is multiplied by radius (3438) and divided by karṇa. Arc of this jyā is found in kalā. This is natāmśa of sun (distance from kha-svastika = zenith).

If shadow end is south from the equinox mid-day shadow, then sun is having north krānti.

Then krānti kalā of equinox day (akśāmśa) is added to natāmśa (kalā) which gives sun's krānti. (for north latitude). Sun's equinox shadow and mid day shadow on desired day being in one direction, difference of krānti and natāmśa is taken. They are added when in different direction.

According to sūrya sīddhānta, palabhā (on equinox day) is found out from akśajyā of the place. Lambajyā in found by taking square root of difference of squares of trijyā (1,18,19,844) and akśajyā.

Notes

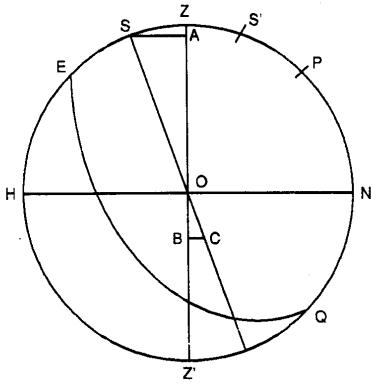



Figure 8

Let HZPN be the observer's yāmyottara maṇḍala and Z be the zenith. Let EQ be the nāḍī-maṇḍala, HON kśitija and P Dhruva (north). Let S be the sun at mid day (In south declination towards south point H from Z). S will be towards N in north declination.

ZS is its natāmśa or distance from zentih (vertical). HS its unnatāmśa (elevation from horizontal) and SE its krānti (distance from equator - shown north here). ZE is akśāmśa.

Draw SA perpendicular to ZO. Then AS is natāmśajyā and OA is unnatāmśa jyā.

Produce ZO to cut the circle at Z′. Cut OB = 12 aṅgula. Draw BC perpendicular to OZ′ meeting SO produced at C. Then OB is śaṅku, BC madhyāhna chāyā (mid day shadow) and OC chāyā karṇa.

Natāmśa ∠SOZ = ∠BOC is given by

$$\text{Sin } \angle BOC = \frac{BC}{OC} = \frac{\text{Chāyā}}{\text{Karṇa}}$$

which is the formula.

Now when S and E are on same side of Z, (as in figure), the shadow BC will be in opposite side of both. In this case, SZ = EZ - ES

Or Natāmsa = Akśāmśa - Krānti

When S is on other side of Z i.e. at S′, the shadow will be in side OZ′H, opposite to equinox shadow. Then,

ES′ = EZ + ES′

Or Krānti = Akśamśa + natāmśa

For same sides it was Akśāmśa - natāmśa

Verses 28-32 : Sun from shadow -

Now I tell the method of finding sun's position from shadow. If natāmśa and akśāmśa are in same direction (i.e. shadow on equinox midday and desired mid day is in same direction from śanku base), then we take the difference of these.

When they are in different directions, then we take the sum. This will give krānti of sun (in case of difference, it is in direction of greater quantity, for sum, it is direction of either.

Krānti jyā is multiplied by trijyā (3438) and divided by jyā of paramakrānti (1370). This will give bhuja jyā of sun. Its arc is found in kalā. If sāyana sun is in first quadrant, this arc itself is position of sāyana sun. If it is in 2nd quadrant, it is substracted from 6 rāśis, in third quadrant added to 6 rāśis. If sāyana sun is in last quadrant, arc is subtracted from 12 rāśis.

Ayanāmśa is deducted from this value to get true sun as measured from meṣa O°. Sphuṭa or true sun is substracted from its mandocca and mandaphala correction is done in reverse manner for madhyama sūrya. By repeated procedures, madhyama sūrya will be more accurate.

Notes : Calculation of sāyāna sun involves two steps (i) Finding krānti of sun as described in verse 27.

(ii) From krānti of sun to its sāyana position, which has been described in chapter 6 verse 96. There the formula has ben used for the reverse process, i.e. to find sun's krānti from position of sāyana sun.

$$\text{Bhujajyā of sun} = \frac{\text{Krānti jyā} \times \text{Trijyā}}{\text{Parama krānti}}$$

This formula has been proved there.

Now sāyana sun is reduced to true sun by reverse process of finding sāyana. Earlier ayanāmśa had bene added (it may be substracted for periods before 493 AD or after 2200 AD according to book - which is not correct). Hence, it will be substracted now.

Madhyama graha from true graha is again a reverse procedure of finding true graha. It has been explained in verse 166 of chapter 5. For sun, only manda correction is done.

Verses 33-34 : Shadow from sun's position of midday

Sun's position will give its krānti as explained above. Akśāmśa of a place is known. If both are in different direction, they are added, to give natāmśa of sun (inclination from vertical).

If both are in same direction, their difference is taken.

(Here direction of akśāmśa is opposite to direction of equinox shadow i.e. direction of equator from the place). Thus in north hemisphere, akśamśa is south).

Thus we get natāmśa at mid day. Its bhujajyā and koṭijyā is calculateed

$$\text{chāya} = \frac{12 \times \text{natāṁśa jyā}}{\text{Koṭijyā}}$$

$$\text{chayā karṇa} = \frac{12 \times \text{radius (3438)}}{\text{Koṭijyā}}$$

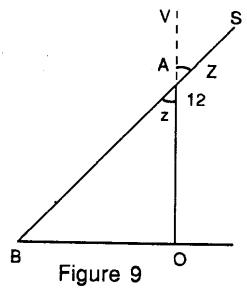



Figure 9

Note : This is obvious if we consider figure after verse 5 or 13, reproduced here. OV is vertical direction at a place where OA is saṅku of length 12. OB is shadow on horizontal plane. Thus $\angle VAS = \angle BAO$ = natāmśa of sun, $\angle BOA = 90°$

Now chāyā BO = OA tan z = $\frac{12 \sin z}{\text{Cos } z}$

$= \frac{12 \times \text{R Sin } z}{\text{R Cos } z} = \frac{12 \times \text{natāmśa jyā}}{\text{Koṭijyā of natāmśa}}$

chāyā Karṇa AB = $\frac{OA}{\text{Cosz}} = \frac{12 \times R}{R \cos z}$

$= \frac{12 \times \text{radius}}{\text{Koṭijyā}}$

Verses 35-37; Unmaṇḍala śaṅku

Unmaṇḍala is great circle passing through east, west points and north and south poles. (Defined in verse 23 - figure 7). Its northern part lies above horizon in north hemisphere places (like India)). Unmaṇḍala is horizon of equator, its śaṅku is formed when sun (or a planet) enters unmaṇḍala. Then perpendicular from it to east west line is unmaṇḍal śaṅku. When sun is in north krānti, it

rises earlier than equator, thus at unmaṇḍala, it has risen at equator horizon and gone above horizon at local place.

$$\text{Unmaṇḍala śaṅku} = \frac{\text{Palabhā} \times \text{Krānti jyā}}{\text{Pala karṇa}}$$

$$\text{yaṣṭi} = \frac{\text{unmaṇḍala śaṅku} \times \text{trijyā}}{\text{Carajyā}}$$

When sun is north from equator, yaṣṭi + U. śaṅku = madhyāhna śanku. For sun in south, yaṣṭi – U. śaṅku = M śanku.

Notes—

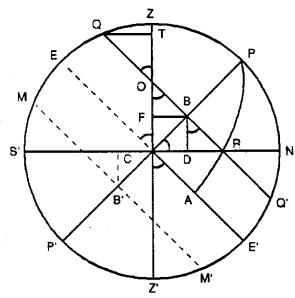



Figure 10 - Unmaṇḍala śaṅku

ZSZ′N is the meridian of a place of latitude Φ. S,N. is north south line on horizon.

ECE′ is diurnal (ahorātra) circle's diameter when sun in on equator.

QQ′ is diameter of ahorātra when its krānti is δ .

P,P′ are north and south pole, joining line is diameter of the circle passing through east and west points on horizon, so perpendicular to plane of paper like equator circle.

PCP′ is the north south line of equator and unmaṇḍala is horizon circle there. C is east point, CR is agrā.

Perpendicular from planet at unmaṇḍal to horizon, is equal to its projection BD in meridian plane. Thus BD is unmaṇḍala śaṅku.

On diurnal circle projection, sun moving from Q′ above, rises at horizon at point R. At position B it is on horizon of equator and rises there. Thus sunrise is earlier in north hemisphere when sun has north krānti.

Half of ahorātra vṛtta diameter BQ = Dyujyā

Difference between equator and horizon rise

= BR = Kujyā (in kalā angles)

Difference in rising time in asu = Kalā for equator = CA = Carajyā

EQ = δ (Krānti), BC = R sin δ= Krāntijyā, Akśānśa ϕ = arc S′E or PN or angles BCR etc marked with $\angle$sign.

BF and QT are perpendiculars on vertical line CZ. Q is mid day time of sun, so TC = madhyāhna śaṅku = R sin z

where z = natamśa QZ = $\angle$QCZ

Thus, madhyāhna śanku is TF length more than BD i.e. unmaṇḍala śaṅku.

TF = yaṣṭi (or madhya yaṣṭi at madhyāhna time)

= Height in vertical direction above equator rising point. This height at any other position is called iṣṭa yaṣṭi.

In latitude ΔBCD,

$$\text{Sin } \Phi = \frac{BD}{BC} = \frac{BD}{R \sin \delta}$$

or, unmaṇḍala śanku BD = R sin δ Sin ϕ - - - (1)

or $\dfrac{(R \sin \delta)\ (R \sin \phi)}{R}$ as stated

FT = FO + OT = (BO + OQ) Cos Φ

= BQ Cos Φ

But BQ is at angle δ from equator

hence, BQ = R cos δ

Hence yaṣṭi FT = R Cos δ Cos Φ - - - (2)

$$\frac{\text{yaṣṭi}}{\text{Unmaṇḍala śaṅku}} = \tan\delta \ \tan\Phi = \frac{\text{carajyā}}{R} \quad - (3)$$

by dividing (1) with (2).

Here, yaṣṭi = madhyāhna śanku - Unmaṇḍala sanku. .... (4a)

when sun krānti is north. In south krānti MM′, śanku at B′ will be in opposite direction. Then yaṣṭi = madhya śaṅku + unmaṇḍala śanku ....(4b)

Value of carajyā in (3) has already been proved in chapter 6. It is proved as in ΔPCA, BR//CA

$$\text{Hence } \frac{CA}{CP} = \frac{BR}{BP}$$

CP = R, BR = BC tanΦ = R sinδ tanΦ (from diagram)

BP = R cos δ

Hence carajyā CA = R tan δ tan Φ used in (3)

Verse 38 : Alternative method for madhyāhna śaṅku - Madhyāhna śaṅku =

$$\frac{\text{Unmaṇḍala śaṅku} \times \text{Antyā}}{\text{Carajyā}}$$

Notes (1) Antyā = Trijyā + cara jyā (defined later)

= EC + CA = EA (Fig 10)

Now $\frac{BD}{BR} = \frac{CT}{QR}$ (similar triangles) $= \frac{TO + OC}{QO + OR}$

$$\frac{BD}{CT} = \frac{BR}{QR} = \frac{CA}{EA}$$

or Madhyāhna śaṅku CT

$$= \frac{BD \times EA}{CA} = \frac{\text{Unmaṇḍala śanku} \times \text{antyā}}{\text{Carajyā}}$$

when sun is having south krānti, Antyā = Trijyā - Carjyā.

Trijyā in asu is half day length at equator, carajyā is difference in half day length at own place. Thus antyā in asu is half day length at any place.

(2) Yaṣṭi is a stick with length equal to trijyā = 3438 used to measure vertical height of sun from horizon, as ratio of trijyā - hence it gives sine values. Thus, the height measured from the position of equator sunrise is iṣṭa yaṣṭi. In north krānti, at equator rise time, it is below horizon, so its vertical height at equator sunset time can be measured, which will be almost equal and opposite. For north krānti it can be measured directly. Hence, the name yaṣṭi has been given.

Yaṣṭi and all śaṅku measurements are in the direction of local vertical i.e. line passing from

earth's centre to the surface point. Heights of sun along this line from equator rise time will give yaṣṭi. This gives a measure of equator time i..e udayāntara correction.

Verses 39-44 - Agrā and Karṇa Vṛttāgrā -

Jyā of natāmśa (R sine of angular distance from zenith is called dṛgjyā and its koṭijyā (R cosine) is called śaṅku jyā

$$\text{Madhyāhna agrā} = \frac{\text{Krānti jyā} \times \text{palakarṇa}}{12\ (\text{śaṅku})} \quad \text{- - - (A)}$$

Agrā at madhyāhna is south or north as sun is having north or south krānti.

Karṇa Vrttāgrā

$$= \frac{\text{madhya agrā} \times \text{chāyā karṇa}}{\text{Radius(3438)}} \quad \text{(B)}$$

(Karṇa Vṛttagrā is distance of shadow end at any time in north direction measured from equinox mid day shadow)

Alternatively,

$$\text{Madhyāgrā} = \frac{\text{Krānti jyā} \times \text{trijyā}}{\text{Lambajyā}} \quad \text{(A')}$$

$$\text{Karṇa Vṛttāgrā} = \frac{\text{Krānti jyā} \times \text{chāyā karṇa}}{\text{Lambajyā}} \quad \text{(B')}$$

Sāyana sun in six rāśi's starting from meṣa is in north hemisphere and in six rāśis from tulā is in south.

When sun is in north and karṇa vṛttāgrā is more than palabhā (equinox mid day shadow), then their difference will be south bhuja or bāhu of shadow (bāhu is length of shadow in north south direction). Sun in north and palabhā more than karṇa, then their difference will be chāyā bhuja in north direction.

When sun is in south, then karṇa vṛttāgrā and palabhā are always added to get chāyā bhuja.

(These rules have been stated for places of north hemisphere like India).

Notes :

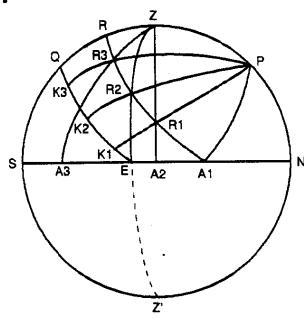



Figure 11 - Karṇa Vṛttāgrā

NZSZ′ is meridian, or ẏāmyottara vṛtta of a place passing through north horizon point N, south point S and khasvastika (zenith) Z - i.e. vertically up point.

NES is horizon circle (east half shown)

P = Pole of equator EQ

A_1R diurnal circle of sun at north krānti (declination)

R_1, R_2, R_3 are its three position.

K_1, K_2, K_3 are positions of sun projected on equator through polar circles.

K_1R_1, = K_2 R_2 = K_3 R_3 = Krānti of sun (almost equal for a day)

PR_1, PR_2, PR_3 are polar distances of sun. ZEZ′ is sama maṇḍala through east and west points of

horizon, zenith (svastika) points. R_2 is sun's position on svastika.

Polar great circles from Z to positions of sun meet equator at A_1, A_2, E and A_3.

Thus natāmśa are ZR_1, ZR_2, ZR_3, angular distance from svastika. A_2R_1, ER_2, A_3R_3 are angular elevations (unnatāṁśa) EA_1, EA_2, EA_3 are agrās of sun.

Now in spherical triangle PZR_1

$$\text{Cos } PZR_1 = \frac{\text{Cos }(PR_1) - \text{Cos }(ZR_1) \times \text{Cos }(PZ)}{\sin (ZR_1) \times \sin (PZ)}$$

$PZR_1 = 90° -$ agrā (a), $PZ = 90° - PN$

$= 90° - \Phi$, Φ = akśāṁśa

$PR_1 = PK_1\text{-}K_1R_1 = 90° - \delta$, δ = Krānti

ZR1 = z natāṁśa

$$\text{Hence, } \sin a = \frac{\text{Sin } \delta - \text{Cos} z . \text{ Sin } \Phi}{\sin z . \cos \Phi}$$

$$= \frac{\text{Sin } \delta}{\text{Sin } z . \cos \Phi} - \text{Cot } z . \tan \Phi \quad \text{- - - - (1)}$$

$$\text{But } \tan \Phi = \frac{\text{Palabhā}}{12}, \text{ Cot } z = \frac{12}{S}, \text{ S = shadow}$$

$$\text{Sin } z = \frac{\text{chāyā}}{\text{chāyā karṇa}} = \frac{S}{K}, \text{ K = chāyā karṇa}$$

$$\text{Hence, } \sin a = \frac{\sin \delta}{\cos \Phi} . \frac{K}{S} - \frac{12}{S} \times \frac{\text{palabhā}}{12}$$

$$= \frac{1}{S} \left\{ \frac{K \sin \delta}{\cos \Phi} - \text{palabhā} \right\}$$

$$\text{or } S \sin a = \frac{K \sin \delta}{\cos \Phi} - \text{palabhā} \quad .....(2)$$

S sin a = bhuja of chāyā measured in north south direction from base of śaṅku.

Thus, karṇa vṛttāgrā = bhuja + palabhā (By definition)

$$K.V. = \frac{K \sin \delta}{\cos \Phi} \quad --- (B')$$

as stated earlier

Relation (2) holds when sun is having north krānti and is north of samamaṇḍala. Then bhujā is in south direction, which may be taken positive.

Bhuja (south) = (Karṇa vṛttāgrā - palabhā), when in north krānti, sun is south of sama maṇḍala angle 'a' is negative (north wards from point E is +ve direction). Then

– Bhuja = K.V. - palabhā

When sun is in south krānti, δ will be negative, a will be negative so

– Bhuja = – KV - palabhā

When north direction values are taken

Bhuja = KV + Palabhā.

These are the rules for bhuja of chāyā.

Here, madhyāhna agrā or madhyāgrā has been the name of agrājyā at sun rise time which may be named A.

Thus A = R sin a_0 where a_0 is agrā at sunrise

Then, natāmśa Z = 90°, cos Z = 0 and Sin Z = 1, equation (1) becoms

$$\text{Sin } a_0 = \frac{\text{Sin } \delta}{\cos \Phi}$$

or, $A = R \sin a_0 = \dfrac{R \sin \delta}{\cos \Phi} = \dfrac{R \times R \sin \delta}{R \cos \Phi}$ (A′)

To find (A) and (B) relations, we have

$$\frac{\text{Palakarṇa}}{12} = \frac{R}{R \cos \Phi}$$

Hence, $A = \dfrac{R \sin \delta \times \text{Palakarṇa}}{12}$ - - - (A)

From (B′), we have K.V. $= \left(\dfrac{R \sin \delta}{\cos \Phi}\right) \times \dfrac{K}{R}$

$= \dfrac{\text{madhyāgrā} \times \text{chāyā karna}}{\text{radius}}$ - - - (B)

Verses 45-51 : Relations in sama maṅḍala– When shadow of śaṅku falls on east west line, then shadow, chāyā karṇa and time (indicated by nata or unnata amśa of sun) - all are in sama maṇḍala i.e east west vertical circle passing through zenith. At this point krānti of sun is equal to akśāmśa of the place.

When north krānti of sun is more than the akśāmśa (for north hemisphere) of the place, shadow is always south of samamaṇḍala.

Shadow is north of sama maṇḍala when sun's north krānti is less than akśāmśa of the northern place or krānti is south.

Summary - 1 - Shadow on sama mandala - then, Krānti = akśāmśa

2. Shadow south ; N. Krānti > akśāmśa (north)

3. Shadow north ; N. Krānti < north akṣāmśa or south krānti

(A) Samamaṇḍala chāyā karṇa $= \dfrac{\text{Palabhā} \times \text{lambajyā}}{\text{jyā of north krānti}}$

$$= \frac{\text{Jyā of north akśāmśa} \times 12}{\text{Jyā of north krānti}}$$

$$= \frac{\text{Palabhā} \times \text{dinārdha karṇa}}{\text{dinārdha vṛttāgrā}}$$

(b) Sama maṇḍala śaṅku

$$= \frac{\text{Jyā of north krānti} \times \text{palakarṇa}}{\text{Palabhā}}$$

(c) Dṛgjyā $= \dfrac{a}{\sqrt{\text{Trijyā}^2 - \text{Samamaṇḍal śaṅku}^2}}$

(d) Sama maṇḍala chāyā $= \dfrac{\text{dṛg jyā} \times 12}{\text{Sama maṇḍala śaṅku}}$

(E) Sama maṇḍala karṇa $= \dfrac{\text{Trijyā} \times 12}{\text{Samamaṇḍala śanku}}$

(f) Sāyana sun bhuja jyā

$$= \frac{\text{Sama maṇḍala śaṅku} \times \text{Jyā of akśāṁśa}}{\text{Jyā of parama krānti (1370)}}$$

Notes : (1) When sun is in sama maṇḍala (east west circle), śaṅku, shadow all are in same plane. Then iṣṭa kāla agrā a = O. Thus from equation (2) after verse 44 -

$$0 = \frac{K\sin\delta}{\cos\Phi} - \text{palabhā}$$

or chāyā karṇa $K = \dfrac{\text{palabhā} \times \text{lambajyā}}{\text{Jyā of krānti}}$ - - - (A)

as lambajyā = $R\cos\Phi$, Jyā of krānti = $R\sin\delta$

Krānti is north then, only sun can enter samamaṇḍala.

Palabhā X Lambajyā = akśajyā X 12

because palabhā =

$$12 \tan \Phi = \frac{12 R \sin \Phi}{\cos \Phi}$$ - - part 2 of (A)

$$K.V. = \frac{K \sin \delta}{\cos \Phi}$$

For madhyāhna, $KV_m = \dfrac{Km \text{ Sin } \delta}{\cos \Phi}$ (KV_m and Km are value at madhyāhna or dinārdha)

or $\dfrac{\text{Lambajyā}}{\text{Krānti jyā}} = \dfrac{\cos \Phi}{\sin \delta} = \dfrac{Km}{KVm}$ part 3 of (A)

(2) Sama śaṅku's and krānti

Figure 10, in yāmyottara plane, indicates position O of sun on samamaṇḍala ZCB. ZCB, unmaṇḍala and equator all bisect each other on east west points, East point is C here.

Sama śaṅku is perpendicular from sun in samamaṇḍala on horizon. It is equal to perp. from O (projection of sun on meridian) to NS, as these are parallel projections.

Thus OC = samaśaṅku

(height of sun in unmaṇḍala)

ZO is distance from vertex along diameter hence angular distance Z is given by

ZO = R (1 – cos z) = versine Z.

OC = R cos Z

In ΔBCO, $\angle$BOC = Φ (latitude or akśāmśa)

$$\text{Sin } \Phi = \frac{BC}{OC}$$

But BC is distance of sun from equator or from centre. Angular distance is given by

R sin δ = =BC

$$\text{Hence, OC} = \frac{BC}{\sin \Phi} = \frac{R \sin \delta}{\sin \Phi}$$

$$\text{Here Sin } \Phi = \frac{\text{palabhā}}{\text{palakarṇa}}$$

$$\text{Hence samaśaṅku OC} = \frac{R \sin \delta \times \text{palakarṇa}}{\text{Palabhā}} \quad \text{- - (B)}$$

R cos z = OC = samaśaṅku

Dṛgjyā = R sin Z = $\sqrt{R^2 - R^2 \cos^2 Z}$

= $\sqrt{\text{Trijyā}^2 - \text{samaśaṅku}^2}$ - - - (c)

(3) For other relations, consider figure 12. OZ is part of samamaṇḍala with centre at P. PG is a śaṅku of length 12 at P. SP and SG are chāyā karṇa and chāyā of śaṅku when sun is at O in samamaṇḍala

Natāmśa z == arc ZO

= ∠ OPR = ∠ SPG = ∠ POC

OC is samaśanku

OR = PC = R sin Z = dṛgjyā

In similar Δ s PSG and OPC,

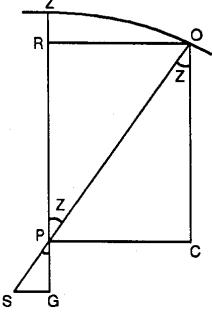



Fig 12 - Samaśanku

$$\frac{SG}{PC} = \frac{SP}{OP} = \frac{PG}{OC} = \frac{12}{\text{Samaśaṅku}}$$

Hence, Samamaṇḍala chāyā SG

$$= \frac{\text{dṛgjyā} \times 12}{\text{samaśaṅku}} \quad \text{- - - (D)}$$

Samamaṇḍala chāyā karṇa SP

$$= \frac{\text{Trijyā} \times 12}{\text{samaśṅku}} \text{.....................(E)}$$

(4) Bhuja of sāyana sun -

We have $R \sin (\text{sāyana sun}) = \frac{R \sin \delta}{\text{Sin (parama krānti)}}$

But samśaṅku $= \frac{R \sin \delta}{\sin \Phi}$ from (B) derivation

or $R \text{ Sin } \delta = \text{Sin } \Phi \text{ X Samaśanku}$

Hence R sin (Sāyana sun)

$$= \frac{\text{Sin } \Phi \times \text{Samaśanku}}{\text{Sin (parama krānti)}} \text{ - - - (F)}$$

Verses 52-62 Koṇa Śaṅku

From sāyāna sun bhuja obtained above, we can find true and madhya sun as before. Now methods for koṇaśanku are explained, which is calculated through agrā etc. Four points midway between east-north, north-west, west-south and south-east are called koṇa (angle directions). There are two great circles perpendicular to horizon and passing through koṇa points (one through NE and SW points and other through rest two points). But they are considered 4, one for each koṇa point.

From Sūrya siddhānta

$$\text{Madhya agrā} = \frac{\text{Krānti jyā} \times \text{Trijyā}}{\text{lambajyā}}$$

$$\text{Karṇa Vṛttāgrā} = \frac{\text{madhya agrā} \times \text{iṣṭa karṇa}}{\text{Trijyā}}$$

When the sun enters one of the koṇa vṛttas, perpendicular from sun on horizon is called koṇa śaṅku. Distance of sun from svastika along koṇa

vṛtta is natāmśa and from horizon, it is unnatāmśa. Jyā of natāmśa (R sin Z) or koṭijyā of unnatāmśa [R cos (90°-Z)] is length of koṇa śaṅku.

Shadow of 12 aṅgula śaṅku, then in opposite direction of koṇa is called koṇa chāyā, when in north part, krānti of sun is equal to akśāmśa, there is no shadow in koṇa directions.

When midday sun has south nata (altitude), then in forenoon, koṇa śanku is āgneya (east south angle) and in forenoon, koṇa śaṅku is nairṛtya (south west angle).

When mid day sun has north nata, koṇa śaṅkus in forenoon and afternoon are called īśāna and vāyavya. Now chāyā and natāṁśa can be found.

(A) Karaṇī = $\dfrac{12^2 \text{ (Trijyā}^2\!/2 - \text{agrājyā}^2)}{(\frac{12^2}{2} + \text{palabhā}^2)}$

(B) Akśaphala or phala = $\dfrac{\text{Agrajyā} \times 12 \times \text{palabhā}}{72 + \text{palabhā}^2}$

(C) Mūla = $\sqrt{\text{Karaṇī} + \text{akśaphala}^2}$

(D) Koṇa śaṅku = akśaphala ± Mūla

(Sum is done when sun is north of east west line samamaṇḍala. If sun is south of sama maṇḍala, difference is taken)

(E) Koṇa chāyā = $\dfrac{\text{Dṛg jyā} \times 12}{\text{Koṇa śaṅku}}$

(F) Koṇa chāyā karṇa = $\dfrac{\text{Trijyā} \times 12}{\text{Koṇa śanku}}$

(G) Dṛgjyā or Koṇa śaṅkujyā

$=\sqrt{\text{Trijyā}^2 - \text{Konaśaṅku}^2}$

Results E to G are quoted from sūrya siddhānta

Notes (1) Equation (1) after verse 44 is

$$\text{Sin a} = \frac{\text{Sin }\delta - \text{Cos z Sin }\Phi}{\text{Sin z Cos }\Phi}$$

where a = agrā at any time, δ = Krānti of sun, z = natāmśa, Φ = akśāmśa of the place. Sun is on koṇaśaṅku, in forenoon, its agrā is 45° north or 45° south from east point (according as krānti of sun is more than north akśāmśa or less).

$$\text{Sin a} = \text{Sin } 45^\circ = \frac{1}{\sqrt{2}}$$

Again we have $\sin\phi = \frac{\text{Palabhā}}{\text{viṣuva karṇa or pala karṇa}}$

$$\text{Sin }\delta = \frac{12 \times \text{agrājyā}}{\text{Viṣuva karṇa}}$$

$$\text{Cos }\Phi = \frac{12}{\text{Viṣuva karṇa}}$$

Then the equation becomes

Sin a x Sin z. Cos Φ = Sin δ - Cos z. Sin ϕ

$$\text{or } \frac{1}{\sqrt{2}} \text{ Sin z } \frac{12}{\text{Palakarṇa}} = \frac{12 \times \text{agrā jyā}}{\text{palakarṇa}}$$

$$- \text{Cos z } \frac{\text{Palabhā}}{\text{palakarṇa}}$$

(The agrā jyā on right side is for sunrise time).

$$\text{or } \frac{1}{\sqrt{2}} \text{ sin z} \times 12 = 12 \text{ A} - \text{Cos z. p.}$$

(A = Agrājyā at sun rise, p = palabhā)

or $\frac{12^2}{2}$ Sin^2 z = 12^2 A^2 + p^2 $\cos^2$ z - 2 X 12 A X p cos z

But $R^2 \sin^2 z = R^2 - R^2 \text{Cos}^2 Z$, Hindu system used.

So, $\frac{1}{2} \times 12^2 (R^2 - R^2 \text{Cos}^2 z) = 12^2 A^2 + p^2 R^2 \text{Cos}^2 z$ $- 2 \times 12 \times A \times R \cos z \times p$

or $12^2 \left(\frac{R^2}{2} - A^2\right) = R^2 \text{Cos}^2 z \left[\frac{12^2}{2} + p^2\right]$ $- 2p \times 12 \times A \times R \cos Z$

Dividing each side by $12^2/2 + p^2$

$$R^2 \cos^2 Z - \frac{2 \times 12 \times A \times p}{\frac{12^2}{2} + p^2} R \cos z$$

$$- \frac{12^2 \left(\frac{R^2}{2} - A^2\right)}{\frac{12^2}{2} + p^2} = 0$$

Third term is karaṇī == N and coefficient of R cos Z in second term is called phala = F, then

$R^2 \cos^2 z - 2 F, R \cos Z - N = O$

or $(R \cos z - F)^2 = N + F^2$

or $R \cos z = F \pm \sqrt{N + F^2}$

But R cos Z = R cos (natāṁśa)

= R sin (unnatāṁśa)

= Koṇa śaṅku

So Koṇa śaṅku = $\sqrt{\text{Karaṇī} + \text{Phala}^2}$ + phala

Thus F is added for north krānti of sun.

(2) As in samaśaṅku, it can be proved easily that, Koṇa chāyā = $\frac{\text{Dṛgjyā} \times 12}{\text{Koṇa śaṅku}}$

and Koṇa chāyā karṇa = $\frac{\text{Trijyā} \times 12}{\text{Koṇa śaṅku}}$

Dṛgijyā = R cos z = OR = PC

PS = Śanku, GS = chāyā,

PG = chāyā karṇa

OC = Koṇa Śaṅku, PO = R

$$\frac{GS}{PC} = \frac{PG}{PO} = \frac{PS}{OC}$$

gives the result.

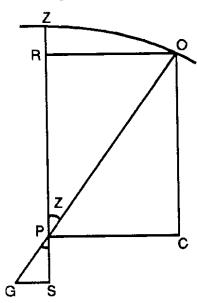



Figure 12 a

Verses 63 to 67 : Calculating natāmśa -

Audayika agrā

A = $\frac{\text{Jyā of sāyana sun} \times \text{Jyā of Parama Krānti}}{\text{Lambajyā}}$

(Already proved)

From this formula, when bhujajyā of sāyana sun is 2431 (R sin 45° = 2431) on equinox day, then sun rises and sets on koṇa circles. When sun is on equator, it is parallel to east west line at all places, hence natāṁśa of koṇa circle in forenoon is equal to natāṁśa of koṇa circle in afternoon. (This can happen at 45° north latitude).

In forenoon, from sāyana sun, krānti, chāyā agrā, time etc are found. Sun position at midday

is approximate and successive approximation is needed for koṇa time.

Verses 68-71 : Shadow from time and vice versa - Now, method is explaine to find shadow length, when time is known or vice versa. By this, true positions of planet or lagna can be known at the time of birth, yajña etc.

Steps - Natakāla is expressed as time or equivalent angle, a planet takes to reach mid day position in forenoon. In afternoon it is time or angle passed from meridian position.

(1) $\frac{\text{Nata kāla}}{\text{half day}} = \frac{\text{Nata kāla in kalā (N)}}{\text{3 rāśi}}$

This is different from nata aṁśa = z which is angular distance from vertical zenith, it is more than the distance from meridian.

(2) Utkrama jyā vers N = R (1-cos N) is found.

(3) Antyā = Trijyā ± carajyā

In north hemisphere, when sun is in north krānti, sum is used. For south krānti of sun, difference is taken

(4) Unnata jyā = Cos N = Antyā - Vers N

Cos N is called iṣṭa antyā also.

(5) cheda = $\frac{\text{Cos N} \times \text{Dyujyā}}{\text{Trijyā}}$, called iṣṭa hṛti also

(6) Mahāśaṅku or śaṅku R cos Z

$= \frac{\text{cheda} \times \text{lambajyā}}{\text{Trijyā}}$

(7) Dṛgjyā = $\sqrt{\text{Trijyā}^2 - \text{śaṅku}^2}$

(8) $\text{Chāyā} = \dfrac{\text{Dṛgjyā} \times 12}{\text{śaṅku}}$

$\text{Chāyā karṇa} = \dfrac{\text{Trijyā} \times 12}{\text{śaṅku}}$ already found

Notes (1) : Formula (6) can be written as

Śaṅku

$$= \frac{(\text{Antyā} - \text{vers N}) \times \text{Dyujyā}}{\text{Trijyā}} \times \frac{\text{lambajyā}}{\text{Trijyā}}$$

$$= \frac{(\text{Trijyā} \pm \text{Carajyā} - \text{vers N}) \times \text{Dyujyā}}{\text{Trijyā}} \times \frac{\text{lambajyā}}{\text{Trijyā}}$$

$$= \frac{\text{Trijyā} - \text{vers N} \pm \text{carajyā}}{\text{Trijyā}^2} \times \text{Dyujyā} \times \text{Lambajyā}$$

$$= \frac{\text{R cos N} \pm \text{Carajyā}}{R^2} \times R \cos\delta \times R \cos \Phi$$

or $R \cos z = (R \cos N \pm \text{Carajyā}) \times \cos \delta \times \cos \Phi$

δ = Kranti, z = natamśa and Φ = aksāmśa

This formula is to be proved.

(2) Figure 11 after verse 44 may be referred again

Natakāla - Natakāla is the time in which sun or any other star or planet comes to yāmyottara (north south vertical circle) in forenoon. In afternoon, it is time lapsed since it had come on yāmyottara. These are called pūrva and paścima nata - incline to east or west.

Unnata kāla is opposite to natākāla i.e. time taken to rise from horizon in forenoon or the time after which the planet will set in west sphere.

Unnata kāla = 1/2 day time - natakāla

When polar circles to equator are drawn through position of sun, the arcs on diurnal circle of the planet are proportional to arcs of equator which are proportional to rising time in asu when arc is in kalā or minute. Rotation of earth is along equator with fixed speed and time for 1′ rotation = 1 asu,.

Thus in figure 11, natakāla at R_1, R_2, R_3 is the time for planet to reach point R of yāmyottara. Natakāla corresponding to points R_1, R_2, R_3 all east from yāmyottara are angles ZPR_1, ZPR_2, ZPR_3 which are proportional to arcs QK_1, QK_2, QK_3 on equator.

Time from E to Q is half day and angle is 90° = 3 rāśi

Hence, $\dfrac{\text{Natakāla}}{\text{half day}} = \dfrac{QK_1}{QE}$ for point K1; sun at R_1

$$= \frac{\text{natāmśa}}{3 \text{ rāśi}} \quad \text{- - - Result (1)}$$

(3) For sun at R_1, in spherical triangle ZPR_1

$$\text{Cos } \angle ZPR_1 = \frac{\text{Cos}(ZR_1) - \text{Cos}(PZ) \times \text{Cos}(PR_1)}{\text{Sin}(PZ) \times \text{Sin}(PR_1)}$$

or cos (nata kāla)

$$= \frac{\text{Cos } z - \cos(90° - \Phi)\cos(90° - \delta)}{\text{Sin}(90° - \Phi)\text{ Sin}(90° - \delta)}$$

$$= \frac{\cos z - \text{Sin } \Phi \text{ Sin } \delta}{\text{Cos } \Phi \text{ Cos } \delta}$$

$$= \frac{\text{Cos } z}{\cos \Phi \,.\, \cos \delta} - \tan \Phi \tan \delta \quad \text{- - - (A)}$$

Now carajyā = R tan Φ tan δ - - - (B)

Adding (A) and (B),

$$R \cos (\text{nata}) + \text{carajyā} = \frac{\text{Cos } z}{\text{Cos } \Phi \cos \delta}$$

or Śaṅku = R cos z

$$= [\, R \cos (\text{nata}) + \text{carajyā}] \times \text{Cos } \Phi \cos \delta$$

In Indian system sin and cos are to be multiplied by R. Results for obtaining chāyā and karṇa have already been proved.

Verses 72-75 : Time from shadow.

For this, same formula are used in reverse order -

$$\text{Step (1) Dṛgjyā} = \frac{\text{Chāya} \times \text{Trijyā}}{\text{Chāyā karṇa}}$$

$$\text{(2) Mahā śaṅku} = \sqrt{\text{Trijyā}^2 - \text{Dṛgjyā}^2}$$

(3) Cheda or iṣṭa hṛti

$$= \frac{\text{Śaṅku} \times \text{Trijyā}}{\text{Lambajyā}} \text{ or } \frac{\text{śaṅku} \times \text{palakarṇa}}{12}$$

$$\text{(4) unnatajyā cos N} = \frac{\text{Cheda} \times \text{Trijyā}}{\text{Dyujyā}}$$

(5) Nata Utkrama jyā = vers N = Antyā-Cos N

(6) Arc N is found from this. Its value in kalā is equal to asu of natakāla.

Nata--asu divided by 6 gives nata pala. When sun is in forenoon, this is time before noon and in afternoon, it is time after noon.

Notes : Methods can be proved in same way, as previous formula.

Verses 76-77 - When nata utkramajyā is less than 27 kalā, there is a separate method.

$$\text{Natāsu} = \sqrt{\text{Antyā}^2 - \text{Unnatajyā}^2} \times \frac{\frac{1}{2}(\text{Trijyā} + \text{antyā})}{\text{Antyā}}$$

Note : Utkrama jyā is 29 kalā for 2nd khaṇḍa of 7-1/2°. For smaller values (leess than 7° natāmśa) this is an approximate method.

Verse N = (1-cos N) = $N^2/2$ for small N

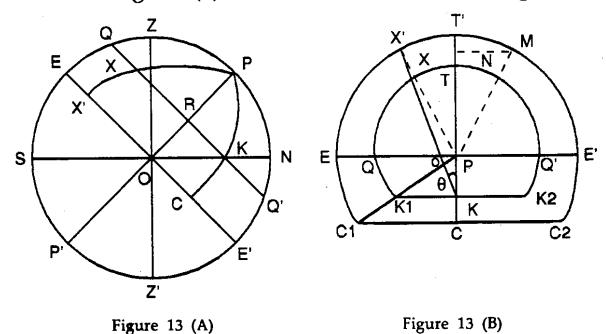
or, $\frac{N^2}{2}$ = Antyā − unnatajyā

For derivation of this approximate formula and to explain the physical significance of terms used at each stage, it is necessary to show diagrams.

Natakāla has been explained in both circles, yāmyottara (meridian circle) in Fig 13a and equator (viṣuva) circle in figure 13b.

In Fig. 13(a), EOE′ is diameter of equator,

Figure 13 (A)
Yāmyottara Vṛtta

Figure 13 (B)
Viṣuva Vṛtta

QQ′ is diameter of ahorātra vṛtta (diurnal circle). NS is diameter of horizon in north-south direction. In north krānti, sun comes on horizon at K, hence

in 1/2 day QK is increased from QR (6 hours) by RK. QR = semi diameter of diurnal circle = Dyujyā, RK = extra length of half day or advance sun rise time = Kujyā.

The corresponding lengths on equator circle are propositional to time (arc in kalā = time in asu). Here OE = Radius of celestial circle = 3438′ kalā. OC = carajyā. Distance of position X from mid day position Q is called nata kāla. Corresponding nata kala on equator is measured by arc EX′. EX′ = vers N as measured from diameter end E.. Length from centre is OX′ = Cos N.

EC = Antyā = distance along meridian diameter from corresponding positions of sunrise and mid-day = EO + OC = Radius + Carajyā

Iṣṭa antyā for position X of sun is its distance along meridian diameter between corresponding positions of sunrise and instant position of equator.

Iṣṭa antya = X′C = CE - EX′ = Antyā - nata utkramajyā

QK = Hṛti, XK = Iṣṭa hṛti

Dyujyā = R cos Φ , where Φ is latitude, Corresponding distances on equator and diurnal circle are propositional, hence

$$\frac{\text{Iṣṭa hṛti}}{\text{Iṣṭa antyā}} = \frac{\text{Hṛti}}{\text{antyā}} = \frac{\text{Kujyā}}{\text{Carajyā}} = \frac{\text{Dyujyā}}{\text{Radius}}$$
$= \text{Cos } \Phi$ - - (1)

Now same positions are represented in Fig 13 (B) but in equator circle and projections on it. Projection of P is at O itself.

QTQ′ = diurnal circle, ET′E = equator circle - half portions above horizon EQE′ are shown. For

positon M, when sun has zero krānti, both circles are one and nata angle N = ∠MOT′ = arc T′M. Nata utkramajyā = T′N, Unnatajyā or nata koṭijyā = ON and natajyā = MN. When N is small, T′M

$$= NM \text{ (approx)} = \sqrt{OM^2 - ON^2}$$

$$\text{or Nata asu} = \sqrt{\text{Trijyā}^2 - \text{Unnatajyā}^2}$$

In this position antyā = Trijyā

Hence the formula, nata = $\sqrt{\text{antya}^2 - \text{Unnatjyā}^2}$

$$\times \frac{\frac{1}{2}(\text{Trijya} + \text{antyā})}{\text{antyā}} = \sqrt{\text{Trijyā}^2 - \text{Unnatajyā}^2}$$

This case is proved.

When sun is having north krānti, horizon point on diurnal circle K_1 corresponds to horizon point C, on equator; so that OK_1C_1 and OK_2C_2 are in one line. Thus horizons are K_1 K K_2 and C_1 C C_2 on diurnal and equator circle.

Here T′C == Antyā, T′O = Trijyā

At Nata N, position of sun is at X and X′ on equator.

Arc X′T′ = ∠X OT′ = N

But sun is seen at X making angle θ at horizon at K.

$$T'K = \frac{T'O + T'C}{2}$$ approx. as K is almost in middle of PC.

Since angle is small

$$XT = \frac{R + A}{2} \times \theta, \; A = \text{antyā}$$

However, we are measuring angle from C in formula

$$\sqrt{\text{antyā}^2 - \text{unnatajyā}^2} = A\,\theta$$

$$\text{Hence, } N = \frac{A\,\theta}{A} \times \frac{R + A}{2}$$

$$\text{or, Hence } N = \sqrt{\text{antyā}^2 - \text{Unnatajyā}^2} \times \frac{1}{2}\,\frac{(R+A)}{A}$$

(2) Since we are making measurements from distance $T'K = \frac{R+A}{2}$, A cos θ may be more than R. as A > R. Then angle is measured by substracting R from A cos θ, as the jyā is same in next quadrant also.

Verses 78-80 : Some precautions

When nata utkrama jyā is more than trijyā, we deduct trijyā from it and arc of remaining part is taken. It is added to 5400 kalā to find nata asu.

When nata asu is more than 5400 asu, we deduct 5400 asu and find jyā of remaining arc. This added to trijyā is nata utkramajyā.

Nata asu multiplied by sāvana dina (21,659 asu) and divided by chakra asu (21600) gives sūkśma nataśu.

Notes (1) Calculation for 2nd quadrant is same as explained in note (2) after verse 77.

(2) We are taking a sāvana dina as 21600 asu instead of 21659 asu, hence this proportionate correction is done.

Verses 81-84 : Sun from agrā and sama śanku.

Now I tell the method to find sāyana sun from karṇāgrā and samamaṇḍala śaṅku

$$\text{Krānti jyā} = \frac{\text{Karṇāgrā} \times \text{lambajyā}}{\text{Chāyā karṇa}} \quad \text{- - - -} \quad (A_1)$$

$$\text{Jyā (sāyana sun)} = \frac{\text{Krānti jyā} \times \text{Trijyā}}{\text{Jyā of paramakrānti}} \quad (A_2)$$

According to the quadrant of sāyana sun, sāyana sphuṭa sun is found. By deducting ayanāmśa, sphuṭa sun is found as before.

Alternatively,

$$\text{Samaśaṅku} = \frac{\text{Trijyā} \times 12}{\text{Samaśaṅku chāyā karṇa}} \quad \text{- - -} \quad (B_1)$$

$$\text{Jyā (Sāyana sun)} = \frac{\text{Samaśaṅku} \times \text{akśajyā}}{\text{Jyā of paramakrānti}} \quad \text{- - -} \quad (B_1)$$

From (B_1), sun is obtained as before.

Notes : (1) Formula A_1 and A_2 have been obtained in verse 40-41 or in 53.

(2) Formula B_1 and B_2 have been given in verse 47-50.

Verse 85 : According to ancient scientists, shadow end of the śaṅku moves on a circular path on a horizontal plane. This is not correct for all places and all times. This will be discussed in golādhyāya. Now we discuss the method to find time in night with help of conjunction of planets and stars.

Note : Locus of shadow has been discussed after verse 5. Its formula for radius of circle has been given by Vaṭeśvara and Bhāskara II. This is correct for only central portion of the hyperbola, which is real locus.

According to Vaṭeśvara, one formula for diamter of shadow circle is $\frac{(\text{R} + \text{agrā})\,(\text{R} - \text{agrā})}{\text{Mid day śaṅkutala}}$

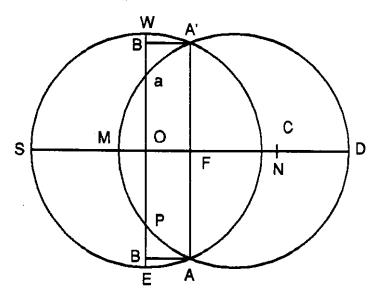



Figure 14
Diameter of shadow circle

+ mid day śaṅku tala.

In figure 14, circle ENWS with centre O is the horizon, with east, north, west and south points. A is the point where sun rises, A′ is the point where sun sets and M is the foot of perpendicular on horizon from mid day sun. Then circle through A′, M and A is locus of shadow, approximately for central portion A′MA.

AB, the distance of A from east west line EW, is sun's agrā (at rising time).

MO = Z_m, = R sine of sun's zenith distance at midday. MF, Distance of M from rising setting line AA′ is sun's śaṅkutala at mid day.

MF = MO+OF = MO + BA = Zm + agrā

C is centre of circle A′MA. Let OC = x, Then

$MC^2 = AC^2$ (both radius)

or $(MO+OC)^2 = FA^2 + FC^2$

or $(Zm + x)^2 = R^2 - (agrā)^2 + (x\text{-}agrā)^2$

where R is radius of circle E NWS.

Solving it for x, we get

$$2x = \frac{R^2 - (\text{agrā})^2}{Zm + \text{agrā}} + \text{agrā} - Zm$$

$$\text{or } 2\,(x + Zm) = \frac{R^2 - (\text{agrā})^2}{Zm + \text{agrā}} + Zm$$

$$= \frac{(R + \text{agrā})\ (R - \text{agrā})}{\text{mid day śaṅkutala}} + \text{mid day śaṅkutala}$$

This gives the diameter, as x + Zm = radius of shadow circle.

Another formula for this diameter is

$$\frac{(\text{shadow})^2 - (\text{bhujā})^2 + (\text{bhuja mid day shadow})^2}{\text{bhuja mid day shadow}}$$

This can be proved from same diagram.

Verses 86-87 : Lapsed or remaining part of night is found by observing madhya lagna in sky from position of nakśatras (position of their stars given in a later chapter). Ayanāṁśa is added to madhya lagna. From rising times at equator, lapsed part of lagna in the fractional rāśi is found. Then remaining rising time for sāyana ravi at night in the part rāśi is found. These two are added along with rising times of complete rāśis between daśama lagna and sāyana sun. From this sum, half solar day is substracted. Remainder is the lapsed time in ghaṭī etc of night. Half day added to the sum is the iṣṭa time from sun rise.

Similarly, remaining part of 10th lagna rāśi, lapsed part of sāyana sun rāśi and complete rāśis from 10th lagna to sun (rising times) - all added and half day of sun deducted gives the remaining part of night.

Notes : Method of 10th lagna has already been explained in chapter 6.

Verses 88-92 : Rising times of nakśatras in Orissa -

Method to find lagna has already been explained from time of day and night. Now for 22° Ayanāmśa, rising times of different nakśatras in Orissa are stated, by which true madhya lagna can be found in sky. This will be very useful for sky watcheers who can satisfy their curiosisty.

At mid day time, that nakśatra is in mid sky in which sun is present. 7th rāśi of lagna at that time is asta lagna (setting rāśi). The lapsed times of lagna rāśis are stated according to the nakśatra, which has risen in middle sky - starting from śravaṇa.

(22) Śrāvaṇa - meṣa 94 pala (23) Dhaniṣṭhā - meṣa 230 pala (24) Śatabhiṣa - Vṛṣa 280 pala (25) Pūrvabhādrapada - mithuna 24 pala (26) Uttarabhādrapada - mithuna 174 pala (27) Revatī - Karka 49 pala (1) Aśvinī - Karka 187 pala (2) Bharaṇī - Karka 256 pala (3) Kṛttikā - siṁha 67 pala (4) Rohiṇī - Siṁha 177 pala (5) Mṛgaśirā - Kanyā 2 pala (6) Ārdrā - Kanyā 58 pala (7) Punarvasu - Tulā 2 pala (8) Puṣya - Tulā 144 pala (9) Aśleṣā - Tulā 184 pala (10) Maghā - Vṛscika 32 pala (11) Pūrvā phālgunī - Vṛścika 197 pala (12) Uttarā phālgunī - Vṛścika 285 pala (13) Hasta - Dhanu 62 pala (14) Citrā - Dhanu 198 pala (15) Svātī - Makara 12 pala (16) Viśākhā - Makara 151 pala (17) Anurādhā - Makara 266 pala (18) Jyeṣṭhā - Kumbha - 67 pala (19) Mūla - Kumbha 231 pala (20) Pūrvāṣāḍha - Mīna 80 pala (21) Uttarāṣāḍha - Mīna

152 pala. From the difference of rising times of these nakśatras, time can be found.

Verses 93-94 - Conclusion -

Bhāskarācārya II has described many types of quantities from bhuja, koṭi and karṇa, etc. in Tripraśnādhikāra chapter of his siddhānta śiromaṇi and has clarified many doubts by questions and answers. This already exists in siddhānta śiromaṇi with his own commentary vāsanā bhāṣya. hence I am not repeating all due to fear of big size of book.

I have described only those topics in detail, which I have verified personally and have separate views. This subject can be understood only through a good grasp of gola (spherical trigonometry) and gaṇita (mathematical methods). Then derivation of formula will not be difficult. Hence I have not enlarged the bulk of book by writing proofs.

Verses 94-95 : Prayer and end -

May lord Jagannātha fulfil my ambitions who is rejoicing with Lakśmī of unsteady eyes and is residing at Nīlācala (Purī) at 276-1/2 yojana north from equator i.e. 19°48′ N latitude and 200 yojana east from Indian prime meridiaṇ (passing through Ujjain).

Thus ends the seventh chapter explaining three questions (Tripraśna) along with views of sages; in Siddhānta Darpaṇa written for correspondance in calculation and observation, and education of students, by Śrī Candra Śekhara, born in famous royal family of Orissa.

Appendix to Tripaśnā dhikāra

(1) (a) Local time, Standard time and true time : These three are basis of corrections to planet positions, in chapter 2. True time is time corresponding to nata kāla; position of sun. Local mean time is average time of a locality, assuming 24 hours in each day. Standard time is local mean time of a position taken as standard for a country or a time zone. This time differs from Greenwich mean time by exact multiples of half hours. Like standard time of India is local mean time of place 82°30′ east of Green-wich i.e. 5-1/2 hours more than G.M.T.

(b) Definitions - Sīdereal time - Point of equinox from which sāyana position of sun is measured on krānti vṛtta (ecliptic) is moving backwards on ecliptic. Position of sun from this point along ecliptic is rāśi of sāyana sun or longitude. Position of sun along equator is right ascensian. If measured relative to local horizon of earth, position of sun along equator is nata kāla or sidereal time, measured from zenith position of sun i.e. 12 hrs noon. Hence right ascension, also is written in hours. (It may be called viṣuva aṁśa or hour angle).

When motion of equinox is assumed uniform, time measured from it, is uniform sidereal time. From the true position of equinox, it is called true sidreal time. The differnce beetwen them is less than 1/10 seconds and normally ignored.

Sidereal time is west wards, because equinox point is moving west wards like sun due to eastward daily motion of earth. It is the time in

hours after the instant equinox point has crossed the meridian (north south vretical circle of a place). Its circle is completed in 24 hours by definition, hence 1 hour movement = 15° (=360° ÷24) Position of planet in hours of right asencion is 15° per hour counted from equinox position along ecliptic.

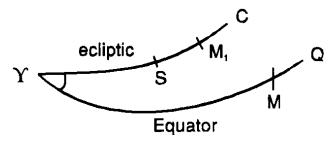



Figure 15

(c) Mean time -

Mean sun M is a fictitious point which moves along equator with average angular velocity n of actual sun.

Since sun completes one rotation in a sidereal year both along ecliptic and along equator, its mean speeds are same in both the circles. Mean sun on ecliptic is M_1 and true sun S. - both coincide at perigee or apogee (mandocca). ϒ is point of intersection of ecliptic and equator.

ϒ M = right ascension of mean sun

ϒ M_1 = mean longitude of sun

ϒ S = true longitude of sun

ϒ M = ϒ M_1 = nt after time t.

Mean time at any place is called local mean time (LMT) Since it will continuously vary at every place, local mean time of Greenwich is considered standard for the world called Greenwich mean time (G MT)

Figure 16

Let A be a place east from G and B another place further east. Longitude difference of A and B is expressed in hours (1 hour = 15°).

Let AB = l hours = $15\ l°$

If S and S′ are local sidereal times at A and B any instant.

$S' = l + S$

because ♈ will cross meridian at B, l hours before meridian of A its west ward motion.

Similarly if M and M′ are local mean times at A and B at any instant.

$M' = l + M$

l is same in both formulas because hour angle and mean sun both increase 360° in 24 hours.

To avoid inconvenience due to differences in the local times of various places in a country, the local time of a chosen meridian is regarded as standard time. All the places in that country keep this time and not the local time. Thus the standard time of India is exactly 5-1/2 hours ahead of GMT i.e. time of a place 82°30′ east of Greenwich. In very large countris like Russia or USA, the country is divided into zones, each having a differnt standard time. For further convenience, the standard times of these time zones differ from GMT by an integral number of hours or half hours.

Hour angle measured at Greenwich from 12 hours noon time is called Greeenwich mean astronomical time. (GMAT) and measured from 0 to 24 hours. Meantime reckoned from mean mid

night at Greenwich is called Greenwich civil time (GCT), GMT or universal time (UT). This also is measured from 0 to 24 hours.

GMT = GMAT + 12^h

Same is for other places also.

1. (d) Mean and Sidereal conversion

In one solar year (tropical), sun crosses ♈ again after one circle.

It takes K = 365.2422 mean solar days, i.e. K revolutions of earth with respect to sun. Hence there are K + 1 revolutions of earth with respect to ♈ or any star. Thus

K+1 sidereal days = K mean solar days

K+1 sidreal hours = K mean solar hours etc.

1 Sidereal days = $1 + \frac{1}{K+1}$ mean solar day

= $23^h\ 56^m$ 4.1 S mean solar units.

Mean solar day = $1+\frac{1}{K}$ sidereal days = $24^h 3^m$ 56.5s sidereal hour etc.

1 (e) Years : Sidereal year is time taken by sun for one complete revolution with respect to stars on ecliptic which are fixed.

Tropical year is average interval between two successive returns of sun to the first point of Aries (♈). As ♈ moves backwards in about 26,000 years, tropical year is slightly shorter.

(a) Tropical year = 365.2422 mean solar days

(a) Sidereal year = 365.2564 mean solar days

(2) (a) Equation of time : From the watches we get mean solar time only and we can get the local mean time after longitude correction from standard time. To know the true solar time or apparent time we have to add some correction (+ ve or - ve) called equation of time. Let

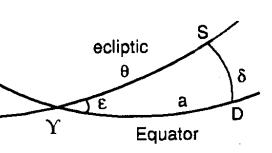



Figure 16

E = Equation of time.

a = Right ascension of sun (distance from ♈ along quator = ♈ D)

θ = true longitude of sun = ♈ S along ecliptic.

l = mean longitude of sun.

Then, by definition,

E = West hour angle of sun - West hour angle of the mean sum

= (S-a) - (S - RA. of mean sun)

= (S-a) - (S-l) = l-a

This can be written as

E = - (a-θ) - (θ-l)

Here - (a-θ) is called the equation of time due to obliquity, because if equator is not oblique a = θ measured along any circle and this term a-θ = 0

Similarly, - (θ-l) is called the equation of time due to eccentricity.

In the spherical triangle ♈ SD of figure 16, $\angle$ D = 90°,

so Cos ε = tan a cot θ

where ε is angle beetween equator and ecliptic

or tan a = cos ε tan (θ-a+a)

Expanding this by Taylor's theorem and neglecting higher powers of ε and θ–a,

$$\tan a = (1-\frac{\varepsilon^2}{2}) [\tan a + (\theta - a) \sec^2 a]$$

$$\text{or } \tan a = \tan a + (\theta - a) \text{Sec}^2 a - \frac{\varepsilon^2}{2} \tan a, \text{ approx.}$$

$$\text{i.e. } \theta - a = \frac{\varepsilon^2}{2} \text{Sin } a \cos a = \frac{\varepsilon^2}{4} \text{Sin } 2a,$$

Since a = l nearly, we can write

$$\theta - a = \frac{\varepsilon^2}{4} \sin 2l$$

which is the required value of – (a-θ). Often a-θ is called the reduction to the equator, because a-θ added to ecliptic co-ordinate θ reduces it to equatorial coordinate.

Again θ = v+D

where D is position of perigee and v is true amomaly, true position of sun measured from perigee.

l = m + D

where m is mean anomaly.

So, θ-l = v-m = 2 e sin m, nearly.

Thus E = 1/4 ε^2 Sin 2 l - 2 e sin (l-D)

where E, ε and e (eccentricity) all are measured is radians giving numerical values. Expressing it in minutes.

E = 9^m. 9 sin 2l - 7^m 7 sin (l+78°)

2. (b) A more accurate value-Put y = $\tan^2 \frac{\varepsilon}{2}$

Then

$$\cos \varepsilon = \frac{1-y}{1+y}$$

and so $\tan a = \frac{1-y}{1+y} \tan \theta$

Using exponential formula, $\tan x = \frac{e^{2ix} - 1}{e^{2ix} + 1}$

where e = base of natural logarithm, we have

$$\frac{e^{2ia} - 1}{e^{2ia} + 1} = \frac{1 - y}{1 + y} \cdot \frac{e^{2i}\theta - 1}{ex^{2i}\theta + 1}$$

or $e^{2ia} = \frac{e^{2i\theta} + y}{1 + y e^{2i\theta}} = \frac{e^{2i\theta}(1 + y e^{-2\theta})}{1 + ye^{2i\theta}}$

Taking logarithms this gives

$2 i a = 2i\theta + (ye^{-2i\theta}$ $1/2\ y^2e^{-4i\theta} + 1/3\ y^3 e^{-6i\theta})$

$-$ [$ye^{2\ i\theta}$ - $1/2\ y^2\ e^{4i\theta} + 1/3$ y3 $e^{6i\theta}$ - - -)

$= 2i\ \theta$ - $2i\ (y \sin 2\ \theta)$ - $1/2\ y^2\ \text{Sin}4\theta + 1/3\ y^3$ Sin 6θ)

or $\theta\text{-a} = y \sin 2\ \theta - 1/2\ y^2 \sin 4\ \theta + \frac{1}{3} y^3$ Sin 6θ - - - - (1)

Again θ - l = v-m

$= 2\ e \sin M + \frac{5}{4} e^2 \sin 2M + \text{- - - -}$

$= 2e$ Sin (l-D)) + $5/4\ e^2$ Sin 2 (l-D) + - - -

(Introduction to chapter 6)

Eliminating θ, we get

$E = \text{l-a} == \tan^2 \frac{\varepsilon}{2}$ Sin 2 l - 2 e Sin (l-D)

+ 4e $\tan^2 1/2\ \varepsilon$ Sin (l-D) Cos 2l

- $5/4\ e^2$ Sin 2 (l-D) - $1/2 \tan^4 \frac{\varepsilon}{2}$ Sin 4l

The equation of time vanishes four times in a year.

$E = 9^m\ 9 \text{ Sin } 2l - 7^m\ 7 \text{ Sin } (l+78°)$

If we draw sine curves for $y == 9^m.\ 9 \text{ Sin } 2l$ and $y = 7.7^m \sin (l + 78°)$ and substract one ordinate from the other, we get the graph of E, From graph is can be seen that it vanishes four times around 23 march, 22 June, 22 September, 22 December.

Sin 2 l attains max numerical values of 1 four times in a year and is alternately positive and negative at three times. Hence first term twice has value + 9.9 minutes and twice - 9.9 minutes atternately negative and positive. Thus E is alternately positve and negative, because second term is smaller numerically. Hence E is zero four times a year from theory of equations.

(3) (a) Parallax : At any instant, the moon has slightly different directions as seen from different places on the earth. Sun's direction changes much less with the change in position of the observer, because sun is more distant. In case of stars, which are far more distant, the difference in their directions as seen from different places of the earth is too small to be measured. But seen from different places in the earth's orbit, (i.e. at different times of the year), the change in the direction of the comparatively nearer stars is measurable.

The change in the direction of a celestial body as seen from different positions is called parallax.

For calculation of sun, moon and planets, we choose earth's centre as the standard position (origin of coordinate axis) from which distances are

calculated. Due to observation from surface of earth, there is parallax error, called geocentric parallax.

For calculation of star position, sun's centre is the standard position and difference in direction due to measurement from different positions of earth's orbit, is called stellar parallax.

As geocentric parallax depends upon the distance of the observer from earth's ceentre, we begin by considering the shape of the earth.

3 (b) Shape of the earth - Surface of earth deteremined by ocean level is called the geoid, heights of places above mean sea level being negligible. It is an oblate spheroid i.e. rotation of ellipse along its minor axis coinciding with polar axis of the earth. Semi major axis a of the generating ellipse (equatorial radius) is 3963.95 miles and the semi minor axis b is 3950.01 miles. The fraction (a-b)/a is called the compression; Eccentricity of this ellipse e = 0.082., compression = 1/297.

Let C be the centre of the earth, O the observer at any place on its surface, OZ the normal at O to the surface and OZ′ the direction which produced backwards passes through C. Then OZ is the direction of the astronomical Zenith, OZ′ that of the geocentric zenith. Angle between these directions ZOZ′ is called the angle of the vertical indicated by V.

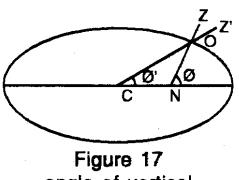



Figure 17
angle of vertical

If Φ and Φ' are the angles made by normals NOZ and line COZ′ from centre with major axis-

Φ = geographical latitude of O

Φ' = geocentric latitude of O

$\nu = \Phi - \Phi'$

If ellipse is referred to C at origin, it is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Then, $\tan \Phi = \dfrac{a^2 y}{b^2 x}$ - - (1)

and $\tan \phi' = \dfrac{b^2}{a^2} \tan \Phi$

Thus, $\tan v = \tan(\Phi - \Phi') = \dfrac{\tan\Phi - \tan\Phi'}{1 + \tan\Phi \tan\Phi'}$

$$= \frac{(a^2 - b^2) \tan\Phi}{a^2 + b^2 \tan^2\Phi} = \frac{(a^2 - b^2)\sin\Phi\cos\Phi}{a^2\cos^2\Phi + b^2\sin^2\Phi}$$

$$= \frac{(a^2 - b^2)\sin 2\Phi}{a^2 + b^2 + (a^2 - b^2)\cos 2\phi} = \frac{m \sin 2\Phi}{1 + m\cos 2\Phi}$$

where $m = \dfrac{a^2 - b^2}{a^2 + b^2}$ which is small

$$\frac{1 + i\tan\nu}{1 - i\tan\nu} = \frac{1 + m(\cos 2\phi + i\sin 2\phi)}{1 + m(\cos 2\phi - i\sin 2\phi}$$

or, $e^{2iv} = \dfrac{1 + me^{2i\phi}}{1 + me^{-2i\phi}}$

Taking logarithms

$2\,iv = \log(1 + me^{2i\phi}) - \log(1 + me^{-2i\phi})$

$= me^{2i\phi} - \frac{1}{2} m^2 e^{4i\phi} + \text{—} \{me^{-2i\phi} - \frac{1}{2} m^2 e^{-4i\phi} + \text{--}\}$

Hence, $v = m\sin 2\phi - 1/2\, m^2 \operatorname{Sin} 4\phi + \frac{1}{3} m^3 \operatorname{Sin} 6\phi$

Distance of the observer O from centre C is indicated by ρ

$$\frac{x/a}{a\cos\phi} = \frac{y/b}{b\sin\phi} = \frac{1}{\sqrt{(a^2\cos^2\Phi + b^2\sin^2\Phi)}}$$

$$\text{So } \rho^2 = x^2 + y^2 = \frac{a^4\cos^2\phi + b^4\sin^2\phi}{a^2\cos^2\phi + b^2\sin^2\phi}$$

$$= \frac{a^2\ [1 - (2e^2 - e^4)\ \mathrm{Sin}^2\ \Phi]}{1 - e^2\sin^2\Phi}$$

on writing $b^2 = a^2\ (1\text{-}e^2)$ and simplifying

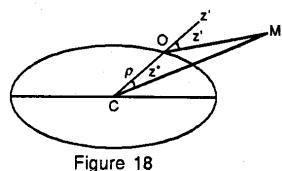



Figure 18
Geocentric parallax

3 (c) Geocentric parallax in zenith distancee

In figure 18, let C be centre of earth, O is observer and M the centre of moon (or sun or planet), Let CO = ρ, CM = r

If Z' is a point on CO produced, apparent zenith distance z′ of M is $\angle Z'OM$ and true (i.e. geocentric) zenith distance zo of M is $\angle Z'cM$.

Hence z′ – zo = parallax in zenith distance

= $\angle OMC$ = p

From plane triangle OCM

Sin p = (ρ/r) Sin z′ ....(1)

Maximum value of parallax p is when z′ = 90°, it is called horizontal parallax p_n of M at O.

Sin p_n = ρ/r

If O is at equator, then the parallax is biggest as ρ has highest value a, equatorial radius. The horizontal parallax at equator Po is

$$\text{Sin Po} = \frac{a}{r}$$

When moon (or the sun) is at its mean distance, r_o from earth, mean equatorial horizontal parallax P is, Sin P = a/r_o

For parallax, earth can be considered almost a sphere then, astronomical and geocentric zeniths coincide, $z'=z$, ρ = constant = a. We take r = r_o approx, then approximate value of parallax is

$$p = P \sin z \quad - - - (2)$$

Since z' > zo, moon, sun or planet is distanced away from zenith by distance P sin z approx due to geocentric parallax. This is also called diurnal parallax as it goes through a complete cycle of change through a day. Parallax is maximum when moon or sun rise on horizon, reduce to zero, when on zenith and again become maximum when they set in west horizon.

3 (d) Distance and size of moon is calculated by parallax method only.

O_1 and O_2, places on same meridian are chosen. Apparent zenith distances of M are

$z_1' = \angle Z_1' O_1M$

$z_2' = \angle Z_2' O_2M$

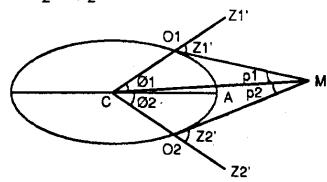



Figure 19
Moon's Distance

p_1 and p_2 are parallax angles O_1 MC and O_2 MC, when C is centre of earth

If CA is in the plane of equator,

$\angle O_1C\,O_2 = \angle\, O_1\,CA + \angle O_2\,CA = \Phi_1 + \Phi_2$

where Φ_1 and Φ_2 are geocentric latitudes of O_1 and O_2

Then $z'_1 + z'_2 = \angle O_1\,CM + p_1 + \angle O_2\,CM + p_2$

$= \Phi_1 + \Phi_2 + p_1 + p_2$

Thus $p_1 + p_2 = z'_1 + z'_2 - (\Phi_1 + \Phi_2) = \theta$..(1)

Because all values on right side are known, θ is known.

$\text{Sin } p_1 = (\rho_1/r) \text{ Sin } z'_1$ - - - (2)

$\text{Sin } p_2 = (\rho_2/r) \text{ Sin } z'_2$ - - - (3)

Eliminating p_1 and p_2 from the three equations, we can know moon's distance r = OM

It is more convenient to find value of p_2 first and then calculate r. From (1) and (2)

$\text{Sin } \theta \cos p_2 - \text{Cos } \theta \text{ Sin } p_2 = (\rho_1/r) \text{ Sin } z_1'$

or $\text{Sin } \theta \cos p_2 = \frac{\rho_2}{r} \sin z'_2 \cos \theta + \frac{\rho_1}{r} \text{ Sin } z'_1$

Eliminating r between this and equation (3), we get

$$\tan p_2 = \frac{\rho_2 \text{ Sin } z'_2 \text{ Sin } \theta}{\rho_2 \text{ Sin } z'_2 \cos \theta + \rho_1 \sin z'_1}$$

This gives p_2 and then (3) gives r.

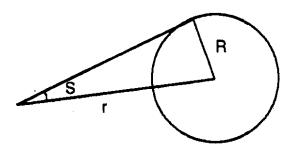



Figure 20
Moon's diameter

In figure 20, let moon's observed angular semi-diameter be S and let its linear radius be R miles. If the distancee of moon is r miles as determined above, Sin S = R/r from which R can be determined.

In India, parallax in zenith distance is called 'nati' and parallax in longitude is called 'lambana'. Lambana can be measured along equator or along ecliptic. Parallax calculation of moon and sun is necessary for calculation of solar eclipse.

3 (e) Lunar parallax along equator and krānti- Mean equatorial horizontal parallax of moon is 57′ and for sun it is 8″.80 i.e. 1/388.6 of moon's parallax. Hence, accuracy is needed only in calculation of moon's parallax.

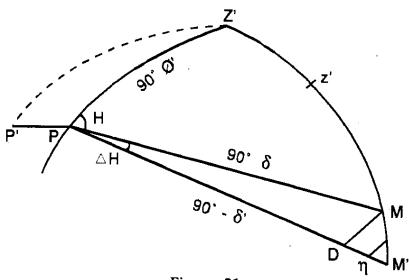



Figure 21
Parallax in natakāla and krānti

In fig. 21, M and M′ ar true and apparent (due to parallax) positions of Moon (or sun)

$$MM' = \frac{\rho}{r} \text{Sin } z'$$

where, ρ = distance of obsrever from centre of earth, r = distance of moon from centre of earth and, z′ = geocentric zenith distance Z′M.

Right ascension and krānti of M and M′ are a,, δ and a′ δ'. Let H and ΔH be their hour angles (nata kāla).

MD is perpendicular to PM′ and $\angle$ MM′D = η

Small Δ MM′D can be taken as a plane triangle, so

$$\Delta a = a' - a = -\Delta H = -\frac{MD}{\sin PM} = \frac{MM' \sin \eta}{\sin PM}$$

$$= \frac{\rho}{r} \cdot \frac{\sin z' \cdot \sin \eta}{\cos \delta}$$

By sine formula in Δ Z′ PM′,

Sin η Sin (z′ + MM′) = cos Φ Sin (H+ΔH)

Hence Δ a = ρ/r Sin z′ Cos Φ Sin (H+ ΔH) X cosec (z′+MM′) / Cos δ

or Δ a = ρ/r cos Φ Sin H secδ - - (1)

neglecting small quantities of second order

Similarly $\Delta\delta$ = δ' - δ = - M′D =- MM′ cos η = ρ/r Sin z′ cos η

From cosine formula in Δ P′ZM

Sin (z′+MM′) cos η = Cos δ' Sin ϕ' − sin δ .cos Φ' cos $(H + \Delta H)\}$

Substituting this value of cos η and neglecting small quantities of second order

$\Delta\ \delta$ = - ρ/r (cos δ sin Φ'- sin δ cos Φ' cos H) - (2)

Regarding earth as a sphere of radius a, we can write ρ and Φ' instead of a and ϕ.

Similarly parallax in longitude (along ecliptic) and latitude (śara) can be calculated by considering P as pole of the ecliptic. Then great circle through P and Z′ will cut the ecliptic at T called 'tribhona' lagna as it is 90° less than the rising point of

ecliptic on horizon or lagna. Hence T = Lagna -90°. If t is distance between Z and ecliptic (at T), then it is sara of z or declination of T (tribhona). PZ' = 90° -t then. In stead of nata kāla H we take distance of moon from tribhona i.e. v and β is latitude in stead of kranti .

Then (1) becomees, Δ 1 = lambana

$\Delta l = - \rho/r \cos t \text{ Sin } v. \text{ Sec } \beta$ - - - (3)

At eclipse time, β = almost 0 and sec β = 1.

Equation (2) becomes

$\Delta\beta = \rho/r (\cos \beta. \text{ Sin } t - \text{Sin } \beta. \cos t. \cos v)$ ---(4)

At eclipse time β = 0 (almost), so cos β = 1, Sin β = 0

$\Delta\beta = - \rho / r \sin t.$ - - - - (4a)

(f) Stellar parallax :

In figure 22, let X' be a star, S the Sun and E the earth. Let EX be parallel to SX'. Then EX is the true direction of the star, viz its direction as seen from the sun and EX' is the apparent direction, viz, the direction of X' as seen by the observer on the earth. The difference between these directions is the angle X'EX which is equal to the angle SX'E.

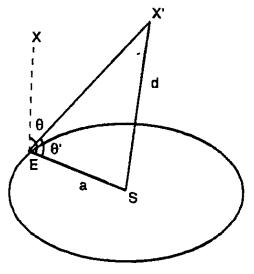



Figure 22 - Stellar purallax

Let $\angle$ SEX′ = θ' $\angle$ SEX = θ

SE = a and SX′ = d

Then from the triangle EX′S in which $\angle$EX′S = θ-θ', we have

$\text{Sin } (\theta-\theta') = (a/d) \text{ Sin } \theta$ - - - - (1)

Let a/d = Sin $\prod$: then II is called the star's parallax (helio centric or annual parallax). Neglecting second and higher powers of the small quantities $\theta-\theta'$ and II, (1) becomes

$\theta-\theta'$ = II sin θ

which gives the displacement of the star due to parallax.

EX, EX′ and ES are in same plane, so X, X1 and S are on the same great circle in celestial sphere of the observer. Thus the displacement of star XX′ on sphere = $\prod$ Sin XS (S is direction of sun on sphere).

Parallax in longitude and latitude -

In figure 23, X is true position of star in celestial sphere, as seen from Sun at S.

X′ is its apparent position affected by parallax as seen from earth (ceentre of the sphere).

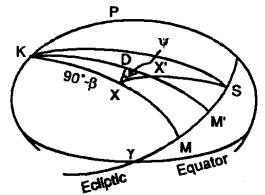



Figure 23
Longitude and latitude of stellar parallax

MM′ is ecliptic and K its pole. M, M′ are the points on ecliptic at which KX and KX′ cut. XD is perpendicular from X on KM′

Let $\angle$ X′ X D = Ψ

Parallax is II,

λ, β and λ', β', are longitude and latitude of X and X′, then

$\Delta \lambda = \lambda' - \lambda = \text{X D Sec}\, \beta = \text{XX}' \cos\Psi \sec \beta$

$= \Pi \sin \times \text{s.} \cos \Psi \sec \beta$

$= \Pi \sin \text{ MS Sec } \beta$

from the Δ XMS, in which ∠X = 90° - Ψ

i.e. $\Delta\lambda = \text{II Sin } (\theta - \lambda) \text{ Sec } \beta$ ....(2)

where θ is the longitude of the sun.

Similarly, $\Delta\beta = \beta' - \beta = -\text{ X}'\text{D}$

$= -\text{ XX sin } \Psi$

$= -\text{ II Sin XS. Sin } \Psi$

$= -\text{ II Sin } \beta \cos (\theta - \lambda)$ .... (3)

by applying sine cosmic formula to ΔKXS

Parallactic eclipse : If we take X, true position of star as origin, XK as the y-axis, where K is pole of ecliptic and XD (perpendicular to XK) as x-axis, the coordinates (x,y) of the apparent position X′ of the star are given by

$x = \text{XD} = \text{II Sin } (\theta - \lambda)$

and $y = -\text{ X}'\text{D} = -\text{ II Sin } \beta.\ \cos (\theta - \lambda)$

Eliminating θ, we see that locus of (x,y) is the ellipse

$$\frac{x^2}{\Pi^2} + \frac{y^2}{\Pi^2 \sin^2 \beta} = 1$$

During the course of a year, the star appears to describe this ellipse, which is known as the parallactic eellipse.

If M, M′ are taken as positions of X, X^1 on equator and T is position of sun on equator, then right ascension and declination can be similarly calculated.

Δ a = II (cos a cos ε Sin θ - Sin aδ cosθ) Secδ

$\Delta\delta$ = II (Cosδ Sin ε Sinθ - Cos a Sinδ Cos θ - Sin a Sin δ cos ε. sin θ).

The parallax is used to measure stellar distances. Star is seen from the two positions of earth six months away (i.e. 180° away in its orbit). Direction of a star is seen with respect to a far i.e. faint star. Nearest star has parallax of only 0".76 corresponding to a distance of

$$93,000,000 \div \frac{0.76 \times \pi}{60 \times 60 \times 180} \text{ miles} = 2.55 \text{ X } 10^{13} \text{ miles}$$

This is used to define steller distance in units of parsec which is the distance for which stellar parallax will be 1". Another unit is light year, which is the distance travelled by light in 1 year at speed of 1,86,000 miles/sec

1 par sec = 19 X 10^{12} miles.

1 light year = 6 X 10^{12}

Stellar parallax is not used in siddhānta texts, but have been indicated only to show the other kind of parallax. Only in golādhyāya it has been mentioned (also in discussion of śīghra paridhi in chapter 51 that stars are 360 times the distance of sun. This distance is much more and its parallax is no way connected to change of śīghra paridhis in different quadrants.

(4) (a) **Refraction :** The apparent direction of any planet or star changes due to bending of rays coming from that on earth due to refraction in its

atmosphere. This is called 'Valana' in siddhānta astronomy and is calculated empirically.

Effect of parallax (nati in krānti or lambana in longitude) is to shift the planet away from zenith. But due to refraction (valana), the planet appears higher i.e. closer to zenith. Both are maximum at horizon and zero for zenith.

It is difficult to make exact calculation on the basis of refraction rules, even according to modern theories of physics. We obtain some formula after some simplifying assumptions about variations in density and refractive index of different layers of atmosphere. In siddhānta books, calculations are based on practical observations and the correction is assumed to vary according to natajyā as in parallax.

According to modern electromagentic theory, refraction of light is due to its reduction of speed, when it enters a material medium from vacuum. Since it is an electromagnetic wave, its speed is reduced due to dielectric properties of the medium, which has effect like resistance. The reduction in speed is more in denser medium. Ratios of speeds is called refractive index.

$$\frac{\text{Speed of light in vaccum}}{\text{Speed in dense medium}} = \mu = \text{Refractive index.}$$

Since speed of light is maximum in vacuum, μ is always greater than 1. When it comes from a lighter medium to material of higher deensity, then also its speed is reduced

$$\frac{\text{Speed in medium A}}{\text{Speed of light in medium B}} \quad \frac{\mu_1}{\mu_2} = \text{constant}$$

μ, μ_1, and μ_2 are constants for the mediums and increase with their density. μ_1 and μ_2 are refractive index of mediums A, B.

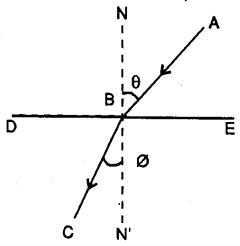



Figure 24 - Plane refraction

Due to wave nature of light, a ray AB entering a denser medium at B, bends towards normal NN′ to the boundary surface DE. If its angle of incidence with normal is θ and angle of refraction Φ then (figure 24)

$$\frac{\sin \theta}{\sin \Phi} = \mu = \frac{\mu_1}{\mu_2}$$

This is a constant depending only on the optical properties of the two media.

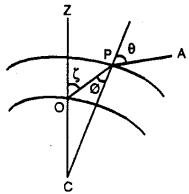



Figure 25 Cassini refraction

4. (b) Atmosphere assumed homogenous shell- This is called Cassini's hypothesis and is

simplest assumption. In figure 24, let O be the observer on the earth, A a star (or planet) and APO a ray which reaches O after refraction at P on the upper surface of the atmosphere. Let μ be the refractive index of the atmosphere. Then the angles being as marked in the figure.

$$\text{Sin } \theta = \mu \text{ Sin } \Phi \text{ - - - (1)}$$

But from the plane triangle OPC, if radius of earth is a, and the height of the atmosphere is h, so that CO = a, CP = a+h, we have

$$\text{Sin } \theta = \mu \text{ Sin } \Phi$$

$$\frac{\sin \zeta}{a + h} = \frac{\sin \Phi}{a} \quad \text{- - (2)}$$

$$\text{Refraction R} = \theta - \Phi \quad \text{- - (3)}$$

To eliminate θ and Φ, from (1) and (3)

$$\text{Sin (R+}\Phi) = \mu \text{ Sin } \Phi$$

or approximately, for small R, Sin R=R, Cos R = 1

$$\text{R cos } \Phi + \text{Sin } \Phi = \mu \text{ Sin } \Phi$$

$$\text{Therefore R} = (\mu - 1) \tan \Phi$$

$$= \frac{(\mu - 1)\, a \sin \zeta}{[(a + h)^2 - a^2 \sin^2 \zeta]^{1/2}} \text{ by (2)}$$

$$= \frac{(\mu - 1) \sin \zeta}{[\cos^2 \zeta + 2\,(h/a)]^{1/2}} \text{ approximately}$$

$$= (\mu - 1) \tan \zeta \; [1 + (2h/a) \sec^2 \zeta]^{-1/2}$$

approximately

$$= (\mu - 1) \tan \zeta \,[1 - (h/a) \sec^2 \zeta]$$

$$= (\mu - 1) \tan \zeta \,[1 - (h/a)\,(1 + \tan^2 \zeta)]$$

which is of the form

$$R = A \tan \zeta + B \tan^3 \zeta$$

The simple formula R = K tan ζ is true for values of ζ not exceeding about 45°, this formula is true for values upto 75°.

4. (c) Concentric layers of varying density : This assumption also gives the same formula, by an approximate method.

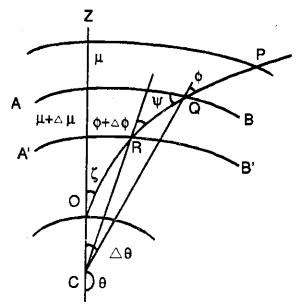



Figure 26
Concentric layers of varying density

Suppose that any layer of the atmosphere is bounded by concentric spherical surfaces AB, A′B′ and that PQR is a portion of a ray of light which finally reeaches the observer O on the surface of the earth.

Let C = Centre of earth, CQ = r, CR=r+ Δr Then the normal at Q to the surface AB is CQ. The angles and refractive indices are as marked in the figure.

From the laws of refraction

$\mu \sin \Phi = (\mu + \Delta \mu) \sin \Psi$ – .. (1)

From plane ΔCRQ

$$\frac{\sin (\phi + \Delta \Phi)}{\mathrm{r}} = \frac{\sin \Psi}{r + \Delta r}$$

Eliminating Ψ, we get

μ r sin Φ = $(\mu + \Delta\mu)$ (r + Δr) sin $(\Phi + \Delta\Phi)$

As this relation is true for any two consecutive layers, μ r sin Φ has the same value for every layer.

But on surface of earth, r = a (radius of earth)

$\Phi = \zeta$ (apparent zenith distance)

$\mu = \mu_0$, (say), depending on density and temperature of atmosphere, so

μ r sin Φ = μ_0 a sin ζ .....(2)

Amount of refraction at Q (say ΔR) = $\Phi - \Psi$ so (1) gives.

$\mu \sin\Phi = (\mu + \Delta\mu)\sin(\Phi - \Delta R)$

$= (\mu + \Delta\mu)(\sin\Phi - \Delta R\cos\Phi)$ approximately.

$= \mu\sin\Phi + \Delta\mu\sin\Phi - \Delta R.\ \mu\cos\Phi$

so, $\Delta R = (\Delta\mu/\mu)\tan\Phi$

Eliminating Φ with help of (2), we have

$$\Delta R = \frac{a\mu_o \sin\zeta}{\mu\,(r^2\mu^2 - a^2\mu_0^2\sin^2\zeta)^{1/2}} \times \Delta\mu \quad ..... (4)$$

To solve the differential equation (4), we assume

$$\frac{r}{a} = 1 + s$$

Where s is small, because the earth's atmosphere extends only to a comparatively small distance from earth's surface. Putting this in (4) and integrating, we get.

$$R = a\mu_0 \sin\zeta \int_1^{\mu_\circ} \mu^{-1} a^{-1} (\mu - \mu_o^2 \sin^2\zeta + 2s\,\mu^2)^{-1/2}\, d\mu$$

$$\text{or } R = \int_1^{\mu_o} \mu^{-2} (\mu^2 - \mu_o^2 \sin^2 \zeta)^{-1/2} \left[1 + \frac{2s\,\mu^2}{\mu^2 - \mu_0{}^2 \sin^2 \zeta}\right]^{-1/2} d\mu$$

neglecting higher powers of S.

It is assumed that z is sufficiently less than 90° to ensure that the denominator $\mu^2 - \mu_0{}^2 \sin^2 \zeta$ is not very small

$$\text{or } R = \mu_0 \sin \zeta \int_1^{\mu_0} \frac{d\mu}{\mu (\mu^2 - \mu_o^2 \sin^2 \zeta)^{1/2}}$$

$$-\mu_0 \sin \zeta \int_1^{\mu_0} \frac{s\,\mu\; d\mu}{(\mu^2 - \mu_0^2 \sin^2 \zeta)^{3/2}} \quad \dots (5)$$

To integrate first term, wee put $1/\mu = t$ then it is $\sin^{-1} (\mu_0 \sin \zeta) - \zeta$

i.e. $\sin^{-1}[(1+x) \text{ Sin } \zeta] - \zeta$, putting $\mu_0 = 1 + x$

To expand the first tern by Maclaurin's theorem,

let $f(x) = \text{Sin}^{-1} [(1+x) \text{ Sin } \zeta]$

$$\text{Then } f'(x) = \frac{\sin \zeta}{\sqrt{1 - (1 + x)^2 \sin^2 \zeta}}$$

Thus $f(O) = \text{Sin}^{-1} (\text{Sin } \zeta) = \zeta$

$$\text{and } f'(O) = \frac{\sin \zeta}{\sqrt{- \sin^2 \zeta}} = \tan \zeta$$

Thus the first term in (5) is equal to $x \tan \zeta$ approximately, neglecting higher powers of x.

Second term in (5), has a small quantity s as a factor. So its coefficient is changed slightly. Putting $\mu = \mu o = 1$ in it, the term becomes

$$-\frac{\sin\zeta}{\cos^3\zeta}\int_1^{\mu_0} s\,d\mu$$

Now by Gladstone and Dale's law

$\mu = 1 + c\rho$

where ρ is the density of the layer with refractive index μ, and c is a constant. This gives

$d\mu = c d\rho$

If ρ_0 is density of the air at surface of the earth, the second term becomes -

$$-c\frac{\sin\zeta}{\cos^3\zeta}\int_0^{s'} s\,d\rho$$

Integrating by parts, and supposing that s = s′ when ρ = O, this becomes

$$-C\frac{\sin\zeta}{\cos^3\zeta}\int_0^{s^1} \rho\,d\,s$$

The intgrated part vanishes at both limits (ρ = 0 at one limit and s = o at other). The remaining integral is equal to mass of a column of air of unit cross section, extending from surface of the earth to the point P = O. It is, therefore, a constant and can be written as B $\tan\zeta\sec^2\zeta$, where B is a constant.

Thus $R = (\mu_0 - 1)\tan\zeta + B\tan\zeta(1+\tan^2\zeta)$

which is of the form

$R = A\tan\zeta + B\tan^3\zeta$

Bradley's formula : He assumed

$r\,\mu^{n+1}$ = constant

Also $\mu r\sin\phi$ = constant - from equation (2)

Therefore, by division

$$\frac{\mu^{n}}{\sin \Phi} = \text{const.} \quad --- (6)$$

By logarithmic differentiation

$$\frac{n}{\mu} = \cos \Phi \cdot \frac{d\phi}{d\mu}$$

From equation (3), $\frac{dR}{d\mu} = \frac{1}{\mu} \tan \Phi$

From these two equations

$dR = (1/n)\, d\Phi$

Integrating from the surface of the earth (where r=a, $\mu = \mu_0$ and $\Phi = \zeta$) to the upper boundary of the atmosphere (where μ=1, r=r′ and $\Phi' = \Phi'$ assumed)

we get $R = 1/n\,(\zeta - \Phi') \quad --- (7)$

From (6), $\frac{\mu_0^{n}}{\sin \zeta} = \frac{1}{\sin \Phi'}$

i.e. $\sin \Phi' = \frac{\sin \zeta}{\mu_0^{n}}$

Then (7) becomes, $R = \frac{1}{n}\,[\zeta - \sin^{-1} \frac{(\sin \zeta)}{\mu_0^{n}}]$

This is known as Simpson's formula

This can be written as

$$\frac{\sin \zeta}{\sin(\zeta - nR)} = \mu_0^{n}$$

or $\frac{\sin \zeta - \sin(\zeta - nR)}{\sin \zeta + \sin(\zeta + nR)} = \frac{\mu_0^{n} - 1}{\mu_0^{n} + 1}$

or $\tan \frac{1}{2} nR = \frac{\mu_0^{n} - 1}{\mu_0^{n} + 1} \tan(\zeta - \frac{1}{2} nR)$

Writing 1/2 nR for tan 1/2 nR we get

$$R = \frac{2(\mu_0{}^n - 1)}{n(\mu_0{}^n + 1)} \tan(\zeta - \frac{1}{2} nR)$$

This is Bradley's formula.

4. (d) Determination of constants - In figure 27, let X_1 and X_2 be true positions of a circumpolar star at its upper and lower culminations (positions on meridian). Then

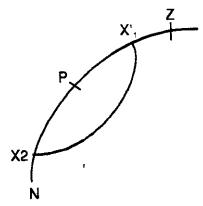



Figure 27

$P X_1 = PX_2 = 90° - \delta$,

$PZ = 90° - \Phi$

Therefore, $ZX_1 = (90° - \phi) - (90° - \delta) = \delta - \phi$

$ZX_2 = 90° - \phi + 90° - \delta = 180° - \phi - \delta$

Hence $ZX_1 + ZX_2 = 180° - 2\phi$ ..... (1)

If the apparent zenith distances at upper and lower culminations are ζ and ζ' then

$ZX_1 + ZX_2 = Z\zeta + Z\zeta'$ .... (1a)

$Z\zeta = \zeta + A \tan \zeta + B \tan^3 \zeta$

$Z\zeta' = \zeta' + A \tan\zeta' + B \tan^3 \zeta'$

Putting this value in (1) we get one equation in ζ and $- \zeta'$. Equation of two more such stars will be used to determine A,B and Φ .

Numerical values of A and B for a pressure of 30″ of murcury and temperature of 50° F (or 10°c) are 58″.294 and – 0″. 0668.

For values of ζ greater than 75°, special tables are used based on observations. The refraction when a body is in the horizon is called the horizontal refraction, and its value is about 35′.

From equation it will be ∞ for $\zeta = 90°$ as $\tan 90° = \infty$, hence equation is not correct for such values.

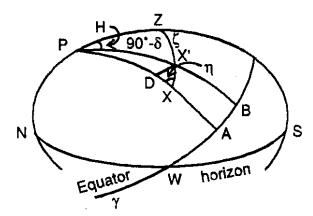



Figure 28

(e) Refraction in viṣuva aṁśa and Krānti - In figure28, let X be the true position of a star and X′ its apparnt position as affected by refraction. Then ZX′ X is a great circle and $XX' = K \tan \zeta$ where ζ is the apparent zenith distance ZX′. Let the hour angle (natāṁśa) and krānti (declination) of X be H and δ for X′ these be H′ and δ'.

Join PX; PX′ and produce them to meet the equator in A and B. Draw X′D perpendicular to PM. Then, since XX′ is small, the trianlge XX′ D may be regarded as a plane triangle.

Now the correction to be added to the apparent right ascension a′ to obtaint the true right ascension (Viṣuvāṁśa) a is a-a′. But

a-a′ = - AB = - X′ D sec X′D

(as X′ D is almost equal to arc X′ D with centre P)

$= - X'X \text{ Sin } \eta \text{ Sec } \delta'$

$= - K \tan \zeta \sin \eta \sec \delta'$

η is given in ΔPZX, by sine relation

$$\frac{\sin (90^\circ - \Phi)}{\sin \eta} = \frac{\sin H}{\sin \zeta}$$

as $PZ = 90^\circ - \phi$, $ZX = \zeta$, so,

$\sin \eta = \text{Sin} \zeta \cos \phi \text{ Sin H}$

Similarly the correction to be applied (added) to δ' is $\delta - \delta'$ But $\delta - \delta' = - DX = - XX' \cos \eta$

$= - K \tan \zeta \, . \cos \eta$

4. (f) Effect of refraction on sun rise and sun- set

Hour angle (natāmśa) H of sun's centre when rising is (Figure 29)

$\text{CosH} = - \tan \phi \tan \delta \quad - - - (1)$

where ϕ is latitude of the place and δ is declination (krānti).

Let $H + \Delta H$ be the natāmśa of true sun when the apparent sun is rising. At this instant, the true sun is really 35′ below the horizon, its true zenith distance being 90° 35′. Hence, from the $\Delta P S'Z$

$\cos (90^\circ \; 35') = \text{Sin} \phi \text{ Sin } \delta + \cos \phi \cos \delta \cos(H + \Delta H)$

or, $- \sin 35' = \text{Sin } \phi \text{ Sin } \delta + \cos \phi \, . \cos \delta \, (\cos H - \Delta H . \sin H)$

nearly

or $-- \text{Sin } 35' = - \Delta H . \text{ Sin H} . \cos \phi \cos \delta$

by (1)

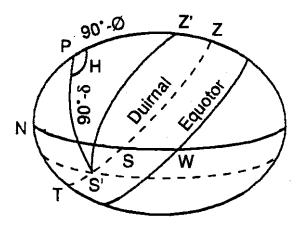



Figure 29

This will give the advance time of sun rise H in radian, it will be divided by sin 1′ to get the value in asu.

(1) is obtained from equation for natakāla cos H. cos Φ . cos δ = cos z – sin φ. sin δ

At sun rise time, z = 90°, cos z = 0

Thus apparent day length is increased and if sun rise at parama krānti time is measured, it gives a higher value of parama krānti. This may be one of the reasons for assuming its value as 24° instead of 23°27′.

4. (g) Shape of sun's disc at sunrise or sunset - Lower limb of the sun is at a greater zenith distance than the upper. Hence due to refraction, the lower limb is raised more than the upper. Thus the sun appears flattened. This effect is maximum when sun is near the horizon.

Let S be sun's centre, a its radius and P any point on sun's limb. (figure 30)

Let ZS = z and let PQ be the perpendicular from P on ZS.

On account of refraction, let P be displaced to P′ and let P′Q′ be the perpendicular from P′

on ZS. Then, since QP is small, the zenith distances of Pand Q are the same. So PQ will be displaced to P′Q′.

Now take SZ as the X axis and perpendicular to it through S as the y axis. Then if the coordinate of P′ are (x,y), we have

$x = SQ' = a \cos \Psi + QQ'$

$= a \cos\Psi + K \tan (z\text{-}a \cos \Psi)$

$= a \cos + K (\tan z - a \cos \Psi \sec^2 z)$ - - - (1)

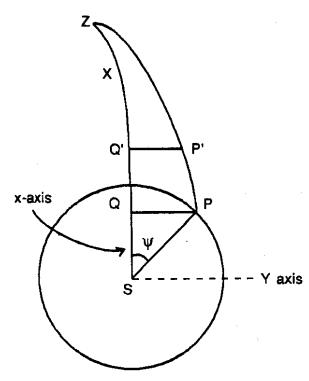



Figure 30 - Sun disc at rising time

$PP' \doteq K \tan z$ where $K = \mu - 1$

Its component along PQ is

$K \tan z \cos \Psi$ - - - (2)

But from right angled triangle ZPQ

$\cos \Psi = \tan PQ. \cot z.$

Hence resolved part of refraction in PQ dirction is

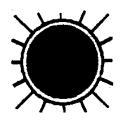
$K \tan PQ = K.PQ$, since PQ is small

$y = P'Q' = PQ - K.\ PQ$

$= (1-K).\ PQ = a\ (1-K)\ \text{Sin}\Psi$ - - - (3)

Eliminating Ψ from (1) and (3) , we see that the apparent figure of the sun is the ellipse

$$\frac{(x - k \tan z)^2}{(a - ak \sec^2 z)^2} + \frac{y^2}{a^2 (1 - k)^2} = 1$$

Thus sun appears elliptical at sunrise and sunset.

Chapter - 8

LUNAR ECLIPSE

Candragrahaṇa Varṇana

Verse 1 : According to views of smārtta, vedic, purāṇa knowers, there are unlimited good results from auspicious works at the time of grahaṇa (eclipse) like bath, homa, charities etc. People repose faith on tithi calculations after seeing eclipse as predicted. Due to this importance, eclipse (solar and lunar) is described now.

Notes : (1) This chapter describes the general methods applicable both to solar and lunar eclipse. Calculation of solar eclipse needs some special methods, which will be discussed in next chapter, named sūrya grahaṇa.

(2) Auspicious effects of grahaṇa are subject of 3rd part of Jyotiṣa called saṁhitā and need not be discussed here. However, calculation of grahaṇa is a very complicated process. If such a rare event occurs as predicted by calculations, it is an excellant proof of correctness of theories and formulas.

Verses 2-6 : Possibility of eclipse.

Lunar eclipse - At the ending time of Purṇimā (when moon-sun = 180° exactly), difference of moon with rāhu and ketu is calculated. When this differncee is less than 13°, then lunar eclipse is possible.

Solar eclipse - Similarly, at the end of amāvasyā (when moon - sun = 0°), moon and its pāta (rāhu or ketu) are calculated. Difference of moon from any of the pāta being less than 18°, solar eclipse is probable.

We calculate amānta time (when sun=moon), from earth's centre. Paścima nata of candra X 1/3 is substracted from this time and we again correct the true moon at this corrected amānta time.

Again we calculate, vitribha (tenth lagna) for this time. 1/60 of its natajyā is added to second true moon of this time, when moon and nata are in same direction. We substract, when they are in different directions. If this is less than 34 then, solar eclipse is probable.

Sometimes, when south nati (in meridian circle) is less than 1°30′ then solar eclipse is probable. When dṛgvṛtta is krānti vṛtta, then difference of candra and its pāta being less than 7°, solar eclipse is possible.

Notes : (1) Reason of eclipse - When moon passes into the earth's shadow, it fails to receive light from sun. This causes an eclipse of moon. This can happen only when the sun and moon are on opposite sides of earth, i.e. on full moon time (Pūrṇima when moon-sun = 180°.)

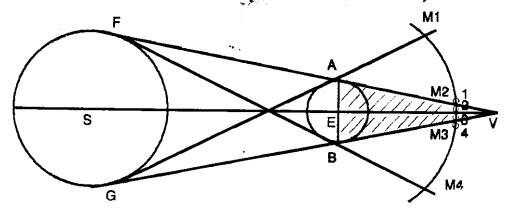



Figure 1 - Lumar eclipse

Let S be the centre of the sun, E of earth. The cone touching sun and earth has its vertex at V. Then the portion of cone from earth upto V is the shadow cone of earth called umbrā (bhūbhā). This is completely dark as no light from sun reaches in that portion.

Another cone is formed by tangents in transverse direction with vertex in opposite direction between earth and sun. The portion of this cone after earth and beyond umbrā (shadow) is partly dark and called penumbra (avatamasa).

M_1, M_4, are points on moon orbit at boundary of penumbra, M_2, M_3 on boundary of umbra. Between M_1 M_2 or M_3 M_4 portion, brilliance of moon is reduced, which are described as colour of eclipse but no eclipse is formed. In portion of orbit M_2 M_3 completely within the shadow cone of earth (bhūbhā), there is an eclipse.

At point 1, moon's disc just starts contact with, shadow, this is called first contact or 'sparśa' (touch) kāla. At point 2 moon's disc just enters completely in the shadow called second contact or 'nimīlana' or 'sammīlana' (closing the eyes). When complete eclipse is about to end i.e. moon's disc starts coming out of shadow at point 3, it is called third contact or 'Unmīlana' (opening the eyes.)

At point 4, moon completely comes out of shadow. It is called fourth contact or mokśa kāla (freedom time.)

When the moon is not completely eclipsed, the times of maximum eclipsed portion correspond to 2nd and 3rd contacts.

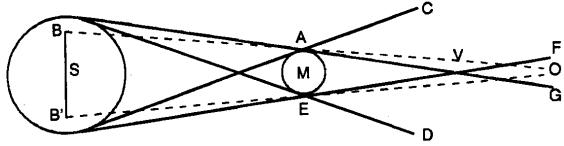



Figure 2 - Solar Eclipre

(2) Reasons of solar eclipse : An eclipse of sun is caused by moon coming in between the observer and the sun. If the whole of sun is hidden behind the moon, we have a total eclipse. If moon covers only part of suns disc, we have a partial eclipse. When apparent diameter of moon is smaller than sun in a total eclipse, the eclipsed part of sun is surrounded by visible circle of sun, it is called annular eclipse. These are called 'sarvagrāsa,. and, 'khaṇḍa grāsa' or kaṅkaṇa grahaṇa respectively. This can happen only on amāvasyā, when sun and moon are in same direction.

In figure 2, if observer is anywhere inside the shadow cone of moon AVE, the whole of sun is hidden from his view. If he is in the extended cone FVG, only the central part of sun is hidden by the moon. If the observer is within penumbra CAV or VAD (except FVG portion), he will see a partial eclipse of sun. It can be seen that at point O in extended shadow cone only the inner portion BB' of sun is obstructed. In this case, moon is smaller, so its shadow cone doesn't reach earth's surface.

In this eclipse also, sparśa or first contact is time when eclipse starts. 'Nimīlana' is time when maximum eclipse starts (or total eclipse) i.e. 2nd contact'. Unmīlana or 3rd contact is when maximum

or total eclipse is about to reduce. 'Mokśa', or 4th contact is time when sun is completely visible.

(3) Why eclipse doesn't occur on every pūrṇimā or amāvasyā?-

The inclination of moon's orbit to the ecliptic is about 5°. Hence the maximum distance of moon's centre from the ecliptic is 5°. Now the axis of the earth's shadow lies in the plane of the ecliptic. Moon's diamater is about 1/2° and diameter of earth's shadow at distance of moon is about 1-1/2°. So moon will touch the shadow, when its centre is at a distance from centre of shadow by less than 1/2 (1/2° + 1-1/2°) = 1° approx. Thus, for most of the time, moon passes clear out of the shadow.

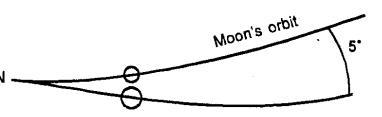



Figure 3 - Lunar eclipse not occurring

Eclipse is possible only when moon is near N, the point of intersection of its orbit with ecliptic. The northern point of intersection, from where orbit goes north of ecliptic is called rāhu and other southern pāta is caled ketu. Hence, rāhu and ketu are said to cause eclipse.

For solar eclipse also, sun and moon should be in the plane of ecliptic, so that moon's shadow touches the earth. Thus on every amāvasyā, when moon and sun are in same direction from earth, solar eclipse doesn't occur. Shadow of moon is almost a point when its shadow cone touches the earth or it may not touch at all. Thus its radius may be taken as zero, at distance of earth (from moon). Earth's radius makes an angle of about 1°

at moon. Hence as distance between shadow centre and earth centre less than 1°, solar eclipse is possible. Thus within similar distance of moon from its node, solar eclipse happens.

In solar eclipse, sun is not covered, it is only locally obstructed, like obstruction of a cloud. Away from shadow cone at a short distance, sun is visible because parallax shift of moon is 57′ compared to 8″.8 of sun, which is not obstructed there.

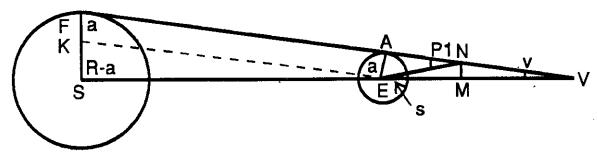



Figure 4 - Earth's shadow in moon's orbit

(4) Size of earth's shadow in moon's orbit.

S and E are centres of sun and earth

V is vertex of shadow (umbral) cone of earth.

FA is one of generators of cone and v its semi vertical angle.

Let moon touch the umbral cone at N and NM be perpendicular to EV.

Then s, the angle subtended by NM at E, is the angular radius of earth's shadow at the distance of moon.

$\angle ENA = P_1$ = horizontal parallax of moon approximately as AE is almost perpendicular to EM.

S = sun's angular semidiamter, P = horizontal parallax of the sun = a/r.

a = radius of earth

r = distance of S from E. (sun from earth).

R = radius of sun.

Then s = $P_1 - \nu$ from ENV of which P_1 is an exterior angle.

= P_1 - ∠KES, if KE | | AF

= P_1-KS/SE nearly as SF is almost perp. to SE

= P_1 - (R-a) /r

= P + P_1 - S.

or s = P + P_1 - S

This gives the theoretical value of s, but it is found that actual observations give the value 2%. larger, because earth's atmosphere absorbs light.

Angular radius of the penumbra at the distance of moon can be shown similarly to be P+P_1+S (S is angular semidiameter).

Approximate value of radius of shadow is about 42′ after adding 2% for atmosphere. It varies with change in distance of sun and moon from earth.

As moon moves 360° in 29-1/2 days with respect to sun, i.e. with respect to shadow, it will be fully in shadow till it covers (diameter of shadow - diameter of moon) = 2X42′ – 30′ = 54′ approx. The time in covereing the distance.

$$\frac{54}{60 \times 360} \times \frac{59}{2} \times 24 \text{ hours} = 1\frac{3}{4} \text{ hours approx.}$$

This is the maximum duration of a total lunar eclipse.

(5) **Ecliptic limits of Moon**—Figure 5 is celestial sphere part for observer. N is node of moon's orbit. C is centre of earth's shadow on

ecliptic. M is centre of moon. Moon's orbit meets ecliptic at N which is its node. Angle betwen the orbits is i.

In the diagram moon is just touching shadow. If C was C_1 when M was at N, then NC_1 is called the lunar ecliptic limit. If shadow is nearer then moon will definitely pass through the shadow. If C is away, moon cannot touch it and there will be no eclipse. Since M moves about 13 times faster, only moon's motion is being discussed.

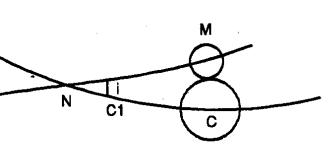



Figure 5 - Ecliptic Limit of moon

As sun is diametrically opposite to C_1 and other node of moon's orbit is opposite to N_1 lunar ecliptic limit is also the distance of sun's centre from nearer node of moon's orbit at the instant, moon is crossing the ecliptic.

Let NC = x when moon is crossing the ecliptic. Let n, n_1 be the angular velocities of the sun and the moon (radian per hour) in planes of their orbits. Let the time counted from moon's centre passing through node be t hours. Then at time t,

$NC = x + nt$, $NM = n_1t$

Taking NCM as a plane triangle,

$CM^2 = (x+nt)^2 + (n_1^2t^2) - 2n_1t\,(x+nt) \cos i$ - (1)

CM is a minimum when t is given by

$2\,(x+nt)\,n + 2n_1^2t - 2n_1\,x \cos i - 4\,n_1nt \cos i = 0$

by differentiating equation (1) with respect to t.

Substituting the value of t given by this in (1) and simplifying, minimum value of CM is

$$\frac{x\, n_1 \sin i}{(n^2 + n_1^2 - 2nn_1 \cos i)^{1/2}} \quad \text{- - - - (2)}$$

When moon just grazes the earth's shadow in its course along its orbit, the minimum value of CM - must be equal to the sum of the radii of shadow and moon. Hence (2) is equated with

$$\frac{51}{50}(P + P_1 - S) + S_1$$

where S and S_1 are angular semi diameters of sun and moon, P and P_1 are equatorial horizontal parallax of sun and moon.

As all the quantities P, P_1, S and S_1 are variable, the lunar ecliptic limit also varies. Its greatest value, called the superior ecliptic limit is 12°.1 and the least value, called the inferior ecliptic limit is 9°.5 These limits are for a partial eclipse.

By equating (2) to the difference of radii of the shadow and the moon, we can find limits for a total lunar eclipse.

(6) Commencement of solar eclipse

When partial eclipse of sun starts, the transverse common tanquent BA touching sun and

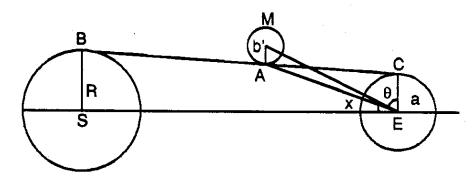



Figure 6 - Start of solar Eclipse

moon at B and A respectively just touches earth somewhere, say at C.

Let a, b and R be the linear radii of earth, moon and the sun.

ES = r, EM = r_1 and ∠ MEC = θ,

∠MES = x

Then $r \cos(\theta + x) + R = a$ - - - (1)

$r_1 \text{Cos}\,\theta = a + b$ - - - (2)

Divide (1) by r and (2) by r_1 and substract.

We get $\cos\theta - \cos(\theta + x) = \frac{a}{r_1} + \frac{b}{r_1} - \frac{a}{r} + \frac{R}{r}$

or $2 \sin\frac{x}{2} \sin\left(\theta + \frac{x}{2}\right) = \frac{a}{r_1} + \frac{b}{r_1} - \frac{a}{r} + \frac{R}{r}$

As x is small and θ is nearly 90°, this gives, approximately,

$x = P_1 + S_1 - P + S$

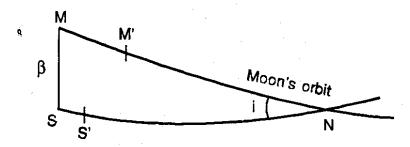



Figure 7 - solar ecliptic limit

Solar Eclipstic Limits - The solar ecliptic limit is the distance of the sun from the nearer node of the moon's orbit, at the moment of new moon, if a solar eclipse is just possible on this occasion.

Let MN be moon's orbit and SN the ecliptic, so that N is a node of the moon's orbit and let the inclination of the moon's orbit to the ecliptic be i. = ∠MNS.

Let M and S be centres of moon and sun at the instant of a new moon occuring when the sun is near N. By the definition of a new moon (amāvasyā), longitudes (rāśi) of M and S are the same. Let β be the latitude of moon when at M.

Let M′, S′ be the positions of the moon and the sun t hours later, and MSN is taken as a plane triangle.

Let $MM' = x$

Then change in moon's longitude in t hours is $x \cos i$.

Then change in the sun's longitude in t hours, i.e. SS′ is $m x \cos i$, where

$$m = \frac{\text{rate of change of sun's longitude}}{\text{rate of change of moon's longitude}}$$

Then $S'N = SN\text{-}SS' = \beta \cot i - mx \cos i$ and $M'N = \beta \operatorname{cosec} i - x$

If $M'S' = D$, we have

$$D^2 = (\beta \cot i\text{-}mx \cos i)^2 + (\beta \operatorname{cosec} i - x)^2$$
$$- 2 \cos i\, (\beta \cos i - mx \cos i)\, (\beta \operatorname{cosec} i\text{-}x) \quad \text{- (1)}$$

Only variable in this is x. Differentiating it with respect to x, minimum value of D is given by

$$(\beta \cot i - mx \cos i)(-m \cos i) - (\beta \operatorname{cosec} i - x)$$
$$- \cos i\, (-\beta \cot i - m\, \beta \cos i \operatorname{cosec} i + 2\, mx \cos i)$$
$$= 0$$

$$\text{or } x = \frac{\beta \sin i}{1 - 2\, m \cos^2 i + m^2 \cos^2 i}$$

Substitution in (1) shows that the smallest value of D is

$$\frac{(1 - m)\,\beta \cos i}{(1 - 2m \cos^2 i + m^2 \cos^2 i)^{1/2}}$$

When numerical values of m and i are substituted, it is seen that the value of this expression is very nearly β cosi, i.e. the value after supposing m = 0 (i.e. very small speed of sun). Putting, therefore,

$$\beta \cos i = S + S_1 + P_1 - P$$

We have the condition that the sun just misses being eclipsed. This gives

$$\beta = (S + S_1 + P_1 - P) \text{ Sec } i$$

as critical value of β within which β should be for an eclipse to be seen in some part of earth.

Solar ecliptic limit is the corresponding value of

$$SN = (S+S_1+P_1-P) \text{ cosec } i$$

Its greatest value is 18°.5 i.e. the superior solar ecliptic limit; its least value is 15°.4 which is the inferior ecliptic limit of sun.

Thus the text mentions only the superior ecliptic limits of sun and moon as 18° and 13°. If distance of sun from the node is more than this; eclipse is impossible, if it is less than the lower ecliptic limit 15°.4 or 9°.5 eclipse is certain at new moon or pūrnimā. If distance of sun is within inferior and superior ecliptic limits on new or full moon, solar or lunar eclipse may or may not happen. Further checking should be done at the ending times of pūrṇimā or amāvasyā by lambana (parallax) of sun and moon and their true diameters and speed.

(7) **The other condition of solar eclipse** is for a local place. The solar eclipse may happen, but it will be visible for only a small belt on earth's surface through which moon's shadow cone passes.

When sun and moon are in same direction from earth's centre, the eclipse will be visible from a place where difference in parallax of moon and sun is less than the sum of their semi diameters (= 34′) approx.

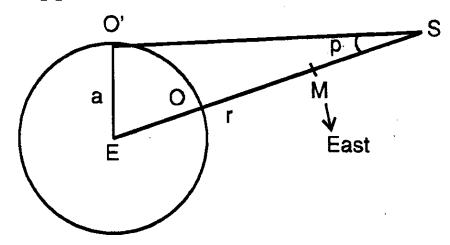



Figure 8 - parallax in amāvasyā time

Figure 8 shows the position of true amāvasyā, when moon M and Sun S are in same direction from earth's centre E. When observer is at O in this line, i.e. when moon and sun are on zenith, then the same position remains. When observer is at O′, moon is ahead of sun towards east by p = ∠ O′SE of parallax. Thus moon will be in same direction with sun slightly before true position, at true time it goes ahead. Thus for east nata amavasyā time is before true time and in west nata it will be after true amāvasyā time.

Sūrya siddhānta has assumed horizontal parallax as 1/15th of the daily motion of a planet, on assumption that the (speed X distance) for the planet is constant. Linear speed of every planet is

assumed to be same and it comes ot to be (15 X radius of earth) as explained in 2nd part of this book. For moon this gives correct parallax but gives great error for other planets, due to wrong assumption of distances. Maximum parallax are compared below in vikalā.

Planet	Bhāskara II	Modern value	
		Minimum	Maximum
Sun	236.5	8.7	9.0
Moon	3162.3	3186	3720
Mars	125.7	3.5	16.9
Budha	982.1	6.4	14.4
Guru	20.0	1.4	2.1
Śukrā	384.5	5.0	31.4
Śani	8.0	0.8	1.0

Siddhānta darpaṇa has corrected the values for sun and moon (through still assuming same linear speeds)

Horizontal parallax for moon = 3388″.22

Horizontal parallax for Sun 31.63

Change in sun's parallax is an improvement caused due to taking higher value of sun's diameter as mentioned in Atharva veda. But still it is about 3.6 times the true value.

Changed formula for parallax are

$$\text{Sun max parallax} = \frac{\text{Daily speed}}{164}$$

$$\text{Moon max parallax} = \frac{\text{Daily speed}}{14}$$

Thus the parallax of moon is the distance travelled by it in 4 ghaṭī (60 ghati in a day/15)

according to sūrya siddhānta and in 4/17 ghaṭi according to this book.

For rough calculation, appendix 3(e) after chapter 7 gives the formula (3) as

$$\Delta l = -\frac{\rho}{r} \cos t \sin v$$

ρ/r = max. parallax, v is distance from 'Tribhona' lagna, which is taken as zenith as first step.

Then the correction in ghaṭi is

4.28 X cos t. X Sin H

where H = nata kāla

For 45° nata (middle position between meridian and west horizon), sin H = $1/\sqrt{2}$, H = 15/2 = 7.5 ghati.

This positive correction for paścima nata will be 2.5 ghati or 1/3 of nata kala if t = taken 30° (nata of tribhona)

Parallax in śara = ρ/r sin t.

Parama nati = 1°/60 approximately, hence 1/60 of natajyā of vitribha or tribhona lagna is added for calculating śara difference of moon. Assuming nil śara at eclipse time, this can be maximum of 34′ for an eclipse to be possible at that place.

(8) **Other condition for solar eclipse** - When sun is moving on east west vertical line, its krānti being equal to latitude of the place, its difference with moon when apparent longitudes are equal is the north south difference i.e. nati (parallax in śara or latitude). When it is less than 1/2 (sum of diameters of sun and moon) or 1°30′ then only solar eclipse can happen.

When Difference of moon and its pāta is less then 7° then also solar eclipse can happen. This is same as 1°30′ difference from ecliptic.

(9) Greatest and least number of eclipses in a year -

Ecliptic limits are as follows -

	Superior	Inferior
Lunar ecliptic limits	12°.1	9°.5
Solar ecliptic limits	18°.5	15°.4

1 Lunar month = 29.5 days

So, time from full moon to next new moon = 14.75 days.

Node of moon moves backwards, making one revolution in about 19 years. Hence sun makes one complete revolution with respect to node in 346.6 days. Thus, with respect to node, sun moves $\frac{360^0 \times 14.75}{346.6}$ or about 15°.3 in half a lunar month.

Figure 9 - No of eclipse

(A) Least number of eclipses -

Figure 9 shows the ecliptic and N, N′ are nodes of moon's orbit. Let $NS_1 = NS_2 = N'S_3 = N'S_4$.

= inferior solar ecliptic limit i.e 15°.4.

Let NM_1 = NM_2 = $N'M_3$ = $N'M_4$
= inferior lunar ecliptic limit i.e. 9°.4

Inferior limits have been chosen to find the most infavourable cases in which no eclipse occurs beyond these limits.

Movement of sun is in direction of arrow. S_1 S_2 = 2X15°.4 i.e. 30°8 but sun moves with respect to N by 2X15°.3 between two consecutive new moons. Thus in travel from S_1 to S_2 at least one solar eclipse is bound to occur, because there will be a definite new moon in 30.6 days and sun will be within limit of eclipse.

Suppose now that the eclipse occurs when sun is near N, then the sun will be outside NM_1 and NM_2 at previous and next full moons (i.e. 15°.3 away) while NM_1 = NM_2 = 9.5 only. Hence, there will be no lunar eclipse in previous or coming full moons. Thus there will be only one eclipse (solar) while sun crosses N.

Sun will be at N′ after about 1/2 X 346.6 = 173:.3 days after it has crossed N. Now 6 lunar months occupy 6X29.5 = 177 days. Therefore, about 4 days after the sun at N′,, there will be a new moon. Then sun is only 3.7 X 360°/346.6 =3.84 from N′ i.e. will within ecliptic limit of $N'S_4$. Thus there will be a solar eclipse. The preceding and succeding full moon occur out side M_3 M_4 as sun moves about 1° in a day. N_1M_3 = 9°.5 but N′S = 14.75 - 3.84 = 10.91 on previous full moon. In next full moon N′S = 14.75 + 3.84 = 18.59. Thus there are no lunar eclipses then.

If the year began shortly after the sun had crossed S_4, the year will end 365.25-346.6 days after

the same point relative to nodes, so the year will have ended much before sun comes near N again.

Hence in such circumstances, there will be only two eclipses in the year, both solar..

(B) Greatest number of eclipses in an year -

Now in figure 9, let $NM_1 = NM_2 = N'M_3 = N'M_4 = 12^\circ.1$ i.e superior lunar ecliptic limit.

$NS_1 = NS_2 = N'S_3 = N'S_4 = 18^\circ.5$, the superior solar ecliptic limit.

Suppose further that there is a new moon as soon as the sun enters S_1N. There will be a solar eclipse then. Time counted from the eclipse is indicated by H = half lunar month.. Then we have to examine solar eclipses at new moons at time 0, 2H, 4H, 6H - - - Similarly lunar eclipses are examined on full moons at times H, 3H, 5H - - -

(i) There is already a solar eclipse at $t = 0$

(ii) At $t = H$, Sun is at $15^\circ.3$ from S_1 and within M_1N at full moon, so there will be a lunar eclipse then.

(iii) At $t = 2H$, sun has advanced $2X15^\circ.3$ from S_1; so it is within NS_2 and there will be a solar eclipse.

At $t = 3H, 4H, - - - 11H$, the sun will be within S_2 and S_3 i.e. outside all the ecliptic limits, and there will be no eclipses.

(iv) At $t = 12$ H, sun will have advanced $12X15^\circ.3$ i.e. about 184° from S_1 i.e. 4° from S_3. So the sun is within S_3N' and there will be a solar eclipse.

(v) At t = 13H, sun will have advanced 4° + 15°.3 from S_3, so it is 19°.3-18°.5 = 0°.8 from N′ in $N'M_4$ and there will be a lunar eclipse.

(vi) At t = 14 H, - - - sun will be 0.8 + 15°.3 = 16°.1 from N′ i.e. will within $N'S_4$ = 18°.5. So there will be a solar eclipse.

At t = 15 H, 16 H, 23 H, the sun will be between S_4 and S_1, i.e. outside all ecliptic limits, and there will be no eclipses.

(vii) At t = 24H, sun will have advanced 2X4° = 8° from S_1, so it is within S_1N and there will be a solar eclipse.

(viii) At t = 25 H, the sun will have advanced 8°+15°.3 from S_1, so it is within NM_2, and there will be a lunar eclipse.

But this eighth eclipse occurs 14.75 X 25 days i.e. 368.75 days after the first eclipse, i.e. about a year and 3-1/2 days after the first. So out of 8 eclipses, Ist solar or 8 th lunar eclipse has to be ommitted in a year. Thus in a year there can be maximum of 5 solar+2 lunar or 4 solar + 3 lunar eclipses depending upon when the year began.

(10) Eclipse cycle : In Chaldea, before 400 BC, (may be in time of Sargon in 2350 BC approx,) a cycle was discovered after which eclipses were repeated. This was called Saros cycle of 18 years 10.5 days or 223 synodic lunar months.

223 synodic months = 6585.321 days

242 dracontic months = 6585.357 days

=19 X 346.62005 days (Dracontic year)

Draconitic year is revolution of sun with respect to lunar node and draconitic month is

revolution of sun with respect to its node. Nodes of moon were called Dragons.

Viśvāmitra had mentioned half cycle in Ṛkveda of 3339 tithis = 111 synodic months + 9 tithis.

Example of the cycle for least no. of eclipses in given below - (No lunar eclipse + 2 solar eclipses)

Years			Dates of solar eclipse	
1915	Feb. 14		Aug. 10	
1933	Feb. 24		Aug. 21	All annulus
1951	March 7		Sept. 1	
1922	March 28		Sept. 21	
1940	April 7	Ann.	Oct. 1	Total
1958	April 19		Oct. 12	
1926	Jan. 14		July 9	
1944	Jan. 25	Total	July 20	Annular
1962	Feb. 5		July 31	

Cycle of years of maximum eclipse

Years	Lunar Eclipse	Solar Eclipses
1917	Jan 8, July 4, Dec 28	Jan 23, Jun 19, July 19, Dec 14
1935	Jan 19, July 16, (Jan 8)	Feb 3, June 30, July 30, Dec 25
1953	Jan 29, July 26, (Jan 19) next year	Feb 14, July 11, Aug 4, (Jan5), next year
	Total Total Total	Part Part Part Annular

Actual determination of eclipse, is by calculating the extent of eclipse according to true speeds and śara as explained later.

Verse 6 : This book has used different methods for lambana correction for sphuṭa amānta (new moon day), true positions of sun and moon, dimensions of sun, moon and shadow, grāsa (covered) amount of moon, sthiti (total eclipse time)

vimarda (total time of complete or maximum eclipse), real true lambana, sphuṭa nati, digvalana and parilekha etc. This will be useful, so learned men should not think it to be incorrect.

Notes : Many of the methods have not been approved by earlier siddhānta works, but these methods give more correct results. Hence this needs to be accepted more eagerly, instead of rejecting it.

His methods for different methods of moon's motion has already been mentioned in chapter 6. Correction of moon's and sun's motion is also due to his revised values of manda paridhis which change continuously in quadrants. For moon, only one maximum value has been indicated and its ratio with least value should be increased. Earlier, either the manda paridhi was fixed or a fixed difference of 40′ was kept at the end of odd and even quadrants.

Lambana and nati formula have been corrected due to changed formula of maximum nati. For moon this is taken as 1/14th of daily motion instead of general formula of 1/15th of daily motion for all planets. For sun it is entirely changed to 1/164 of daily motion, which has no parallel in earlier texts. The correct variation of nati and lambana has been calculated instead of rough linear method.

Value of sun's diameter and consequently its distance from earth has been increased about 11 times the traditional value of 6,500 yojanas to 72,000 yojanas as mentioned in Atharvaveda. This has led to other changes in constants and methods. These corrections have been in right direction and more accurate.

Verses 7-8 - Correct time of parvānta -

On amānta or pūrṇānta day (moon-sun = 0° or 180°), sun and moon will be made sphuṭa (at sunrise or midnight time. For parva ending, only mandaphala correction is needed in moon. On amāvāsyā day, difference of moon and sun is taken, on pūrṇimā, it is moon - (Sin + 180°). Difference rāśi etc is converted to parā (1/60 vikalā) and is divided by difference of sphuṭa gati of moon and sun in kalā. Result will be in vighaṭī (pala).

This time in pala etc is added to parvānta time i.e. to sunrise time for which calculations had been done, if moon is less than sun (or sun+180° on purnimā). If moon is more, it will be substracted. Then we get the correct time (after or before sunrise for ending time of parva (pūrṇimā or amāvasyā). For this time, we again calculate sphuṭa moon and sun and from these values, correct parvānta time is calculated. After repeated applications of the method we get correct parvānta (for centre of earth). After that, other corrections for eclipse are made (like lambana or nati) for observation from surface of earth.

Notes : As first approximation speed at parvanta is assumed to be same as at sunrise time and accordingly correct time is calculated. Our aim is to find the time when moon-sun or moon-(sun+180°) is zero. If moon is less than this value, it will cover up the distance due to higher speed. The difference is in parā (1/60 vikalā), speed diff. is in kalā/day.

$$\text{Hence result time} = \frac{\text{parā}}{\text{kala/day}}$$

$$= \frac{\text{Kala} \times 60 \times 60}{\text{kalā}} \times \text{day} = \text{pala etc.}$$

After finding approximate parvānta time, we get better approximation of sun and moon position (their difference and their speeds. Then we get more correct value of parvānta.

Vrses 9-11 - Samaparva Kāla - When for sun, the mandaphala, gati phala and udayāntara phala - all three are positive or negative, we further correct the samaparva kāla i.e. middle point of eclipse is slightly different from true parvānta above. Steps are as follows -

(1) (Udayāntara + bhujāntara of moon) + (gati phala of sun) = S

(2) S X mandaphala of moon = P

(3) On pūrṇimā, X

$$= \frac{P}{\text{moon diameter (444 yojana)}}$$

On amāvasyā, $X = \frac{P}{\text{Sun diameter (72,000 yojana)}}$

(4) $\frac{\text{X in vikalā}}{\text{Moon gati} - \text{Sun gati}}$ = L in daṇḍa pala etc.

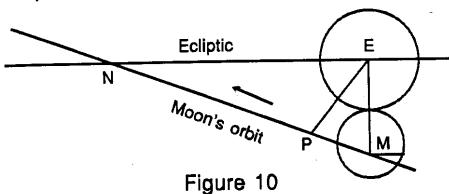
(5) When mandaphala, gati phala and udayantara phala all are positive,

Sama Parvakāla = Parvakāla - L

When the three above are negative

Samaparvakāla = Parvakāla + L

(6) For this difference of time we further correct the positions of sun and moon at parvants.

Figure 10

Notes : (1) Before analysing the formula we should analyse the reasons as to why closest contact will not be at amānta or purnimānta time.

E is shadow of earth centre moving on ecliptic for lunar eclipse. For solar eclipse it is disc of sun. M is centre of moon moving on its orbit in direction MPN.

At point EM, when EM is perp. to NE, ecliptic, longitudes of E and M are same which is ending time of amāvasyā or pūrṇimā as calculated earlier. However, closest approach is at P when EP is perpendicular at P. Thus the real mid point of eclipse will be after pūrṇimānta time. When RM is after crossing N, then it is before parvānta time. This difference is due to inclination of moon's orbit with ecliptic and difference PM is given by udayāntara phala of moon in latitude along ME direction and bhujāntara phala in EN direction.

Another reason of difference is due to different speeds at points of contact before P and after P. Due to that the mid point will be shifted from P in ratio of speed difference given by mandaphala of moon.

Udayāntara and bhujāntara phala of moon are almost for same time difference as sun, as moon

and sun or earth's shadow are in same position almost. The result for shadow at 180° from sun is same. If speed of moon is increasing, the time in covering contact distance towards N after P will be less and mid point will be towards opposite direction i.e. deducted.

Similarly for other results positive, the time is to be deducted. If mandaphala is + ve, gati phala of sun is negative, hence relative motion of moon will be positive and it is to be added.

Thus the difference due to latitude difference is (udayāntara + bhujāntara) of moon + gatiphala of sun. This will be increased in the ratio of mandaphala of moon. For outer contact, moon will cross its (own diameter + shadow portion). For inner contact (maximum) it will cover (shadow - its own diameter). Hence the product is to be divided by angular diameter of moon. In solar eclipse, it is almost equal to diameter of sun.

There appears some error in text. All the quantities are in angular measure, which cannot be divided by yojanas, it should be angular diameter.

When all the three factors causing error are of one sign, correction is proposed, otherwise they almost cancel each other.

Qualiative discussion will be done at the time of calculating duration of eclipse.

Verses 12-15 : Diameters and distances of sun and moon-

In Atharvaveda, while explaining the meaning of 'Aum', diameter of solar disc has been stated to be 72,000 yojanas. Based on this statement, I

have corrected the disc sizes of planets, their orbits etc. through observation and calculation.

Diameter of moon and earth are 1/162 parts and 1/45 parts of sun's diameter. Earlier astronomers also have stated the diameter of earth as 1600 yojana (value obtained here). The values in yojana and angle are stated as follows -

	Diameter in yojana	Angular diameter mean
Sun	72,000	32/32/6 kalā
Moon	444	31/20 kalā
Earth	1600	—

$$\text{Mean sun diameter} = \frac{72{,}000 \text{ yojana}}{2213}$$

$$\text{Mean moon diameter} = \frac{444 \times 6}{85}$$

Mean distance of sun from earth = 76,08,294 yojana

moon = 48,705 yojana.

From this true distance, manda spaṣṭa karṇa also can be calculated.

As in case of moon's angular diameter, earth's shadow's angular diameter also can be known in moon's orbit (approx by multiplying with 6/85).

Notes (1) Comparative sizes of planets

	Āryabhata I, Lalla, Bhāskara I	Sūrya siddhānta, Siddhānta śiromaṇi	Modern values in yojana = 5 miles
Sun's diameter	4410	6500 (6522)	1,73,156
Sun's distance	4,59,585	6,89,378	1,85,80,000

Moon's diameter	315	480	430
Moon's distance	34,377	51,566	47,500
Earth diameter	1050	1600	1586

Diameter of earth is a measure of yojana as its astronomical definition. Hence; it is seen that diameter and distance of moon are almost accurate in sūrya siddhanta or others, but sun's diameter is taken only 4 times the earth or 14 times moon by Āryabhaṭa (13.37 times by Bhāskara II or sūrya sīddhānta). Its real value is 109.18 times earth's diameter or 402 times moon's diameter.

However angular diameters were almost correct.

	Bhskara II	Sūrya Siddhānta	Siddhānta Darpaṇa	Modern Values
Moon	32/1	32/0	31/20	31/7
Sun	32/31/33	32/20	32/32/6	32/4

Angular diameters and their ratios are almost correct. Moon's angular diameter can be directly observed, but it is difficult to see sun directly. Still it can be seen through reflection etc. and due to frequent annular eclipses its mean diameter has been taken slightly more than moon.

Linear diameter is calculated by formula (angular diameter x distance), when angle is in radians. This rato is almost 1/108 , this 108 is an important number for no. of beads in a prayer garland, no. of salutes to guru, aṣṭottarī system of daśā in astrology etc. Moon's distance could be correctly estimated with direct parallax, but direct measurement of sun's distance cannot be done.

The accurate looking figures of distances of sun and moon are derived from round figures of

their circumference of their orbits after division by 2π = 2 X 355/113 almost. On moon's orbit 1′ has been assumeed eequal to 15 yojana by Sūrya siddhāśnta and 10 yojanas by Āryabhaṭa. Linear velocity of planet = (angular velocity X distance) has been assumed constant. Actually areal velocity = angular velocity X $(distance)^2$ is constant according to Kepler's laws for elliptical orbits. Thus all planets are assumed to cover equal distance in equal time and total distance covered by them in a kalpa is equal to orbit or circumference of sky.

Accordingly, orbit of stars has been assumed 60 times orbit of sun. Candraśekhara must have seen distances of farther planets like pluto 40 times sun's orbit. Hence he increased it to another round figure 360 and explained difference of 1° sīghra paridhi difference according to this, which is not correct.

Similarly, he must have come across much larger figure of sun's distance and verified it according to parallax in solar eclipse. But he could increase it only 150 times moon's distnace instead of 400 times as he got diameters of 72,000 yojana from Atharva-veda. Earlier astronomers also must have obsereved it, but they didn't try to change it drastically, as the angular measure is sufficient for prediction of eclipse. Traditional value of sūrya siddhānta appears to be obstruction.

Siddhānta darpaṇa has assumed value of yojana in Atharvaveda as his own yojana which is incorrect as Āryabhaṭa etc. had assumed yojana of about 8 miles; compared to 5 miles yojana of sūrya siddhānta. Of course, he has compared 1600 yojana

diameter with sūrya siddhānta, though no such measure has been found in vedas.

However M.B. Panta (Vedavatī, Pune, 1981) has opined that for steller measures; mahā yojana = 5 X Āryabhaṭa yojana = 40 miles was used. Accordingly, Triśaṅku means 3 X 10^{13}; in mahāyojana units it is $3 \times 10^{13} \times 40$ miles = 207 light years which is really the distance of Triśanku star (Beta Crucis). Similarly Agastya or Argo navis has crossed Jaladhi or 10^{14} distance; which is $10^{14} \times 40$ miles = 690 light years in mahā yojana units (correct distance is 652 light years). Maṇḍala means revolution or circumference, diameter is indicated by width or viṣkambha in jyotiṣa. Hence 72000 yojana maṇḍala means it is circumference. In māhayojana units this value means diameter of 9.1 lakh miles which is slightly more than 8.66 lakh miles, the modern value. This may be correct if we include the corona of sun.

Another indication of yojana measure is given in Ṛkveda (1-123-8)

सदृशीरद्य सदृशीरिदु श्वो दीर्घ सचंते वरुणस्य धाम ।
अनवद्यास्त्रिंशतं योजनान्येकैका क्रतुं परियन्ति सद्यः ॥

Sāyaṇa has interpreted it that dawn goes ahead of sun by 30 yojanas and along with it moves round.

Similar verse is in RK 6-59-6 which, dawn goes ahead 30 steps i.e. units of length. In modern astronomy, dawn is taken 18° ahead, Tilaka in his Arctic home in vedes, page 85, has taken it 16°, probably for central India at 24°N. However, in sandhyā of each yuga, its value has been taken as 1/12th of yuga value. Thus dawn of day time of 12 hours is 1 hour, i.e. 1/24 of a day. This is 15° (360°/24) in angles. Thus circumference of earth is

30X24=720 yojanas and sun's circumfereence is 72,000 yogana i.e. 100 times in round figures. In round numbers 108 japa is counted as 100 hence it gives almost correct dimensions of sun.

Ratio of moon's diameter with earth's diameter has been slightly increased and it is more correct according to modern values. Increase of parallax from 1/15 of earth radius to 1/14th is also more correct and might have been confirmed by observation.

(2) Diameter of earth's shadow in moon's orbit - 85 yojanas in moon's orbit have been taken as 6 kalā i.e. 1kalā = 14.2 yojana. Hence linear diameter of earth's shadow multiplied by 6/85 gives its angular diameter; because it is in moon's orbit.

Verses 16-21 - True values of diameter and distance— If manda kendra (of sun and moon) is in 6 rāśis beginning with makara, manda koṭiphala is added to trijyā and substracted from trijya if manda kendra is in other six rāśis (kārka to dhanu). Result is substracted from double of trijyā, by remainder we divide the square of trijyā (118, 844). Result will be sphuṭa maṇda karna of sun and moon. If this method is used for star planets like maṅgala, it will give their radial distance from sun as centre.

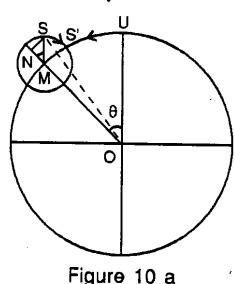
This sphuṭa karṇa in kalā is multiplied by madhya yojana karṇa and divided by trijyā to give sphuṭa manḍa karṇa in yojanas. Madhya bimba kalā divided by sphuṭa yojana and multiplied by madhya yojana gives sphuṭa biṁba kalā.

(Quoted from Siddhānta Śiromaṇi) - Manda karṇa is found like śighra karṇa. It is substracted from 2 X trijyā and by remainder, we divide square of trijyā. Result in kalā is manda karṇa of sun and moon which is the distance from centre of earth.

Manda karṇa kalā multiplied by madhya yojana karṇa and divided by trijyā gives sphuṭa yojana karṇa. Diameter of sun is 6522 yojana and of moon is 480 yojana (values of Bhāskara, not of this book - Quotation ends).

Method of Bhāskarācārya also gives accurate value, still I have calculated sphuṭa karṇa from koṭi phala (instead of mandaphala because, for 3 rāśi difference between sphuṭa graha and mandocca, manda sphuṭa karṇa is equal to koṭi.

Note : (1) **True method** - Madhya graha M is at angle θ from direction of ucca U. True planet S on manda paridhi has moved by same angle θ = $\angle$SMN in opposite direction. SN is $\perp$ on OM extended.

Figure 10 a

NS = manda bhuja phala = r sin θ

where MS = r = radius of mandaparidhi

R = 3438′ = OM is radius of madhya graha.

MS′ // SN is mandaphala

$$\frac{MS'}{NS} = \frac{OM}{ON} = \frac{R}{R + r\cos\theta}$$

because MN = r cos θ

ON is called koṭi of karṇa, at 90° it is zero. Manda Karṇa OS = K is true distance of planet S.

$K^2 = ON^2 + SN^2 = (R + r\cos\theta)^2 + (r\sin\theta)^2$

$= R^2 + r^2 + 2Rr\cos\theta$ - - - (1)

(2) Bhāskara approximation - His formula

$$K = \frac{R^2}{2R - K}$$

appears meaning less as it can be used only if K is already known in right side also. However the first K is an approximation by koṭi of karṇa only = R + r cos θ. This relation holds good and gives a better approximation from formula.

$K^2 + R^2 = r^2 + R^2 + 2\,Rr\cos\theta + R^2$ - - - from (1)

$= r^2 + 2R\,(R + r\cos\theta)$

$= 2\,RK$ approx neglecting r^2

(3) Siddhānta Darpaṇa formula has two unncessary steps for manda kendra 270° to 90°, we first add manda koti phala to trijyā, then substract the sum from 2 X trijyā. This is equivalent to substracting mandakoṭiphala from trijyā

$$2R - (R + r\cos\theta) = R - r\cos\theta$$

$$\text{Now, } \frac{R^2}{R - r\cos\theta} = R\left(1 - \frac{r\cos\theta}{R}\right)^{-1}$$

$$= R + r\cos\theta + \frac{r^2\cos^2\theta}{R} + \ldots\ldots\ldots\ldots \quad (2)$$

$$\text{Now from (1), } K = R\left(1 + \frac{2r}{R}\cos\theta + \frac{r^2}{R^2}\right)^{1/2}$$

$$= R + r\cos\theta + \frac{r^2}{2R} + \ldots\ldots\ldots\ldots \quad (3)$$

average value of $\cos^2\theta = 1/2$, hence, expression (2) is almost equal to K.

Verse 22 : Mean angular diameters (bimba kalā of moon and sun multiplied by true daily

motion and divided by mean daily motion gives true diameter in kalā.

Note : Linear diameter is fixed = D yojana

Angular diameter B varies with distance, Bo is mean value

Bo x R = BxK = D - - - - (1)

True motion xK = Mean daily motion XR-(2)

Dividing (1) by (2), we have

$$\frac{B_0}{\text{mean motion}} = \frac{B}{\text{True motion}}$$

$$\text{or } B = \frac{B_0 \times \text{True motion}}{\text{mean motion}} \quad \text{- - - - (3) Proved}$$

To prove (2), Let θ and θ' be the manda kendra for today and tomorrow at sunrise

True longitude for sunrise today

$$= \text{Apogee today} + \text{arc}\left(\frac{R \sin\theta \times R}{\text{manda karṇa today}}\right) \quad \text{(Fig.10a)}$$

True longitude tomorrow sun rise

$$= \text{Apogee tomorrow} + \text{arc}\left(\frac{R \sin\theta' \times R}{\text{manda karṇa tomorrow}}\right)$$

Taking difference of these two equations

Daily motion for today = Daily motion of apogee

$$+ \frac{(\theta' - \theta) \times R}{\text{manda kendra for today}} \quad \text{approx - - - - (4)}$$

Here, manda kendra difference in one day has been ignored, (θ'- θ) = daily motion of manda kendra i.e. mean daily motion.

This is formula (2), if we ignore very slow motion of apogee.

Verse 23 - Formula are

(a) $\text{Diameter in kalā} = \dfrac{\text{Diameter in yojana} \times R}{\text{Spaṣṭa karṇa yojana}}$

This follows from (1) above.

(b) $B = Bo \pm \dfrac{\text{gati phala}}{110}$ for sun

Addition is done when manda kendra is in 2nd and 3rd quadrant, otherwise substraction is made.

Proof : $\dfrac{B}{B_0} = \dfrac{\text{True motion}}{\text{Mean motion}}$

or $\dfrac{B - B_0}{B_0} = \dfrac{\text{True motion} - \text{mean motion}}{\text{mean motion}}$

$= \dfrac{\text{gati phala}}{\text{mean motion}}$

or $B - Bo = \dfrac{B_0 \times \text{gati phala}}{\text{mean motion}}$

$= \dfrac{\text{gati phala} \times 32/32/8}{\text{mean motion } (59/8)}$

(Putting values of Bo and mean motion)

$= \dfrac{\text{gati phala} \times 11}{20}$

approx if both are in kalā

If gatiphala is in vikalā then, the correction is

$$\frac{\text{gati phala} \times 11}{20 \times 60} = \frac{\text{gati phala}}{110} \text{ approx.}$$

Verse 24 : For moon

$b = bo \pm \dfrac{\text{moon gati phala}}{25}$ (c)

Proof - As in above formula correction is

$$\frac{b_0 \times \text{gati phala}}{\text{mean motion}} = \frac{\text{gatiphala} \times 31/20}{790/35}$$

= gatiphala/25 approx

(C) $B = \dfrac{\text{True sun speed} \times 11}{20}$

(D) $b = \dfrac{\text{True moon speed} - 7}{25}$

Proof (i) Formula (c) is obvious

$$B = \frac{Bo \times \text{True speed}}{\text{Mean speed}} = \text{True speed} \times \frac{32/32}{59/8}$$

$$= \text{True speed X } \frac{11}{20} \text{ approx.}$$

(ii) $b = \dfrac{b_0 \times \text{True speed}}{\text{Mean speed}} = \dfrac{\text{True speed} \times 31/20}{790/35}$

$$= \frac{\text{True speed} \times 31/20}{31/20 \times 25 + 7} = \frac{\text{True speed} - 7}{25} \text{ approx}$$

as 7 is very small compared to speed of about 800 kalā per day.

Verse 25 : Shadow length of earth (conical from centre)

$$= \frac{\text{True sun karṇa} \times \text{diameter of earth}}{\text{Sun diameter} - \text{Earth diameter}}$$

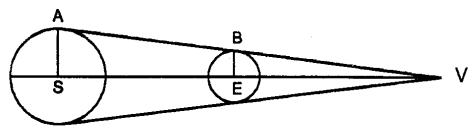



Figure - 11

Note : In figure 11, BE is Parallel to AS, so

$$\frac{AS}{BE} = \frac{SV}{EV} = \frac{SE + EV}{EV} = \frac{SE}{EV} + 1 \text{ or } EV = \frac{SE \times BE}{AS - BE}$$

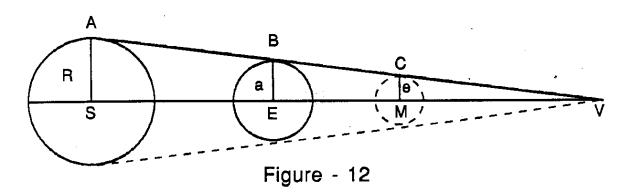
Verse 26 : According to Siddhānta Śiromaṇi

Diameter of earth's shadow in moon's orbit

= Earth diameter —

$$\frac{(\text{Sun diameter} - \text{Earth diam}) \times \text{moon distance}}{\text{sun distance}}$$

Note : This is called reduction in earth's diameter; because sun is bigger and earth's shadow converges into a cone.

Figure - 12

In fig 12, S,E, M are centres of sun, earth and shadow, Common tangent line A B C meets SEM at V. Radius of sun, earth, shadow are R, a, e. Distance of sun and moon from earth are r, r_1 Shadow cone from moon MV = x.

Now in similar triangles ASV and BEV

$$\frac{AS}{SV} = \frac{BE}{EV}$$

$$\text{or } \frac{AS}{BE} = \frac{SE + EM + MV}{EM + MV}$$

$$\text{or } \frac{R}{a} = \frac{r + r_1 + x}{r_1 + x} = \frac{r}{r_1 + x} + 1$$

$$\text{or } \frac{r_1 + x}{r} = \frac{a}{R - A} \text{ or } x = \frac{ar}{R - a} - r_1 \text{ - (1)}$$

In similar triangles BEV and CMV

$$\frac{BE}{CM} = \frac{EV}{MV} = \frac{EM + MV}{MV}$$

$$\text{or } \frac{a}{e} = \frac{r_1 + x}{x} = \frac{r_1}{x} + 1$$

$$\text{or } \frac{r_1}{x} = \frac{a - e}{e} \text{ or } x = \frac{e\,r_1}{a - e} \text{ - - -(2)}$$

Equating values of x from (1) and (2)

$$\frac{ar}{R - a} - r_1 = \frac{e\,r_1}{a - e}$$

$$\text{or } \frac{e}{a - e} = \frac{ar}{r_1 (R - a)} - 1 = \frac{ar - r_1 (R - a)}{r_1 (R - a)}$$

$$\text{or } \frac{a}{e} = 1 + \frac{r_1 (R - a)}{ar - r_1 (R - a)} = \frac{ar}{ar - r_1 (R - a)}$$

$$\text{or } e = \frac{ar - r_1 (R - a)}{r} = a - \frac{R - a}{r} \times r_1 \text{ --- (3)}$$

After multiplying by 2, result is proved.

Verse -27 Earth shadow in kalā

$$= \frac{\text{Moon true motion}}{7} - \frac{\text{Sun true motion} \times 78}{145}$$

Note : Formula (3) in previous verse can be written as

$$\frac{2e}{r_1} = \frac{2a}{r_1} - \frac{2R - 2a}{r} \text{ - - - - (1)}$$

This gives angular diameeter in radians. Multiplied by Trijyā = 3438′ it will give diameters in kalā. First term in kalā in right side of (1) is

$$\frac{2a \times 3438}{r_1}$$

$$\text{But } \frac{r_0}{r_1} = \frac{\text{True speed}}{\text{mean speed of moon}}$$

(See equation (4) after verse 22), r_0 = mean distance

Hence this becomes

$$\frac{2a \times 3438 \times \text{True speed}}{r_0 \times \text{mean speed}}$$

$$= = \frac{1600 \times 3438}{48{,}705 \times 790/35} \times \text{true speed}$$

(Because 2a = 1600 yojana

r_1 = 48,705 yojana,

mean speed of moon = 790/35 kalā

$$= \frac{\text{True speed}}{7} \text{ approx.}$$

Second term in (1) is similarly

$$\frac{(2R - 2a) \times 3438 \times \text{True speed of sun}}{\text{Mean distance} \times \text{mean speed of sun}}$$

= True speed

$$\times \frac{(72{,}000 - 1600) \times 3438}{7608{,}294 \times 59/8} \text{ giving values}$$

$$= \frac{78}{145} \text{ approx.}$$

Hence the formula

Vereses 28-30 : Meaning of rāhu

Lunar eclipse is caused when moon enters earth's shadow, and solar eclipse is caused by covering of sun by moon. This is possible only when sun, moon and shadow of earth are near node (pāta) of candra whose names have been given rāhu and ketu (half part of rāhu itself). Hence it is said that rāhu devours sun or moon in eclipse.

In siddhānta śiromaṇi - If eclipse is caused by same rāhu, why there are different direction (of beginning of) eclipse, different times (short or long periods) and different coverings (total or partial eclipse). So persons assuming eclipse by rāhu have false pride of their knowledge of sphere; actually they are fools and against (true meaning of) Veda, purāṇa and samhitā.

Rāhu is shadow planet (a fictitous point), which covers moon by entering earth shadow (being near it); and covers sun by entering moon. For such ability, sun has given boon to him. This type of interpratation is not against scriptures.

Veerse 31-32 : Reasons of eclipse

Lunar eclipse - Shadow of earth is in opposite direction of sun and moves east wards in ecliptic like sun. At the end of full moon, when moon is in opposite direction of sun, its speed is more than shadow speed, so it enters the shadow and crosses it. After entring shadow, its light (from sun) is lost. Thus lunar eclipse is seen.

Solar eclipse : At the end of amāvasyā, when moon and sun are in same direction (same rāśi), then moon moving east covers sun and with faster speed crosses out in east direction. Sun being very

luminous cannot bee seen. While covering it, only lightless moon is seen.

Verse 33 : Digamśa correction of rāhu

As we add or substract fourth phala (śīghra correction) in pāta of star planets like maṅgala, similarly digaṁśa phala of rāhu is calculated as of moon and it is added or substracted.

Note : Digamśa phala = 1/10 of mandaphala of sun. This is correction in moon's orbit due to variation in annual attraction of sun, which changes the direction of moon's orbit. Hence it changes direction of rāhu also. This correction has been described in chapter 6.

Verses 34-35 - Śara of moon

Sphuṭa pāla is deducted from sphuṭa moon at corrected parvānta time. Bhuja jyā of this arc is calculated. This is jyā of vikśepa kendra. We add 1/38 part of it. From half of result, arc is found. This arc divided by 6 gives śara of moon.

When moon - pāta (or vipāta candra) is in six rāśis beginning with meṣa, śara is in north direction, otherwise in south direction.

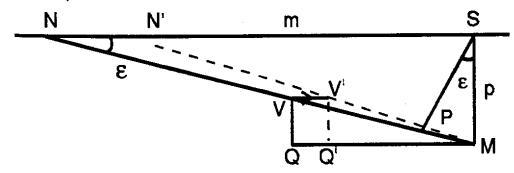



Figure - 13

Notes - Maximum śara of moon has been stated as 309′ in siddhānta darpana i.e. inclination angle $\varepsilon = \angle MNS$ is given by R Sin ε = 309′ NS

is ecliptic on which S is position of Sun or earth's shadow. NS = m = distance of moon from node N. NM is orbit of moon with moon at M. Its śara is MS = p.

p = m tan ε as ΔMSN is right angled and almost plane due to small size.

$$\text{Thus} \quad p = \frac{m \,.\, R \sin \varepsilon}{R \cos \varepsilon} = \frac{m \,.\, R \sin 309}{R \sin (5400 - 309)}$$

$$= \frac{m \times 308}{3423} = \frac{m}{11 + \frac{1}{3}} \text{ approx.} \quad \text{- - (1)}$$

However, at the time of eclipse, S has slow motion and is considered fixed and we calculate only the moon's speed. Relative speed of moon is obtained by adding vector VV[1] equal and opposite to motion of S to velocity vector MV of moon. Thus resultant motion of moon is smaller and in direction MV'N' which make angle ε' slightly bigger than ε'

$$\tan \varepsilon = \frac{VQ}{QM} = \frac{\text{motion of śara}}{\text{motion along ecliptic}}$$

$$\tan \varepsilon' = \frac{VQ}{QM - VV'}$$

$$\text{or} \quad \frac{\tan \varepsilon'}{\tan \varepsilon} = \frac{QM}{QM - VV'}$$

$$\text{or } \tan \varepsilon' = \tan (309') \, \frac{790/38 \times \cos (309)}{790/38 \cos (309) - 59/8}$$

thus $\varepsilon' = 333 = 5^\circ\ 33'$

Shortest distance of moon from ecliptic

$= p \cos \varepsilon'$

= SP which is pependicular from S to MV'

Hence effective śara = p cos ε'

$$= \frac{p \cdot R \cdot \sin(5400 - 333)}{R} = \frac{P \times 3420.5}{3438} \quad (2)$$

Equation (1) takes value (3438/3423) times more than the sine value. For half the angle increase is about 1/38 times as approximated here. Hence after increase of 1/38 in m, Sin,. of its half value in taken, then divided by six again. Taking sine almost equal to small angles, formula given is

$$p = \frac{39}{38} \times \frac{1}{12} m = \frac{13}{152} m$$

which is almost same as (2). as may be verified. Due to relation (2), the effective inclination of moon's orbit is reduced by about 18′ to 290′ approx. Hence the value of parama śara was taken as less than the true value, in earlier texts.

Verse 36: Method for śara gati

Instead of finding arc, we multiply the koṭiphala of moon and it is multiplied by pāta and motion of moon and divided by trijyā. Result is current speed of śara. By adding or substracting krānti gati, we get sphuṭa śara from equator.

Notes: (1) There are three confusing words in the verse--Whose koṭiphala is to be taken is not specified----I have interprated it to be koṭiphala of moon's movement along ecliptic i.e. its rāśi etc from pāta.

Whether motion of pāta and moon both are meant---

pāta has very small motion and when motion of sun is being neglected, much smaller motion of

pāta cannot be taken. Hence it is pāta or śara from ecliptic and motion of moon.

Result of this multiplication and correction with krānti both are called sphuṭa śara. First sphuṭa śara is distance of moon from krānti vṛtta. Second sphuṭa śara is distance from equator= distance frojm krānti vṛtta(śara)+ distance of spaṣṭa moon on krānti vṛtta from equator (i.e. krānti of moon).

Translation has been made according to these clarifications.

(2) Sphuṭa śara from equator has already been explained. Now p=sin m. tan ε.

p=śara, m=distance of moon from pāta along ecliptic ε =angle of inclination of moon's orbit with ecliptic, since ε is constant, taking differentials

Cos p. δp= Cosm. δm. tan ε

Here δp and δm are motion of pāta and moon in unit time of hour or day. We are to find δp.

$$\delta p = \frac{R \cos m}{R} . \delta m . \frac{\tan \varepsilon}{\cos p} \quad \ldots\ldots (1)$$

Now cos p= cos ε/sin m according to Napier relations of right angled triangle N E P.

$$\text{Hence, } \frac{\tan \varepsilon}{\cos p} = \sin m \ \sin \varepsilon \approx \sin p \approx p$$

Hence (1) becomes

motion of pāta

$$= \frac{\text{Koṭiphala of moon} \times \text{Moon gati} \times \text{Pāta}}{\text{Trijyā}}$$

verses 37-38: śara from chart

Pāta is substracted from moon and bhuja of the resulting angle is found. From degrees of bhuja

śara etc can be found in charts where śara, śara difference, koṭiphala and bhujaphala etc have been given in appendix: These have been given at intervals of 225.

Alternatively, pāta is substracted from moon. Its bhuja is converted to kalā. Its 1/16 will be subtracted and divided by11. (i.e. moon-pāta). For greater values, this will be incorrect

Notes : Kalā of bhuja is almost equal to jyā for small angles.

It is to be multiplied by

$$\frac{309}{3438} \text{ i.e. } \frac{\text{Parama Krānti}}{\text{Trijyā}}$$

$$= \frac{1}{11 + \frac{39}{309}} = \frac{1}{11 + \frac{1}{8}} \text{ approx.}$$

Substraction of 1/16 part is to convert the bhuja approximately to its jyā:

Verse 39: Extent of eclipse (grāsa)

The planet which is to be eclipsed is called grāhya or chādya and the planet or shadow which covers it, is called grāhaka or chādaka. Half the sum of their angular diameters is 'mānārdha' or 'mānaikyārdha'. If śara is more, there cannot be eclipse. Differance of mānārdha and śara is the grāsa (covering). If grāsa is more than grāhya, then eclipse is total. Remaining part of grāsa is in sky.

Note : Except for the terms, the cause of eclipse has already been explained.

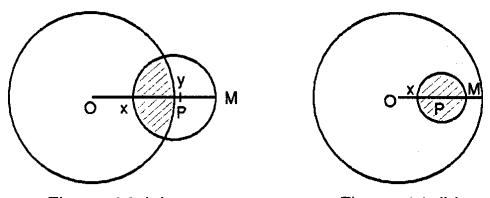



Figure 14 (a) Figure 14 (b)

P is centre of planet to be eclipsed and XM is diameter. Its distance from centre of coverer O is śara-OP. Either the coverer (chādaka) earth shadow or covered (chādya) sun is on ecliptic

In figure 14(a) covered portion

$XY = PX - PY$

$= r_1 - (OP - r_2) = r_1 + r_2 - OP$

Where r1 and r2 are radii of the bimba.

When $XY > 2r_1$ then eclipse is total as in fig 14(b)

Verse 40 : Direction and stages--

Lunar eclipse has contact in east and end in west often (as explained in verses 31-32) Calculation is done as per this rule only. But solar eclipse often starts in west and ends in east direction, However, sometimes south west direction is calculated instead of east west direction

When grāhaka just completely covers the grāhya, it is called 'nimīlana' time. When it is about to start emerging, it is called unmīlana. Time from nimīlana to unmīlana is called 'marda kāla' or 'vimarda kāla' (period of total or maximum eclipse). Total time of eclipse from sparśa to mokśa is called grahaṇa kāla or 'sthityardha' kāla.

verses 41-43 : Total time and time of complete eclipse-

Parvānta time. When moon and sun have same rāśi etc. (in solar eclipse) or their difference is exactly six rāśis; it is the time of eclipse. This time (after minor correction of verse 11) is called sama parva kāla and is middle point of eclipse.

Square of mānārdha (half sum of diameters of grāhya and grāhaka) is substracted from square of sphuṭa śara and of remainder, square root is taken. This is multiplied by 60 and divided by difference of moon gati and sun gati. Result is half time of eclipse (sthityardha kāla). Its double is total time of eclipse.

Similarly, square of half the differençe of diameters of grāhya and grāhaka is substracted from square of sphuṭa śara. Square root of remainder is multiplied by 60 and divided by differance of gati of moon and sun. Result is 'marda- ardha' kāla in ghaṭī etc. Its double is 'marda' kāla or time of complete eclipse.

From samaparva kāla (mid point of eclipse), substraction of sthiti ardha and marda-ardha give sparśa and nimīlana times. When sthiti ardha and marda-ardha are added, it gives unmīlana and mokṣa times:

For more correct times, we calculate sphuṭa candra and śara at time of sparṣa, unmīlan etc and from them again these times are calculated. In solar eclipse, repeated laṁbana corrections are made.

Notes : (1) This is an approximate formula in which śara of moon is considered to be same, hence there is need for successive approximation. First, we derive the approximate formula, assuming

the śara to be minimum distance of moon from mid point of eclipse.

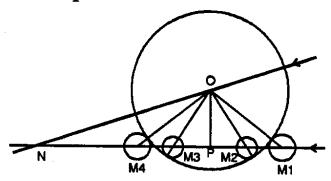



Figure 15 - Period of eclipse

In figure 15, O is centre of earth shadow and ON is ecliptic. Shadow is considered fixed and moon is moving in direction M_1 N with relative speed (moon-sun). This direction is slightly more inclined 5°33′ compared to 5°9′ angle of moon's orbit with ecliptic. as explained in verse 35. M1, M2, M3, M4 are positions of moon at 1st contact (sparśa), 2nd (nimīlana), 3rd (unmīlana) and 4th contact (mokśa)

OP is perpendicular on M_1 N and is almost equal to śara. This value of śara is assumed for all the four positon. Right angled triangle OPM_1 is almost a plane figure.

Hence

$$M_1P = \sqrt{M_1 O^2 - OP^2}$$

$$= \sqrt{\frac{1}{2}(\text{moon bimba} + \text{shadow bimba})^2 - \text{śara}^2}$$

Moon moves along M_1P with relative speed of m-s where m and s are daily motions of moon and sun(or shadow) in 60 daṇḍa.

Hence it will cover M_1P in $\frac{60 \times M_1 P}{m - s}$ daṇḍa

This is same as M_4P distance (as M_1 O=M_4 O).

Similarly, OM_2 or OM_3 = radius of shadow-radius of moon.

$$PM_2 \text{ or } PM_3 = \sqrt{OM_2^2 - OP^2}$$

$$= \sqrt{\frac{1}{2}(\text{shadow diam} - \text{moon diam.})^2 - \text{śara}^2}$$

Hence half time of total eclipse

$$= \frac{60}{m - s} \sqrt{\text{mānāntara}^2 - \text{śara}^2}$$

(2) Let T be time of conjunction, when moon and earth shadow have same longitude, and p the latitude (śara) of moon, North latitude is considered positive, p′ is hourly increase in latitude (increase towards north is positive)

m′= excess of hourly increase in longitude of moon over that of sun.

M= angular radius of Moon, S= angular radius of shadow at the moon.

Then at any time t hours after time of conjunction, T, the distance between shadow and moon in longitude is m′t and the latitude of moon is p+p′t.

Thus distance between centres of shadow and moon

$$=\{m'^2t^2 + (p + p'\,t)^2\}^{1/2}$$

The eclipse begins or ends when the moon's rim just touches the rim of the shadow in entering it or leaving it. Distance between such time is S+M=D say (fig 15) then $\{ m'^2\, t^2 + (p + p'\, t)^2 \}^{1/2}$ = D gives the time of beginning of eclipse. Solving this for t, we get

$$t = \frac{-pp'}{m'^2 + p'^2} \pm \left[\frac{p^2 p'^2}{(m'^2 + p'^2)^2} + \frac{D^2 - p^2}{m'^2 + p'^2}\right]^{1/2}$$

The + sign gives the end and --sign (earlier time) gives the beginning.

Total phase of the eclipse begins or ends when the rims touch the moon being inside shadow (M_2, M3 position of fig. 15) i.e. D= S- M, Putting this value of D in above soulution, we get the times of beginning or end of total phase of eclipse.

Discussion of results:

(1) The eclipse begins at

$$T - \frac{pp'}{m'^2 + p'^2} - \left\{\frac{p^2 p'^2}{(m'^2 + p'^2)^2} + \frac{D^2 - p^2}{m'^2 + p'^2}\right\}^{1/2}$$

(2) Eclipse ends at

$$T - \frac{pp'}{m'^2 + p'^2} + \left\{\frac{p^2 p'^2}{(m'^2 + p'^2)^2} + \frac{D^2 - p^2}{m'^2 + p'^2}\right\}^{1/2} \text{ hours}$$

For full eclipse time D=S+M. For total eclipse D=S-M.

(3) Middle of the eclipse falls at

$$T - \frac{pp'}{m'^2 + p'^2} \text{ hours}$$

(a) If p and p′ are both positive or both negative, middle of the eclipse is before the time of conjunction.

(If one is positive and other negative, middle is after conjunction)

(b) Only when the latitude at conjunction p=o, the middle falls at T, the time of conjunction, because p′ cannot be zero near a node.

In verse 11-- p latitude is positive then udayantara is positive, p′ i.e. speed of latitude is positive when bhujāntara is positive (verse 36). Thus when both are positive or negative time is corrected

(4) If D=p, (1) and (2) reduce to

$$T - \frac{pp'}{m'^2 + p'^2} \pm \frac{pp'}{m'^2 + p'^2}$$

(a) Eclipse begins or ends at the conjunction

(b) Duration of eclipse is $\frac{2\,pp'}{m'^2 + p'^2}$ which may amount to about 22 minutes

(5) The duration is zero, when the expression between the double brackets is zero, i.e. p is greater than D by $\frac{D'^2}{2\,(m'^2 + p'^2)}$, (neglecting fourth powers of p′/m′), which may amount to about 14″ in the mean.

(6) If t is not real, there is no eclipse, or total eclipse, according as D is taken to be S+M or S-M. Then p-D is more than about 14″ calculated above

(3) Conditions of eclipse in equator coordinates

Instant at middle of eclipse is chosen as origin of time in figure 16.

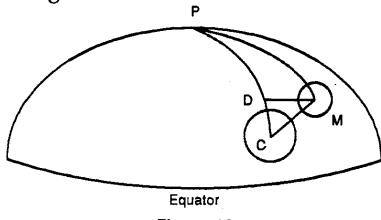



Figure 16

Equatorial coordinates of centre C of shadow at time t hours is a, δ and of centre M of moon be a_1, δ_1. Then, if P is the pole and M D the perpendicular from M on P C (on celestial sphere), CD= δ_1-δ and DM= $(a_1\text{-}a)$ cos δ_1, nearly. So $CM^2 = (\delta_1 - \delta)^2 + (a_1 - a)^2 \cos^2 \delta_1$ - - (1)

If, hourly rates of increase of a, a_1, δ, δ_1 at t =o are (a) o, $(a_1)_0$ $(\delta)_0$ $(\delta_1)_0$ respectively, we can write (1) as

$$CM^2 = [\{ (\delta_1)_0 + \delta'_1 \ t \} - \{(\delta)0 + \delta' \ t\}]^2$$
$$+ [\{(a1)_o + a' \ t \} - \{ (a)_o + (a' \ t) \}]^2 \cos^2 \delta, \ldots\ldots(2)$$

approximately, neglecting the changes in cos δ_1 due to changes in δ_1, because $\cos^2 \delta_1$ in the above equation is multiplied by a factor which is small. Equation (2) is of the form

CM^2=a t^2+ bt +c - - - -(3)

Where a, b, c are known quantities.

If we put CM= $\frac{51}{50}$ $(P + P_1 - S) + S_2$ the two values t_1 and t_2 given by (3) are the times of 1st and fourth contacts (sparśa and mokṣa). For the second and third contacts (i.e. beginning and end of totality, we put

$$CM = \frac{51}{50} (P + P_1 - S) - S_1$$ and solve for t.

Middle of the eclipse is $\frac{1}{2}$ $(t_1 + t_2) = - \frac{b}{2a}$

verses 44-45 --Single time calculation

Method above uses successive approximation. Now method of single time calculation is described. Śara of samaparva kāla is calculated. Its half is

divided by difference of gati of moon and sun; Result in daṇḍa etc is substracted from samaparva kāla śara, if śara is increasing, otherwise it is added. For this parvakāla, new values of moon and its śara are found. Diffenence of this śara and samaparva kāla śara in viliptā is squared and its half is taken. Its square root substracted from samaparva kāla śara is sphuṭa śara. Sthiti kāla calculated from this is correct. Now more accurate value of shadow is stated (in verses 78-84)

Notes --- In figure 16(a).

AB is ecliptic and CD is moon's orbit, relative to shadow centred at S on the ecliptic. S and M

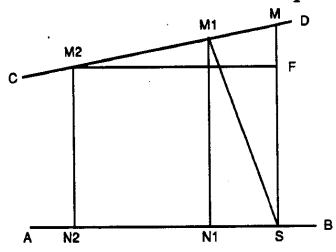



Figure 16a - One time calculation of sthiti ardha

are the centres of shadow and moon respectively at the time of oppositon. SM_1 is perpendicular from S on moon's orbit and M_1N_1 is perp. from M_1 on the ecliptic. Then M_1 is the moon's centre at the middle of the eclipse. ΔMM_1S is almost plane, $\angle MM_1S=90$

MS = moon's latitude at opposition,

$\angle MSM_1$ = i, inclination of moon's orbit to ecliptic.

$N_1 S = M_1 M$ approximatlely as i is small

$= \dfrac{309 \times MS}{3438}$ minutes (Kalā) as R sin i = 309′

$$= \frac{309 \times 60 \times MS}{3438} \times \frac{1}{\text{gati antara (of moon and sun)}} \text{ daṇḍa}$$

$$= \frac{MS}{2 \times \text{gati antara}} \text{ approximately............. (1)}$$

$$\text{or } N_1 S = \frac{MS \times 309 \times 60 \times 60}{3438 \; (790'35'' - 59'\; 8'')} \text{ pala}$$

$$= \frac{MS}{2} \text{ palas ....(2)}$$

This time is subtracted from the sthitiardha.

Since square of śara is used in calculation, average of squares of śara at M and M_1 is taken. Hence, half the square of difference is taken.

verses 46-50 : Grāsa from time.

Now method is described to calculate grāsa from time and vice versa. If time is before mid eclipse, it is substracted from sparśa sthiti ardha time. Remainder in daṇḍa etc. is multiplied by hāra =(moon gati-sun gati) corrected for lambana for solar eclipse, next chapter verse 46-47) and divided by 60. This will be koṭi kalā of lunar eclipse.

In solar eclipse, it is again multiplied by madhya sthiti ardha and divided by sphuṭa sthiti ardha, to get sphuṭa koṭi kalā.

For given time, squares of koṭi kalā and bhuja kalā are added, square root of sum is karṇa. This karṇa substracted from half the sum of bimba kalās gives grāsa.

If the given time is after mid eclipse time, it is deducted from mokśa sthiti ardha. Difference is multiplied by gati antara of sun, and moon (hāra) and divided by 60. We get koṭi. Then śara of given

time is found, from which spaṣṭa koṭi kalā of solar eclipse can be found. Again karṇa is found by adding the squares of bhuja and koṭikalā and taking square root. Karṇa substracted from half sum of bimba, gives grāsa.

From grāsa value, remaining free portion of eclipsed planet can be found.

Notes . (1) Grāsa = covered part (literal. meaning devoured portion)

Amount of grāsa is the length of diameter along the line joining centres of covered and covering discs, which has been eclipsed.

Magnitude of eclipse (in modern astronomy) is grāsa expressed as fraction of diameter. Thus grāsa =radius of shadow + radius of moon

-distance between centres of shadow and moon

Magnitude = grāsa/diameter of moon.

For solar eclipse, instead of shadow, we take moon's disc and covered disc is of sun.

(2) Formula of grāsa has already been established while calculating sthiti or mārda times. To revise, refer to figure 15. If M is any position of moon's centre, MP is distance covered from central point P. If it is before P, then it is at M_1 (contact point or sparśa). Then in the given time after sparśa, moon moves from M_1 to M. The remaining portion is MP till mid time at P.

Thus in time (sthiti ardha-given time) =t planet covers MP which is $\frac{t \times m'}{60}$ where m' is difference of daily speeds of moon and sun.

m′/60 is speed in one daṇḍa. MP is koṭi kalā

$$OM = \sqrt{OP^2 + MP^2} = \sqrt{(\frac{m't}{60})^2 + (\text{śara})^2} = \text{Karṇa}$$

When OM $\leq \frac{1}{2}$ difference of diameters, complete portion of moon is covered. For OM bigger than this value 1/2(sum of diameters)--OM is amount of grāsa. Similar calculation is done for period after midtime.

(3) In solār eclipse there is fast change in śara and valana, hence true kotikalā is found.

$$\frac{\text{spaṣṭa koṭi kalā}}{\text{Madhya koṭi kalā}} = \frac{\text{madhya sthiti ardha}}{\text{spaṣṭa sthiti ardha}}$$

Because, if sthiti ardha increases, the difference with given time decreases and koṭi kalā decreases. Thus, they are inversely proportional.

Verses 51-53 : Time from grāsa

When grāsa is between sparṣa and mid time, then it is substracted from half sum of covered and covering discs. This gives difference karṇa between centres of two discs. From square of this karṇa, we substract square of spaṣṭa śara at that time. Square root of difference will be koṭi kalā.

For solar eclipse this koṭikalā is multiplied by lambana corrected sthiti ardha and divided by madhya sthiti ardha. This gives spaṣṭa koṭi kalā. This is multiplied by 60 and divided by difference of daily speeds of moon and sun. Result in daṇḍa etc. is the time after sparśa.

For position in second half of eclipse, the result is substracted from sthiti ardha time, The remainder will be time remaining till mokśa.

Notes : This is reverse process of the previous method and uses the same formula.

Verses 54--Method for solar eclipse-For solar eclipse, the sthiti ardha for sparṣa and mokśa is called mean sthiti ardha, because special parallax(lambana) correction is done in this. Hence, all processes are done with mean śara (this doesn't change in short period of eclipse). Repeated parallax correction will give correct time.

Verses 55-59 :Direction of eclipse from parallax--

Now I describe valana(parallax) correction in kalās for correction of moon and sun in their eclipses, which arise due to ayana and akśāmśa. Due to these effects, direction of sparśa, mid-point and mokśa of an eclipse is known in east or west portions (kapāla) of sky.

In case of lunar eclipse, sāyana candra, and in case of solar eclipse, sāyana sun is found. Its koṭijyā (in kalā) is multiplied by parama krānti (1410) and divided by 3 rāśis (5400kalā). Result is āyana valana. This is in same direction (east or west part of sky) in which eclipse takes place.

In solar eclipse, rāśi of sun and moon is same, hence āyana valana can be found only from moon.

We calculate the nata kāla in pala from moon midday in lunar eclipse and from solar midday in solar eclipse This multiplied by 90° and divided

by its half day time gives nata in east or west direction in degrees

This nata is multiplied by akśāmśa of the place and divided by 90. Result will be akśa valana in north direction for east nata direction or valana in south direction for west nata.

Akśa and āyana valana are added when in same direction and difference is taken for different directions. Result will be dik-valana in degrees of moon in lunar eclipse and of sun in solar eclipse. This is true valana from which sparśa and mokśa directions can be known. Its measure in aṅgula has been stated while describing parilekha (degrees).

Notes (1) Sphuṭa valana is the angle between east or west point of disc of eclipsed planet with krānti vṛtta (ecliptic). This is made of two components. Due to akśāmśa of the place (distance from equator), krānti vṛtta cuts the horizon in eastern half of sky in north direction from east point. So ecliptic is towards north of east point of disc in east half of sky (and towards south in west half). This is called akśa valana.

Due to angle between ecliptic and equator (causing ayana), ecliptic is inclined further towards north when sāyana makara is on meridian (north south vertical circle). When sāyana makara is on meridian ($\pm 90°$), it is shifted southwards in east half of sky. For west half, the directions are opposite. This component is called āyana valana.

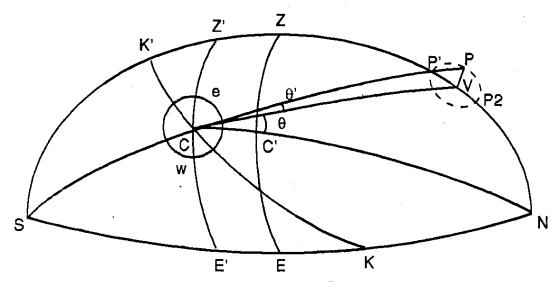



Figure 17 - Ayana and Ākśa valana

NVZS is yāmyottara vṛtta (meridian) at desired place.

NES is east half of horizon (kśitija vṛtta), north, east and south points shown

Z=kha-swastika (Zenith),

ZE = Samamaṇḍala, Z′E′ parallel to ZE through C,–

e, w — are east and west points.

KK′ = Krānti vṛtta

C= centre of planet disc to be eclipsed (chādya)

NCS = Samaprota vṛtta of c (circle of position)

V = North Pole in sky

P = pole of ecliptic (kadamba)

P P_1P_2 is kadamba vṛtta in which P moves round V in a day.

Z C^1 =Nata degree (in time units) of C

P_2 = Kadamba when sāyana karka is at K′ (meridian)

P_1 = Kadamba when sāyana makara is at K′

CV = Polar distance, C P = Kadamba distance

$\angle$ NCV $= \theta =$ Ākśa valana, $\angle$VCP $= \theta' =$ Āyana valana = Āyana valana

$\angle$ NCP $= \theta + \theta' =$ sphuṭa valana $= \angle$ KCE′

In this figure 17, for position P betweeen $\pm$ 90° distance of P_1, āyana valana θ' is also in north direction, hence sphuta valana is $\theta + \theta'$ as shown in figure. For P between $\pm 90°$ of P_2, θ' will be in south direction and sphuṭa valana will be θ-θ'. The direction of valana will be opposite, when planet is in west kapāla (west half of sky).

(a) Ākśa valana - From spherical triangle NCV

$$\frac{\text{Sin NCV}}{\text{sin NV}} = \frac{\text{sin CNV}}{\text{sin CV}} = \frac{\text{sin ZC}'}{\text{sin (polar distance)}}$$

because NZ and NC′ both are right angles, hence angle between them is equal to ZC′, which is natamśa of planet.

sin (polar distance)= R cos δ, (δ = krānti of planet)

= Dyujyā or radius of ahorātra vṛtta.

sin NV = R sin Φ , Φ = akśāṁśa of the planet.

so,

$$\text{sin NCV} = \frac{\text{R sin}.\phi \times \text{sin ZC}'}{\text{Dyujyā}} = \frac{\text{R sin}\,\phi \times \text{sin ZC}'}{\text{R cos}\,\delta} \quad \text{- (1)}$$

Rule for finding natamśa--

This is as per Bhāskarācārya. In half day or half night time, a planet rises 90° from horizon, hence nata kāla multiplied by 90 and divided by half day (or night in lunar eclipse) time gives natāmśa in degrees. This is not the angle from vertical Z point, but the angle between meridian

and samaprota vṛtta, corresponding to nata kāla. (H)

Its relation with natāmśa from Z is z = ZC. This can be found from spherical triangle NZC,

cot Z C x sin ZN

=cos ZN. Cos NZC + Cot ZNC. sin NZC

But ZN = 90° hence sin ZN =1, cos ZN = 0

Hence, cot ZC = cot ZNC. sin NZC

$$\text{or, cot ZNC} = \frac{\text{cot ZC}}{\text{sin NZC}}$$

or, tan ZNC = Sin NZC. tan ZC

But, ∠ NZC = 90° + ∠ EZC = 90° + agrā

Hence sin NZC = cos (agrā)

Hence tan ZNC = cos (agrā) x tan z

Rule for ākśa valana :

Sūrya siddhānta and Bhāskara II both have given the formula (1) i.e. Jyā of natakāla is multiplied by Jyā of akśāmśa and divided by dyujyā or semi diameter of diurnal circle.

In this text, R sin NCV and R sin ϕ both have been approximated to the angles NCV and ϕ and dyujyā is equated to 90°. When δ is small, R cos δ = R sin 90°. nearly. Thus all the 4 jyā are slightly increased to the arcs and the errors almost cancel each other as a rough rule.

(b) Āyana valana is known from spherical triangle PCV in figure 18

$$\frac{\text{Sin} \angle \text{PCV}}{\text{sin PV}} = \frac{\text{sin CPV}}{\text{sin CV}}$$

$$\text{or sin PCV} = \frac{\text{sin PV} \times \text{sin CPV}}{\text{sin CV}}$$

PV= distance from dhruva to kadamba which is equal to parama krānti (angle between ecliptic and equator). CV is distance from dhruva whose jyā is koṭijyā of krānti

$\angle$C P V is the angle between circles from C to ecliptic pole and āyana circle K_1 P.

Positions of planet on ecliptic and equator are L_1 and L_2.

$\angle$ CPV=arc K_1 L_1 = 90° – ML_1 where M is vernal equinox.

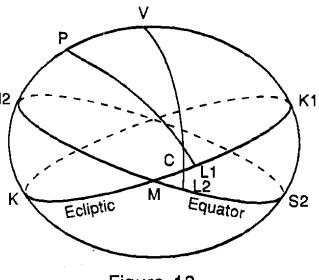



Figure 18

Hence Jyā of CPV is koṭijyā of ML_1 = sāyana graha

$$\text{Thus sin PCV} \quad \frac{\text{Jyā of parama krānti} \times \text{koṭijyā}}{\text{Koṭijyā of krānti}}$$

This is the formula given

Verse 60-65 : Period of lunar daytime:

Solar day time has already been described. The period from moon rise to moon set is its day. At sunset time, sphuṭa sāyana sun and moon are calculated. For sun, rising time (in asu) is calculated for remaining part of rāśi and for moon, it is for lapsed part of rāśi. These two udaya asu are added with rising times (udaya asu) of the rāsis from sun to moon. We add 56 asu to the total and divide by 360 to make them ghaṭī. This time after sun rise moon will rise.

For finding moon-set time, sāyana sun and moon for next sun rise time is calculated. Rising time of rāśis between sun and (6rāśi + moon) is

calculated and 56 asu lambana time is substracted. This time after sun rise, moon will set.

Śara of moon is very little (within 5°9′ and almost zero at eclipse time). Hence time between rising and setting of sun will be its day time, which has been calculated for diurnal circle of sun.

Śara in kalā (minutes of angle) at rising time or setting time is multiplied by palabhā and divided by 12. Result will be added to rising time if śara is south and substracted if śara is north. Reverse is done for correcting moon-set time. Thus we get sphuṭa time of moon rise and moon set.

Alternatively, sphuṭa gati of moon at midnight is divided by 19 and result in pala is added to night time of sun. This gives day time of moon.

Notes : (1) Difference between rising times of sun and moon is the difference between rising times of their rāśis, since sun and moon move in almost same ahorātra vṛtta. Śara at eclipse time is almost zero.

Since, at pūrnima time, moon-sun is less than 180° (it is 180° at end of pūrnimā), when sun has risen, moon will be slightly above west horizon. Thus difference of moon from sun +180° or (moon +180°-sun)distance is to be covered by moon for setting after sunrise.

Due to parallax, angle of moon at horizon seen from surface is 56′ lower than the angle calculated from earth's centre. Thus moon will rise on horizon after covering 56′ more. Hence moon rise time will be later than the rising time of moon-sun by further rising time of 56′.

Similarly setting of moon will be earlier by corresponding rising time of 56′ extra arc.

(2) Alternative formula --- Solar day in asu is more than nākśatra day in asu (21, 600) by the daily motion of sun (59.8″), extra time taken by earth to cover this distance covered by sun in mean time.

Similarly, lunar day is more than nākśatra day by its daily motion in asu i.e. 790′/35″ It is more than solar day by 790 (1-1/13.37) asu For true speed, it is more than solar day by moon gati (1-1/13.4) asu, relative speeds of sun and moon assumed almost content

Due to parallax the decrease in day time is (moon gati/14) both at moon rise time and moon set time. Hence (moon day-sun night)

$$= \frac{\text{moon gati}}{2}\left(1 - \frac{1}{13.4}\right) - \frac{2 \times \text{moon gati}}{14} \text{ asu}$$

(Moon day = $\frac{1}{2}$ moon - day and night)

$$= \text{moon gati}\left(\frac{12.4}{2 \times 13.4} - \frac{1}{7}\right) \text{asu}$$

$$= \text{moon gati} \times \frac{1}{6}\left(\frac{6.2}{13.4} - \frac{1}{7}\right) \text{pala}$$

$$= \frac{\text{moon gati}}{19} \text{ Pala approx.}$$

Verses 66-69 : Explaination of valana correction.

On great circle from north pole to south pole in the sky, pole of ecliptic called 'Kadamba' is situated 23°30′ south from north pole. This is surface centre of ecliptic in north part of celestial sphere.

The south surface centre of ecliptic (krānti vṛtta) is called 'kalamba' which is north from south pole by same 23°30' angle on ayana prota vṛtta (between two 'dhruva') 'Śara' is calculated along kadamba prota vṛtta which is distance from ecliptic.

Moon disc moves fastest of all the planets. Hence only its difference along two circles ayana prota and kadamba prota is calculated.

Distance of moon from 'dhruva' on dhruva prota vṛtta and from 'kadamba' along great circle through kadamba is taken. Their difference (angular) is multiplied by 360 and divided by circumference of moon disc (angular). This gives ayana valana. When moon is in north ayana, it is north valana and it is south valana in south hemisphere from equator.

For akśa valana, Lalla and Śrīpati have calculated versine of nata. But it has been done from R sine of nata by Brahmagupta and Bhāskara II. For āyana valana also two methods exist. One is from koṭijyā of madhya graha and the other from versine of bhuja of sāyana graha. But in my method, no jyā is needed because nati is according to equator and ecliptic arcs. Hence koṭi degree and nata degrees only should be used for āyana and akśa valana.

Note : Correct method and meaning of terms has already been explained.

Verses 70-77 : Diagram of eclipse--

For making a parilekha (diagram), place is made plane like water level and a circle of 18 aṅgula semi diameter is drawn. East and west points are marked as explained earlier (in Tripraśnā

dhikāra) From these two points on circumference also two circles touching each other are drawn, each of 18 aṅgula semi-diameter. In these two circles also, 4 points for cardinal directions and 4 middle angles are marked.

An east west line is drawn through centre of the two circles. A point is marked 1 aṅgula north of north point of eastern circle and another point 1 aṅgula south of south point of western circle. When planet is in west kapāla (west half of sky), a circle of 16/10 angula semi-diameter is drawn from southern point. When planet is in east kapāla, same size circle is drawn from northern point.

These arcs in the respective circles indicate krānti vṛtta (ecliptic)

Both arcs in the respective circles indicate krānti vṛtta. In that signs of 12 rāśis from meṣa are given from west to east after making 12 equal parts. Centre of moon is kept in its correct rāśi of krānti vṛtta and around it, a partly eclipsed moon circle is formed.

A line joining its two horns is drawn. The line joining horns is equal to diameter of moon. With this diameter, circles are drawn at both external points of krānti vṛtta. From this diagram moon will appear to be moving on krānti circle.

In eastern circle, kranti vṛtta is 328 yojana north ($23\frac{1}{2}^{\circ}$ akśāmśa of karka rekhā) from equator which is line between east and west points of the circle. Krānti vṛtta is actually a straight line, but appears curved due to drawing in a plane figure.

Hence jyā or koṭijyā are not needed in ākśa or ayana valana.

The curved shape krānti vṛtta (and equator also) is perpendicular on all yāmyottara (meridian) lines between two poles. Hence, on this krānti vṛtta, distance from prime meridian (Ujjain or Greenwich) is deśāntara jyā. Similarly, akśa jyā is distance on north south line.

Notes : This is like representation of earth in two touching circles in which karka rekhā and makara rekhā are north and south of equator.

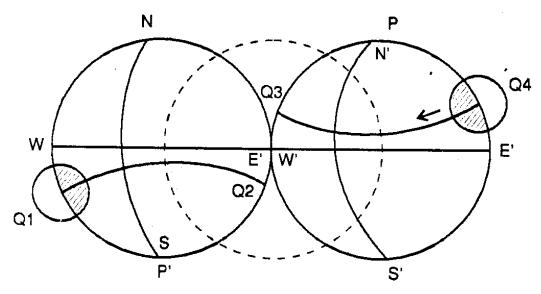



Figure 19

In figure -19 central circle is only for finding east west direction. East, west circles are of 18 aṅgula semi diameter in which all directions have been marked. P′ is 1 aṅgula north of N^1, P^1 is 1 aṅgula south of S. Kranti vrtta. Q_1 Q_2, Q_3, Q_4 are drawn from these with 16/10 aṅgula radius. This is only for explaination and not to the scale. However, this is a copy of school atlas map and reasonings about ākśa valana and āyana valana on that basis are not correct.

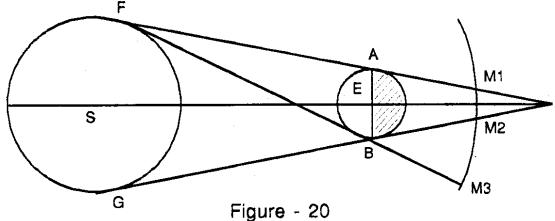
Verses 78-79 : Effective shadow of earth.

In moon's orbit, there is 5 kalā less dark shadow (avatamasa or penumbrā). On adding this, earth's shadow diameter increases by 10 kalā. This semi dark shadow covers moon at other times also, then there is no eclipse but light of moon is dimmed.

1/3 part of this semi shadow (penumbra) is very dark hence it almost merges with main shadow. Hence 1/3 of avatamasa or 10/3 kalā is added to the earth's shadow to find the effective diameter of shadow.

Notes:

(1) M_2 M_3 penumbra in moon's orbit is formed

Figure - 20

by direct tangents GB and transverse targent FB' (this will be very close to B).

$\angle$FBG = 2R/r, where r is distance of sun

R = radius of sun.

Hence penumbra at distance r_1 of moon, making same angle. M_2 BM_3 is

$$r_1 \times \angle \text{ FBG } = \frac{2Rr_1}{r} \text{ yojānas}$$

$$= \frac{6}{85} \times \frac{2 \text{ R } r_1}{r} \text{ Kalā}$$

$$= \frac{6}{85} \times \frac{72000 \times 48,705}{76,08,294} = 32.5 \text{ kalā}$$

Thus the extent of lesser dark shadow is arbitrary. However, in penumbra, moon's light will be definitely lesser.

As explained earlier, the effective increase of earth's shadow is by 2% or about 1 kalā due to absorption by atmosphere.

Verse 80-81 : Size of earth's shadow- It changes both due to sun distance and due to moon's distance, where its size is calculated.

When sun is near mandocca, it is farthest from earth, hence shadow is bigger when gaṭi is small and at 90° from nica, it reduces. Hence 1/28 of ravigati phala is added to shadow or substracted from middle value.

Moon's diameter is multiplied by 35 and divided by 13. In this, gati phala is substracted when positive. This gives true value of earth shadow. Method to find moon's diameter has already been stated.

Note : True dimensions of shadow has already been stated based on true motions of sun and moon both in verse 27. This correction is based on arbitrary assumption of avatamasa' i.e. darker part of penumbra.

Verses 82-83 : Calculation of true earth shadow–Due to relative rotation of sun around earth, earth shadow also rotates in same directon with same speed, but always remains opposite. It covers moon according to its value in moon's orbit. There is difference of 1/20 parts due to variation in distance from sun. Due to varying distance of

moon also its value changes. But this is very small compared to variation due to sun, hence it is neglected.

Now method to calculate effective earth shadow is explained. From sun's diameter (72,000 yojanas), its 1/10th (7,200yojana) and earth diameter (1600yojana) are substracted. Remainder (63,200) is multiplied by mean moon distance (48,705 yojana) to get (3,07,81,56,000). This product is divided by true distance of sun. The result substracted from earth diameter is the diameter of shadow in moon's orbit. This diameter multiplied by trijyā (3438) and divided by true distance of moon gives angular diameter.

Note : 'Avatamasa' (dark part of penumbra) is 10/3 kala which is 1/12 of earth shadow (about 40′) Hence (sun diameter-earth diameter) is reduced by 1/10 of sun diameter. Rest of the process is already explained in verse 26, whose diagram will make it clear.

Verses 84-86 : Colour of eclipse

From the shadow of earth, 40 kalā deducted gives the value of andhatamasa (dark penumbrā). (shadow is as calculated above). When śara of moon is small, moon enters this dark penumbra and looks very dark.

When lunar eclipse is very little, sky turns blue. In half eclipse, sky appears black. In more then half eclipse, it looks red black. In total eclipse, moon becomes pale yellow due to its entry in earth's shadow. In solar eclipse, there is no change in colours; we seen only moon which is relatively dark.

Moon is always smaller than sun, even in angular diameter. Hence horns of sun are sharp in solar eclipse. But moon is cut by bigger circle of earth's shadow, so its horns are rounder in eclipse.

Note : This is subjective description, hence no comments

Verses 87-88 : Close

Being dark in colour, shadow of earth is like rāhu, in which moon enters at eclipse time and gives mantra siddhi to vaiṣṇava and tāntrikas. They may do good to us.

Thus ends the eighth chapter describing lunar eclipse in detail in siddhānta darpaṇa written for education of children and correspondance between theory and observation by Śri Candraśekhara born in famous royal family of Oriśsa.

Eighth chapter ends.

Chapter - 9

SOLAR ECLIPSE

Solar Eclipse

Verse 1 - In last chapter, eclipse of moon and sun both have been discussed in a general way. For solar eclipse, in addition, it is necessary to calculate laṁbana and nati and biṁba of moon (angular diameter) also is different for the purpose of solar eclipse. These three will be specially discussed in this chapter.

Verse 2 : Reason of lambana and nati

At the end of amāvasyā, rāśi etc of moon and sun are same, even then they are seen in same direction only at the time of mid-day. On other times, they are not in the line passing through centre and surface point of observation. Why this happens for times other than mid day, will be described in this chapter. When sun and moon are in mid point of sky, their direction from centre and surface of earth is same.

Verses 3-6 - Meaning of lambana and nati

Sphuṭa ending time of amāvasyā calculated from sphuṭa moon and sun is called śamaparva Kāla'. This time after lambana correction is the middle time of grāsa (eclipse) in solar eclipse. This is sphuṭa amānta time for the place.

At amānta time calculated from earth's centre, the difference between directions of sun and moon

is called lambana. This difference arising due to observation from earth's surface, and in east west direction is called lambana'. Its component in north south direction is called 'nati' or 'avanati'. When sun and moon are in mid sky, the line from earth centre to their centres passes through the surface point, hence there is no lambana or nati.

When moon and vitribha lagna (lagna-90° on ecliptic) is same, there is no sphuṭa lambana, only nati is possible. When north krānti of vitribha lagna is same as (north) akśāmśa of the place, then it has no nati also.

When vitribha lagna's north krānti is more than akśāmśa of the place, moon (at vitribha lagna) has north nati. If north krānti of vitribha is less than north krānti of the place, or krānti is south, then moon has south nati.

In amāvasyā (corrected with lambana), moon and sun have same rāśi etc, hence nati in north south direction is easy to calculate.

Verses 7-15 : Sphuṭa lambana by successive approximation - Instantaneous position of sun is found by method explained in sphuṭādhikāra and from that, lagna of samaparva kāla is calculated. By deducting 3 rāśis (vitribha), again krānti is found for that. This krānti and akśāmśa (direction of equator) being in different direction, difference is taken. They are added if they are in same direction.

Result will be natāmśa of vitribha lagna; On substracting this from 90°, it gives unnatāmśa. Jyā of this unnatāmśa is called sphuṭa dṛg-gati.

Earth half diameter (800 yojana) assumed to be in sun or moon orbit, its angular diameter is

found in kalā. (For sun's orbit, it is divided by 2213 and for moon's orbit multiplied by 6/85 according to verse 15 of previous chapter). Result is called 'Kuchanna Kalā'. This is equal to the parama nati of sun and moon. Difference of these two is the parama (maximum) nati in solar eclipse.

$$\text{Parama nati of sun} = \frac{\text{Daily motion of sun}}{164}$$

$$\text{Parama nati of moon} = \frac{\text{Daily motion of moon}}{14}$$

Difference of parama nati of moon and sun in vikalā is divided by difference of daily motions of moon and sun in kalā. Result in daṇḍa etc. will be parama laṁbana time.

Parama laṁbana time in daṇḍa etc. is multiplied by vitribha śaṅku of desired time and divided by trijyā (3438). Result is antyā of laṁbana. Jyā of antyā (in asu) is called para.

Alternatively; sphuṭa dṛg gati (vitribha śaṅku) is multiplied by 100 and divided by 216. That will give the same para.

Now jyā of difference of vitribha lagna and sun is multiplied by para and divided by trijyā. Result is laṁbana jyā. Its arc in asu is sphuṭa laṁbana. If sun is west from vitribha lagna, this laṁbana time in asu is added to samaparva kāla, otehrwise it is substracted. Result is sphuṭa samaparva kāla.

For this sphuṭa samaparva kāla, we again calculate sphuṭa sun and vitribha lagna and laṁbana is calculated from their difference again. After repeated corrections, when there is no

difference between two samaparva kāla, that is the correct lambana.

Notes - (1) Approximate use of this method has already been made in verse 4 of previous chapter to find possibility of solar eclipse.

First we derive the equation of parama nati (already explained in appendix to tripraśnādhikāra).

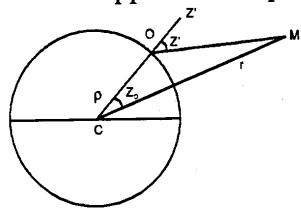



Figure 1 - Parallax of moon

C is centre of earth and M is moon. From a local place O, the moon's zenith distance is z' and z_0 is zenith distance from centre of earth. If OC = ρ, radius of earth for the place and CM = r, distance of moon from earth centre, then in ΔOCM

$$\frac{r}{\sin COM} = \frac{\rho}{\sin OMC}$$

But Sin ∠ COM = Sin (180°-z′) = Sin z′

∠ OMC = ∠ Z′ OM – ∠ OCM =z′–zo = p i.e. parallax.

Thus Sin p = (ρ/r) sin z.

Maximum parallax $P = \frac{\rho}{r}$ occurs when z=90° i.e.Sin z = 1.

This is parallax when moon is at horizon

$$\text{Thus parama lambana } P = \frac{\text{radius of earth}}{\text{Distance of moon}} \quad (1)$$

This is angular radius of earth if it is kept in moon's orbit, hence it is called 'Kuchanna' expressed in kalā (minutes) i.e. ku = earth, channa = removed (to moon's orbit). Similarly parama lambana of sun is earth's angular radius if it is viewed in sun's orbit.

Alternative formula - For moon P in kalā

$$= \text{radius of earth} \times \frac{3438}{48705} = \frac{6}{85} \times \text{radius of earth}$$

(Verse 15 of previous chapter).

But radius of earth = 800 yojana, moon's daily motion in kalā is 790/35″ which is slightly less than earth radius. Hence

$$p = \frac{\text{mon's daily motion}}{14} \qquad (2)$$

Similarly parama lambana P′ of sun is (mean value)

$$P' = \text{radius of earth} \times \frac{3438}{76,08,294} = \frac{\text{Earth radius kalā}}{2213}$$

(Result mentioned in verse 15 of previous chapter)

$$= \text{Sun daily moiton} \times \frac{\text{Earth radius}}{2213 \times \text{sun daily motion mean}}$$

$$= \frac{\text{Sin daily motion} \times 800}{2213 \times 59/8} = \frac{\text{Sun daily motion}}{164} \qquad (3)$$

(2) Explaination of the terms :

In figure 2, LNE is horizon and Z is zenith in celestial sphere.

MVS is ecliptic and K its pole.

M is meridian point of ecliptic and V is vitribha lagna, i.e., shortest distance from Z. Since ZV is perpendicular, it bisects the ecliptic above horizon, hence V is at 90° from horizon point L called lagna. Thus it is called vitribha or 3 rāśi less (than lagna).

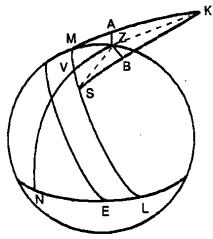



Figure 2 - Explaination of drgjya etc

ZA is perpendicular to MK, so sin Z A is dṛg gati of madhya lagna M (or smaller dṛggati).

ZB is perpendicular to SK - so that R sin ZB is dṛggati of sun S (or larger dṛggati).

ZS is zenith distance of S; R sin z is drg jyā.

ZM is zenith distance of M, R sin ZM is madhya jyā.

Distance from Z in direciton of ecliptic is thus dṛg gati. Distance from Z in direction perpendicular to ecliptic is dṛkkśepa. Thus dṛkkśepa of M, V and S are AM, ZV, SB. Total distance from z is dṛgjyā.

$(R \sin ZS)^2 = (R \sin ZB)^2 + (R \sin SB)^2$

or drg jyā^2 = dṛg gati^2 + dṛk kśepa^2 - - -(4)

$(R \sin SB)^2 = (R \sin MA)^2 = (R \sin ZM)^2 - (R \text{ Sin } AZ)^2$

or dṛk kśepa^2 = dṛgjyā of madhyalagna^2 -drggāti of madhyalagna^2 - - - -(5)

(3) Lambana antara of Sun and moon

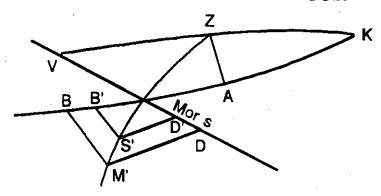



Figure 3 - Lambana of solar eclipse

Portion ZVSB of figure z is repeated here as ZVMA. M is the common geocentric position of moon or sun at end of amāvasyā.

S′ and M′ are apparent positions of sun and moon due to parallax, when viewed from surface. Thus MM′ = p, MS′ = p′

VM is ecliptic and K its pole. V is vitribha lagna, Z is zenith.

M′D and S′D′ are perpendicular on ecliptic.

M′B and S′B′ are perpendicular on KM produced

Arc MD or M′B is lambana i..e parallax of moon along ecliptic.

Similarly arc MD′ or S′B′ is lambana of Sun. Arc D′D is lambana of solar eclipse or difference of lambanas of moon and sun.

ZA is perpendicular to KM. Then from similar triangles MBM′ and ZAM we have

$$R \sin (BM') = \frac{R \sin ZA \times R \sin MM'}{R \sin ZM}$$

But BM′ = MD = R sin BM′ approx

$$\text{or } R \sin MD = MD = \frac{\text{Dṛg gati} \times R \sin p}{R \text{ Sin } z}$$

= Dṛg gati × parama lambana P (6)

as From (1) $P = \frac{\text{Sin p}}{\text{Sin z}}$

Similarly MD′ = Drggāti X P′ (7)

where P and P[1] are parama lambana of moon and sun.

Thus DD′ or lambanāntara = MD-MD′

= Dṛg gati × (P-P′) - - - from (6) and (7)

= Dṛg gāti × parama lambana antara (8)

Parama lambana (antara) in time units is the time in covering that distance by moon. Relative speed of moon is moon gati - sun gati = m′ kalā

Hence Parama lambana time

$$= \frac{\text{Parama lambana kalā}}{\text{m}' \text{ kalā/day}} \text{ day}$$

$$= \frac{\text{Parama lambana vikalā}}{\text{m}' \text{ kalā}} \text{ ghaṭī}$$

Thus DD′ in ghaṭī

= dṛg gati X parama lambana antara ghaṭī (8a)

(4) Vitribha Śaṅku and dṛggati - In figure 2

ZV = nati of vitribha lagna

= nati of equator - krānti - - - (9)

In figure (3), KV = KM=90°

In similar triangles KZA and KVM

$$\frac{\text{R sin KZ}}{\text{R sin KV}} = \frac{\text{R sin ZA}}{\text{R sin VM}}$$

or Dṛg gati R sin ZA $= \frac{\text{R sin KZ} \times \text{R sin VM}}{\text{R}}$

But R Sin KZ = R sin (90° - ZV) = vitribha śanku.

R sin VM = iṣṭa śanku of sun or moon.

Thus Dṛggati = $\frac{\text{Vitribha śanku} \times \text{iṣṭa śaṅku}}{\text{Radius}}$ (10a)

or $\frac{\text{vitribha śanku} \times \text{Jyā of viśleṣāmśa}}{\text{Radius}}$ (10b)

where VM = difference of sun and vitribha called viśleśāmśa

From (8), laṁbana

= $\frac{\text{Param laṁbana} \times \text{vitribha śaṅku}}{\text{radius}}$ × Jyā of-viśleṣmśa (11)

Parma lambana = $\frac{56 \times 60}{731}$ daṇḍa

56 kalā is moon's parama laṁbana, sun lambana is negligible, it is converted to vikalā, 731 is difference of moon and sun gati

= $\frac{56 \times 60 \times 360}{731}$ asu = 1658 asu (taking 56/6/35 for 56)

$\frac{\text{Parama lambana}}{\text{radius}} = \frac{1658}{3438} = \frac{100}{216}$ - - - (11a)

Actually it comes 207, but after parallax in moon rise it is 216.

(5) Summary of procedure -

Natāmsa of vitribha śanku is found from its krānti and akśamśa - - - - equation (9)

'Para' is calculated from 100/216 × vitribha śanku - - - (11a)

or $\frac{\text{Parama laṁbana} \times \text{vitribha śaṅku}}{\text{radius}}$ - - - - (11)

Then from equation (11)

Lambana = Para × Jyā of viśleṣāmśa in asu

or Lambanajyā $= \dfrac{\text{Para} \times \text{Jyā of viśleṣāmśa}}{\text{radius}}$ in kalā

(12)

For local place on surface, moon will be in same direction as sun before geocentric amānta when sun is in east. Because both move in east direction and in east half of sky moon appears further east due to parallax. In west sky, moon will be towards west, and it will reach sun's apparent positon towards east after lambana time.

Lambana will change at new position, hence the procedure is repeated for further accuracy.

Verse 16-22 : Accurate lambana in a single step.

Now I tell the method to find accurate lambana in a single step.

At samaparva kāla, koṭijyā and bhuja jyā of difference between sun and lagna is found. Square of (difference of para stated above and bhuja jyā) and square of koṭi jyā are added. Square root of sum is karṇa. Koṭi jyā multiplied by para and divided by karṇa gives mean lambana time in asu. From this mean lambana, samaparva kāla is corrected and dṛggati of that time is multiplied by madhyama lambana and divided by initial dṛggati (Here drggati means dṛg gati of tribhona lagna i.e. vitribha śanku). This is madhyama sphuṭa lambana, as stated by Bhāskara II.

If this is more than madhyama lambana, their difference in asu is squared, multiplied by madhya lambana. Result is added to madhya sphuṭa lambana. We take any of these - Ist sphuṭa lambana of Bhāskara or second sphuṭa lambana - multiply

it by mean gati difference of sun and moon and divide by (first sphuṭa gati of moon - sphuṭa sun gati.) By this Bhāskarīya lambana becomes more sphuṭa. With this value of sphuṭa laṁbana in ghaṭī, parva kāla is corrected as before.

Alternatively, at sphuṭa parva kāla, koṭijyā of difference of sphuṭa sun and lagna is multiplied by 276 × dṛggati and divided by trijyā. This gives laṁbana in pala.

This is multiplied by difference of mean gati of sun and moon and divided by difference of sphuṭa gati. This will give difference of sphuṭa and samaparva kāla.

Notes (1) Bhāskara formula -

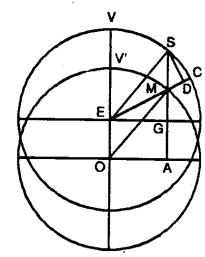



Figure 4 - Bhāskara sphuṭa laṁbana

E is earth, circle with centre E is sun orbit, circle with centre O is moon's orbit, deflected due to parallax.

Vitribha lagna V and V' of the orbits are in same direction from E, so that when sun is at V and moon at V', there is no lambana.

Maximum lambana for this position of ecliptic is when distance from vitribha of sun and moon is 90°. This is OE. Directions OM and ES of sun

from their vitribha is same, hence these lines are parallel and equal. In parallelogram ESMO, SM also is parallel and equal to OE. Thus SM is equal to parama lambana or 'para' in short.

SM // VE, hence is perpendicular to horizontal lines at G and A.

Now in similar triangles SDM and EMG,

$$SD = \frac{EG \times SM}{EM}$$

$$\text{or } R \sin (\text{lambana}) = \frac{R \sin (s \approx v) \times \text{para}}{\text{Karṇa}} \quad \text{- - - (1)}$$

Karṇa EM

$$= \sqrt{MG^2 + EG^2} = \sqrt{(SG - SM)^2 + EG^2}$$

$$= \sqrt{[R \cos (S \approx V) - \text{para}]^2 + [R \sin (s - V)]^2} \quad \ldots\ldots (a)$$

$$\text{where para} = \frac{\text{dṛkkśepa śaṅku} \times R \sin 25\text{-}5/7^\circ}{R} \quad \text{- - (b)}$$

We get maximum lambana when dṛkkśepa śaṅku = R i.e. vitribha coincides with zenith and ecliptic is vertical. Then it is 1/14 of daily movement of moon which is 360° in angles. Thus max. lambana = 360°/14 = 25-5/7°.

Putting values of (a) and (b) in (1) we get the formula.

(2) Further corrections : Vitribha lagna is 90° from lagna by definition, hence V-L = 90°, L = lagna.

Then Sin (s ~ V) = Sin [(V-L) - (S-L)] = cos (S-L) and cos (S ~ V) = Sin (S ~ L)

Further correction is based on sphuṭa gati difference of sun and moon as we had assumed

average gati in formula (b) above. Laṁbana angle = madhya laṁbana × madhya gati diff.

= sphuṭa lambana time × sphuṭa gati diff.

This ratio is basis for further correction.

Verses 23-39 : Nati correction in śara

After finding mid point of eclipse by methods described above, we have to find sphuṭa sthiti ardha in which śara of moon is to be corrected by nati.

For this; last sphuṭa gati of moon is found for eclipse purpose as explained in chapter 6. That will be multiplied by sphuṭa lambana in ghaṭī and divided by 60. Quotient will be added or substracted in sphuṭa sun of samaparva kāla as lambana correction. Pāta of moon (rāhu or ketu) is corrected with digamśa phala (1/10 of sun mandaphala - chanpter 6) and is substracted from sphuṭa moon. From this difference (candra-rāhu), śara is calculated.

Then from sphuṭa sun (sāyana) of that time, lagna and vitribha lagna of sphuṭa parva kāla is found and their krānti is calculated.

South śara is added to akśāmśa (where equator is towards south) and north śara substracted to get śarākśa.

Śarākśa and vitribha krānti are added, if in same direction or substracted for different directions. Result is nata (north south distance of moon from zenith). Jyā of this arc is natajyā. This is also called versine of madhya lagna or udayajyā.

Madhya jyā is multiplied by udaya jyā and divided by dyujyā. Square of quotient and square

of madhya jyā are added. Square root of sum is dṛkkśepa.

Alternatively, vitribha lagna is assumed sun, and for that position krānti and cara are calculated.

15 ghati + cara = dinārdha and its difference with vibribha lagna is natāsu. From this vitribha natāsu; dṛkjyā is found through utkrama jyā, cheda, iṣṭa hṛti, and vibribha śaṅku as explained in seventh chapter. (verses 45-51)

Arc of this dṛgjyā is vitribha natāmśa. When vitribha krānti is north of local akśāmśa, then natāmśa is north, otherwise it is south. This natāmśa and śara of moon will be added, if in same direction, otherwise difference taken. Jyā of the resulting arc is dṛkkśepa.

Thus there are two types of dṛkkśepa. Both are separately multiplied by moon gati in kalā and divided by trijyā. Results are added when manda kendra of moon is in six rāśis starting from karka (90° to 270°) or substracted for other six rāśis. By this, dṛkkśepa becomes sphuṭa.

When grāsa is less than 1 kalā or more than 28 kalā, then the second dṛkksepa is used which is corrected with vitribha natāmśa. For grāsa between 1 to 28 kalā first dṛkksepa is used which is corrected with śarākśa vitribha krānti (1/60 of total eclipse is one kalā - when moon has śara, eclipse will be less than half and hence śara is used for correction).

Difference of sun (21/38) and moon parma nati (56/28/13) i.e. (56/6/33) multiplied by dṛkkśepa and divided by trijyā gives śphuṭa nati.

$$\text{Also sphuṭa nati} = \frac{\text{Dṛkkśepa} - \frac{\text{Dṛkksepa}}{225}}{61}$$

This is in direction of dṛkksepa.

Śara of moon and sphuṭa nāti are added if in same direction and difference is taken for opposite direction. Resulting direction will be direction of greater value of śara or nati. From that, grāsa and sthiti ardha are calculated according to method stated in last chapter for lunar eclipse.

When north śara of moon is more than the akśāmśa of the place, akśāmśa will be substracted from it. Difference will be north śarākśa. When north krānti of vitribha is less than this śarākśa but more than akśāmśa, then akśāmśa is substracted from vitribha north krānti and result added to śarākśa gives north nata.

Vitribha north krānti if less, is deducted from akśāmśa and from remainder; śarākśa is substracted to get south nata. If śarakśa is more than remainder, their difference will be north nata.

Sum of vitribha south krānti and akśāmśa if less than śarakśa, their difference is north nata. These corrections are necessary for all places having more than 1° akśāmśa.

Thus in almost all places except equator region, natāmśa is calculated from vitribha krānti corrected with śarākśa; and dṛkkśepa, madhya jyā, nata in north south direction are calculated.

Notes : (1) Moon's latitude from ecliptic depends upon its distance from pāta (rāhu or ketu). Its effective latitude for solar eclipse is latitude corrected for nati.

Now nati itself depdnds upon moon's distance from zenith towards south - consisting of two components -

Distance of vitribha from zenith (it is only in north south direction - it is sum of akśāmśa and kranti.

Distance of moon from ecliptic - i.e. śara, Thus total distance in north south direction is algebraic sum of.

Krānti of vitribha$\pm$akśāmśa of place$\pm$śara- -(1)

These are added if in same direction and substracted if in different direction.

Total śara = śara$\pm$lambana - - (2)

(2) Nati of moon :

In figure 5, Z is zenith, V is central ecliptic point (Tribhona lagna), S the sun, S′ apparent sun due to parallax, and S′A the perpendicular from S′ on the ecliptic. Then from similar triangles SS′A and SZV,

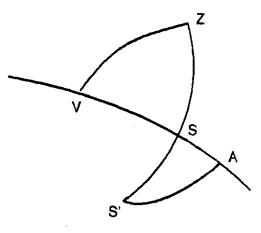



Figure 5 - Nati of moon

S′A or sun's nati (approximately R sin S′A)

$$= \frac{R \sin ZV \times R \operatorname{Sin} SS'}{R \sin SZ} \quad \ldots\ldots. (3)$$

$$= \frac{\text{Sun's dṛkkśepa} \times R \sin SS'}{R \sin SZ}$$

But R sin SS′

$$= \frac{\text{parama nati of sun} \times R \sin SZ}{R} \quad - - (4)$$

$$= \frac{\text{Earth's semi diameter in yojana} \times R}{\text{Sun's mean distance in yojanas}} \times \frac{R \sin SZ}{R}$$

Hence, Sun's nāti

$$= \frac{\text{Sun's dṛkkśepa} \times \text{Earth's semi diameter in yojanas}}{\text{Sun's mean distance in yojanas}}$$

$$= \frac{\text{Suns dṛkkśepa} \times \text{sun's true distance in yojana}}{\text{Sun's mean distance in yojanas}}$$

$$\times \frac{\text{Earth's semi diameter in yojanas}}{\text{Sun's true distance in yojanas}}$$

$$= \frac{\text{Sun's drkkśepa} \times \text{Sun's manda karṇa in minutes}}{R}$$

$$\times \frac{\text{Earth's semi diameter in yojanas}}{\text{Sun's true distance in yojanas}}$$

or, Sun nati

$$= \frac{\text{Sun's true dṛkkśepa} \times \text{Earth's semi diameter in yojana}}{\text{Sun's true distance in yojanas.}}$$

Similarly moon's nati =

$$\frac{\text{Moon's true dṛkkśepa} \times \text{Earth's semi diameter in yojanas}}{\text{Mon's true distance in yojanas}}$$

Alternate formulas

From (3) and (4)

Sphuṭa nati (difference of sphuta nati of moon and sun)

$$= \frac{\text{Diff. of parama nati} \times \text{dṛkkśepa}}{\text{Radius}} \quad --- (5)$$

as dṛkkśepa of sun and moon is same when they have same longitude after lambana correction, giving the values—

$$\text{Sphuṭa nāti} = \text{dṛkkśepa} \times \frac{56/28/13 - 21/38}{3438}$$

$$= \frac{\text{dṛkkśepa}}{61}\left(1 - \frac{1}{225}\right) \quad \ldots\ldots (6)$$

(3) Complete procedure for sthitiardha -

(a) First of all, calculate the time of geocentric conjunction (gaṇitāgata or karaṇāgata darśānta or amānta). Then calculate the lambana for that time and treating it as lambana for the time of apparent conjunction, obtain the time of apparent conjunction by the formula -

Time of apparent conjunction = Time of geocentric conjunction ±Lambana for the time of apparent conjunction - - - - (1)

+ or - sign being taken according as the conjunction occurs to the west or east of the central ecliptic point. Next, calculate the lambana for the time of apparent conjunction obtained and then again apparent conjunction is calculated from formula (1).

For the time of this second apparent conjunction, lambana is calculated and again aparent conjunction is calculated (third) by formula (1).

This process is repeated till lambana for the time of apparent conjunction is fixed. Applying this lambana in formula (1) we get the correct time of apparent conjunction. This is the time of spaṣṭa darśānta or spaṣṭa amānta, and also the time of middle of the eclipse.

(b) Spārśika and maukśika sthiti ardhas - Calculate the semi diameters of the sun and moon and also moon's true latitude corrected for nati as explained in notes (1) and (2), for the time of apparent conjunction. This is almost equal to

moon's latitude at first contact time β_1. If S and M are semi diameters of Sun and moon, d is difference between true daily motion of moon and sun in degrees -

$$\text{spārśika sthityardha} = \frac{\sqrt{(S + M)^2 - \beta_1^2}}{d} \text{ ghatīs} \quad \text{- - (2)}$$

In practice, one uses the semi diameters of the sun and moon for the time of apparent conjunction, because, for the time of first contact, there is negligible change.

Therefore, time of first contact

= Time of apparent conjunction - spārśika sthityardha - - - - - - - - - (3)

Next, calculate the moon's true latitude for the time of first contact thus obtained; and then find the spārśika sthityardha by formula (2), then time of first contact by formula (3).

Then calculate the moon's true latitude for the time of first contact (2nd value), then calculate the spārśika sthityardha by formula (2) and time of first contact by formula (3) again.

Repeat this process unitil the spārśika sthityardha and the time of the first contact are fixed.

The sthityardhas and vimardārdhas which are thus obtained are called madhyama (or mean), because they are still uncorrected for laṁbana.

(c) Lambana for times of apparent first contact and separation-Calculate the laṁbana for the time of first contact obtained above and treating it as the lambana for the time of apparent first contact, obtain the time of apparent first contact by the formula—

Time of apparent first contact = time of first contact ± lambana for the time of apparent first contact - - - - - - (4)

+ or - sign being taken according as the first contact takes place to the west or east of the central ecliptic point.

For the time of apparent first contact, thus obtained, calculate the lambana afresh and applying it in formula (4), obtain the time of first contact again.

Repeat this process until the lambana for the time of apparent first contact is fixed.

Similarly, find the lambanas for the times of apparent separation, immersion and emersion

(d) Spārśika and maukśika sthityardhas, corrected for lam̆bana —

The madhyama spārśika and madhyama maukśika sthityardhas corrected for lambana, are called true (sphuṭa) spārśika and sphuṭa maukśika sthityardhas. They are obtained by the formula.

True spārśika sthityardha = time of apparent conjunction - time of apparent first contact

True maukśika sthityardha = Time of apparent separation - time of apparent conjunction.

Similarly,

True spārśika vimardārdha = Time of apparent conjunction - time of apparent immersion

True maukśika vimardārdha = Time of apparent emersion - time of apparent conjunction

Verses 40-42 : **More accurate value of moon diameter (bimba)** - Bimba (angular diameter) of

sun and moon is calculated according to method given in previous chapter on candra grahaṇa. Now method is being given to make it more accurate. This has not been told by any earlier scholar (ācārya).

If manda kendra of moon is in six rāśis starting with karka, its koṭi phala is substracted from trijyā, otherwise they are added. Square of the result is added to square of manda bhujaphala. Square root of sum is substracted from twice the trijyā. By remainder, square of trijyā is divided. Result will be manda karṇa in liptā (i.e. kalā or minute of arc). Mean biṁba kalā of candra (31/20) is multiplied by trijyā (3438) and product (107724) is divided by manda karṇa. It will give sphuṭa biṁbamāna of moon.

Notes (1) If R and r are radius of main circle and manda paridhi, then

Koṭi of karṇa = $R + r \cos \theta$

when θ is manda Kendra

Bhuja of karṇa = $r \text{ Sin } \theta$

Hence, karṇa K is given by

$$K^2 = (R + r \cos \theta)^2 + (r \sin \theta)^2 \quad --- (1)$$

This is the correct formula. However, in place of bhuja phala or koṭi phala we take the lower value

Bhujaphala = $r \sin \theta \times R/K$

Similarly, koṭiphala, $r \cos \theta$ also in reduced in same ratio.

Thus we take

$$K_1^2 = \left(R + r \cos \theta \times \frac{R}{K}\right)^2 + \left(\frac{r \sin \theta \times R}{K}\right)^2$$

$$= R^2 + \frac{2r\,R}{K} (\cos\theta + \sin\theta)$$

$$+ \left(\frac{Rr}{K}\right)^2 (\cos^2\theta + \sin^2\theta)$$

$$\text{or } K_1 = R + \frac{rR}{K} (\cos\theta + \sin\theta) \text{ approx.} \; - - - (2)$$

$$2\,R - K_1 = R - \frac{rR}{K} (\cos\theta + \sin\theta)$$

$$\frac{R^2}{2R - K_1} = \frac{R^2}{R - \frac{rR}{K}(\cos\theta + \sin\theta)}$$

$$\frac{R}{1 - \frac{r}{k}(\cos\theta + \sin\theta)}$$

$$= R \left[1 + \frac{r}{K}(\cos\theta + \sin\theta)\right] = K \text{ from } (2)$$

(2) Mean bimba X mean distance (trijyā)
= True biṁba × true distance (manda karṇa)
= Diameter in length units.

Verses 43-45 : Methods for calculating tamomāna -

(1) At the time of sphuṭa amānta time, we find śanku and dṛgjyā from spaṣṭa sun. Parama lambana (56/28) is substracted from śaṅku. Squares of remainder and dṛgjyā are added and of the sum, square root is taken. This will be tama karṇa (chāyā karṇa). Sphuṭa candra biṁba is multiplied by trijyā and divided by tama-karṇa

This gives tamo-māna or grāhaka (eclipser) value in solar eclipse.

(2) Alternatively, 1/60 of sphuṭa candra bimba is multiplied by śaṅku of sphuṭa parva time and divided by trijyā. Quotient is added to sphuṭa candra bimba to get tamomāna.

(3) Due to hard labour involved in calculating tamo bimba through śanku etc., I have found an easy method also for this. Unnata kāla in ghaṭī at the time of sphuṭa parva kāla is multiplied by 2, the product in vikalā is added to sphuṭa candra biṁba in kalā etc.

Notes : (1) In previous verses sphuṭa candra biṁba has been calculated for its variation in distance from earth's centre. However, due to parallax in observing moon from surface, its angle from vertical is increased, but distance is decreased. Though we correct the angle difference, the distance difference still remains. Since moon is seen at a nearer distance due to parallax, its effective angular diameter will appear increased. We have to calculate the increased biṁba māna. Here tamo-māna is not the value of shadow, because shadow is not the cause of solar eclipse. Moon disc itself appears dark compared to sun and is called tama.

(2) Derivation of formula - Figure 4 after verse 22 may be referred to For clarity, a smaller figure is made here (figure 5.) OZ is vertical and ZSH the great circle from Z through S, centre of sun and Moon. OH is horizon line. Sun and moon are at same place on samaparva kāla, but figure at amānta time is shown when M is separate due to

parallax, so that distance difference is shown. $\angle SOZ = Z$ is distance from vertical. SP is śanku $= R \cos Z$, and $SN = R \sin Z$ is dṛgjyā.

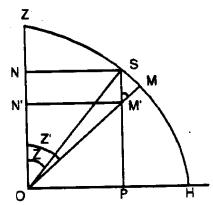



Figure 5a - Tamomāna increase in bimba due to parallax

Due to parallax, moon is lowered to M′ where SM′ is equal to parama lambana of moon (as in figure 5a).

SM′ = P. On celestial sphere moon is seen at M in that direction. SM is small and this arc and straight line are almost same.

In right angled triangle SMM′, $\angle$ SMM′ = 90°, $\angle$ SM′M = z′ when ZOM = z′. At sphuṭa samaparva kāla z = z′.

Thus lambana SM = SM′ sin z′ = P Sin z′ which is according to the formula for lambana. It comfirms that apparent height of moon is lowered by distance SM′ = parama lambana.

Apparent distance from surface is OM′ = tamo-karṇa of moon

$OM'^2 = OP^2 + PM'^2$

$= NS^2 + (SP-SM')^2$

or tamokarṇa

$= \sqrt{\text{dṛgjyā}^2 + (\text{Śanku} - \text{parama lambana})^2}$ - (1)

Sphuṭa bimba of moon has been calculated for the distance of radius OM from earth's centre.

Apparent biṁba at M′ is bigger, which is tamomāna.

Hence, linear diameter being same

linear diameter = tamomāna × OM′

= sphuṭa biṁba × OM

$$\text{or, tamo māna} = \frac{\text{Sphuṭa biṁba} \times \text{Trijyā}}{\text{Tama karṇa}} \quad --- (2)$$

(3) Alternate formula -

$$\text{Increase in sphuṭa biṁba} = \left(\frac{\textit{Trijyā}}{\textit{Tama Karṇa}} - 1\right) \text{ parts of}$$
biṁba

$$= \frac{OM - OM'}{OM'} = \frac{M'M}{OM'} = \frac{P \cos z'}{R - P \operatorname{Cos} z'} \quad --- (3)$$

when z = O, at Z, increase is maximum $= \frac{P}{R - P}$

Absolute in crease is P cos z′

Then fractional increase in sphuṭa biṁba

$$= \frac{P}{R - P} \quad \frac{R \cos z'}{R}, \text{ as } P \cos z' \approx P$$

$$\frac{56/28}{3438 - 56/28} \quad \frac{\text{Śanku}}{\text{Trijyā}} = \frac{1}{60} \quad \frac{\text{Śanku}}{\text{Trijyā}} \quad --- (4)$$

In 15 ghāṭi unnata kāla, increase in moon biṁba is 1/60 of sphuṭa biṁba = 1/60 × 30 kalā approximately, when moon is at Z.

Hence in 1 ghaṭi increase is $\frac{1}{60} \times \frac{30}{15}$ kalā

= 2 vikalā approximately

Thus for each ghaṭi unnata kāla, sphuṭa biṁba increases by about 2 vikalā.

Verses 46-47 : Hāra of solar eclipse.

Sphuṭa candra gati is multiplied by 1/60 of sphuṭa śaṁku of sun and divided by trijyā. In

quotient, final sphuṭa gati of moon is added. Sum substracted from sun gati will be hāra at the time of eclipse (mid time).

Unnata kāla in ghaṭī is reduced by its 1/8, remaining is assumed as kalā and added to final sphuṭa gati of moon and sphuṭa gati of sun is substracted. Result is hāra of sparśa and mokśa time.

Notes : (1) Hāra means multiplier; here the purpose of this multiplier is not mentioned. However, in verses 46-50 of previous chapter on lunar eclipse, hāra is used for calculating amount of grāsa (magnitude of eclipse) at desired time. Hāra in that context is difference of moon's speed and sun's speed. For solar eclipse this needs accurate calculation and correction for lambana.

Hāra = Candragati - sūrya gati - - - (1)

In this, variation due to parallax is only in candragati as the parallax of sun is negligible.

$$\text{Bi\.{m}ba} = \frac{\text{linear diameter}}{\text{true distance}}$$

$$\text{gati} = \frac{\text{linear motion}}{\text{true distance}}$$

Thus biṁba and gati of moon both increase in same proportion due to apparent decrease in distance due to lambana or parallax.

Thus according to first alternative formula in note (3) of previous verse, equation (4) is

Proportional increase in candragati

$$= \frac{1}{60} \times \frac{\text{sphu\d{t}a \'{s}anku}}{\text{Trijy\=a}} \quad \text{- - - (2)}$$

This correction put in equation (1) gives the first formula for lambana corrected hāra

(2) Hāra for sparśa or mokśa time -

At 15 ghaṭi unnata kāla the increase in candragati from (2) is 1/60 part of its gati, when moon is at Z approximately.

This increase is 790′35/60 = 13-1/6 kāla approximately. Hence, proportionate increase of each ghati in moon gati is

$$\frac{13\frac{1}{6}}{15} = 1 - \frac{11}{6 \times 15} = 1 - \frac{1}{8} \text{ kalā approx.}$$

Thus ghaṭi is reduced by its 1/8 and remaining part taken as kalā is the increase in daily motion of moon due to parallax. For this, unnata ghaṭī of sparśa or mokśa time is taken.

Verses 48-49 : Difference in solar eclipse at each place.

In lunar eclipse, shadow of earth and moon - both are at same place (in moon's orbit), hence grāsa is same at all places, because there is no parallax. But in solar eclipse, chādya sun and chādaka moon are very far from each other. Only at a particular place, they may be in one line, but at other place they will be seen in different direction due to laṁbana (or parallax). Thus solar eclipse has different magnitudes for different places). Even due to a small difference in east west or north south direction, there will be difference in total eclipse, annular or partial eclipse. Hence, they are to be calculated separately for each place.

Notes : Location of the point of observation is only reason for solar eclipse, other wise they

are vastly far from each other. This has been explained in beginning of previous chapter and while calculation of solar eclipse also.

Solar eclipse is seen in a very small circle cut in moon's shadow cone by earth's surface. In north south direction from that circle, eclipse will become partial and then non existant.

Due to relative motion of moon towards east the shadow circle on earth's surface moves from west to east and finally leaves. Thus the eclipse is earlier in west and later in eastern places on the strip of earth surface. Thus due to east west difference of places, eclipse times and grāsa times will be different (according to standard time also).

When tip of shadow cone is about to leave earth surface, before and after the strip, when circle on surface is of zero radius, extended shadow cone touches the surface. Then annular eclipse is seen at those places.

Verses 50-53 : Madhya sphuṭa sthiti kāla

According to rules explained in candra grahaṇa chapter, we calculate the sphuṭa śara, half sum of bimba. From hāra of grahaṇa time we calculate the sthiti ardha and marda ardha in ghaṭī. By adding or substracting this from samaparva kāla, we get times of sparśa, mokśa, sammīlana and unmīlana.

Then current lagna and vitribha lagna is found and lambana in east west direction is calculated. Sparśa and mokśa times are corrected with this lambana. For these sparśa times etc, we calculate the lambana again and second value of sthitiardha and sparśa kāla is found. For second values of

sparśa and mokśa times, lambana is again calculated and from that we get third value of sparśa or mokśa. After repeated process, when there is no difference in successive values, we get the true values.

Verses 54-56 : Sphuṭa sthiti kāla by śara correction From difference of sphuṭa parva kāla and these times of sparśa etc., we get the values of both sthiti ardha and marda ardha in ghaṭī etc. Alternatively, we find the sphuṭa śara by single step method (verse 45 of previous chapter), and new values are found. From their ratio, śara for sparśa and mokśa time is found. One difference is + ve and other is negative. Both changed by half the sum give the śara of sparśa, mokśa time.

From this śara, second value of second sthiti ardha is found. From that we find śara for sparśa, mokśa and middle time śara. Then we find the difference of middle śara with the śara of sparśa and mokśa times. By proportionate difference we again find sphuṭa śara ardha. After repeated process sthiti ardha becomes spaṣṭa.

Notes : (1) Correction of sthti ardha for lambana by repeated process has already been explained after verse 53 and in notes after verse 39.

(2) Suppose the śara at middle time be L and sparśa time śara is l_1. By single step method, the spaṣta śara is L′. Thus difference of śara is L′-l, in single step method and L-l_1 in repeated method. Thus the difference of single step method is to be changed by $(L-l_1)/(L_1-l_1)$ for correct difference. Thus we get accurate śara by one step method. If sparśa

time śara is less than middle time śara, mokśa time śara will be more.

Verse 57 : Method for small sthiti ardha We take the difference of sthiti ardha after Ist lambana correction and the sthiti ardha before that correction (initial value). Square of difference in pala is divided by initial sthiti ardha. Result is added to sthti ardha obtained initially.

This process is done only for sthiti ardha less than 1 daṇḍa. From new values we get correct sparśa time etc.

Note : Let the sparśa times counted from middle eclipse time be t_0, t_1 and t_2 before śara correction and after first and second śara corrections. For small sthiti ardha, second corrected time t_2 will be almost correct time. Change in sthiti ardha after Ist correction is

$$t_1 - t_0 = t_0 \left(\frac{t_1}{t_o} - 1\right)$$

It is assumed that sthiti ardha will change in same proportion $\left(\frac{t_1}{t_o} - 1\right)$ in next step also.

$$t_2 - t_0 = t_0 \left(\frac{t_1}{t_o} - 1\right)^2 = \frac{(t_1 - t_o)^2}{t_o}$$

Thus the correction is obtained by dividing square of difference of initial and first corrected sthiti ardha by initial sthiti ardha.

Same process can be used for mokśa time also. Proportional decrease or increase can be assumed only for small sthiti times.

Verses 58-60 - Single step method for sphuṭa sthiti time. We obtain sphuṭa śara for sparśa or mokśa times after adding or substracting madhya sthiti ardha from lambana corrected amānta. If this śara is more than sum of semi diameter of the biṁba; or equal to it, then madhya sthiti ardha is multiplied by grāsa kalā and made half. It is divided by difference of parva kāla śara and śara at sparśa or mokśa time (expressed in kalā). By this, mokśa and sthiti ardha are found in a single step only. From sthiti ardha times obtained, the corrected middle time gives sphuṭa lambana in one step only. Then sphuṭa śara will be found for lambana corrected sparśa and mokśa times in one step only.

Note (1) Grāsa kalā is amount of grāsa expressed as ratio of diameter of eclipsed planet, out of total kalā of 60. Thus

$$\text{grāsa kalā} = \frac{\text{sum of semi diameters} - \text{śara}}{\text{Diameter of eclipsed graha}} \times 60$$

When śara is more than semi diameter sum, then the planet will not be eclipsed and eclipse time will be shortened.

Average value of śara between madhya kāla and sparśa time is taken. When grāsa is small, its value nearer to middle time is taken, as the real sthiti ardha itself is shortened.

Verses 61-62 : Annular eclipse

In solar eclipse when biṁba of sun is more than tamo-biṁba (apparent biṁba of moon increased for parallax, then eclipse will be annular (valaya grāsa). Then, from sum of semi diameters, diameter of moon is substracted. From square of

the difference, square of sphuṭa śara is substracted. From square root of this difference, we find sthiti ardha etc. in pala as per method described in lunar eclipse chapter. This sthiti ardha pala is corrected for lambana and on adding or substracting from samaparva kāla, we get beginning and end times of valaya grāsa.

Notes : This method is same as that of total eclipse time in which difference of semi diameter is taken. In this case, we get valaya grāsa instead of total eclipse, because moon bimba is smaller.

Verses 63-64 - Reason for extra methods

Brahmagupta (son of Jiṣṇugupta) had observed errors in the calculation of eclipse durations, hence in his Brahma-sphuta-siddhānta, stated at the end of tithi chapter, corrections for nāḍī (āyana dṛk karma), bhuja of nata, its jyā etc.

The method described by Bhāskarācārya in his Siddhānta Siromani also doesn't give correct eclipse duration. Hence, on the difficult topic of solar eclipse, I have stated many more things.

Notes : Already many new improvements have been described to get more correct values of moon bimba etc. Now entirely new methods are being described for correct duration of eclipse. After that, modern methods will be described, as comments.

Verses 65-72 : Eclipse duration through yaṣṭi - After calculating sūrya grahaṇa by above rule, we multiply the sphuṭa śara at the time of sparśa, middle and mokśa, separately by the lagna krānti

jyā of their respective times to give yaṣṭi for the three times.

The three yaṣṭis are converted to parā (1/60 vikalā) and divided by hāra (candra gati - sūrya gati) for the time of sparśa etc. When lagna krānti and sphuṭa śara are in different direction, this result in pala etc is added to time of sparśa etc otherwise substracted. Then true sparśa, madhya and mokśa times are obtained.

If this time is more than previous time (i.e. yaṣṭi ÷ hāra is added for different directions of lagna krānti), then it is the true time for sparśa etc. If new time is less then previous, it is multiplied by its lambana jyā and divided by 'para' (stated arlier). Result is added to sparśa time etc., when sun is west from vitribha lagna, otherwise, it is substracted. This will give true times of sparśa, madhya and mokśa. Madhya time will again be corrected with sphuṭa lambana to get correct value.

Then squares of mid time śara and yaṣṭi are added and square root of the sum is sphuṭa madhya kāla śara. Then from the śara, sthiti ardha for sparśa and mokśa are found. They are separately multiplied by sphuṭa lagna dyujya for madhya kāla and divided by trijyā.

When śara of sparśa and mokśa is in same direction, first result is substracted from sparśa time and second result is added to mokśa time. When the two śara are in different direction, reverse is done.

The sparśa and mokśa times are corrected for their lambanas to get true values. But sthityardha

is multiplied by dyujyā of madhya kāla and divided by trijyā.

Lambana for parvānta is found from true sun of that time. At the time of sparśa and mokśa, lambana is calculated from position of moon at that time.

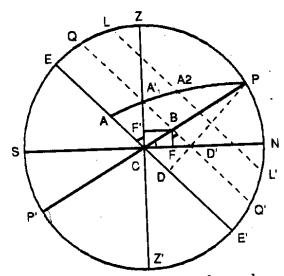



Figure 6 - Śara correction through yaṣṭi

Notes : To explain yaṣṭi, figure 10 after Tripraṣnādhikāra verse 37 is reproduced here. NZSZ′ is yāmyottara vṛtta, NS is horizontal line, ECE′ is diameter of equator.

QQ′ is diameter of diurnal circle of sun and LL′ is diurnal circle of moon further removed from equator as krānti and śara are in opposite direction. These three circles are bisected by perpendicular PP′ through poles - which is diameter of unmaṇḍala.

BQ = Dyujyā = semidiameter of diurnal circle $= R \cos \delta$ = corresponding to equator half day CE = 6 hours

BD′ = Kujyā = Extra length of half day on diurnal circle = BC $\tan \Phi$

$= R \sin \delta \tan \Phi$

CD = Carajyā = Extra length of half day on equator in asu

$$= \frac{BD'}{\cos \delta} = R \tan \delta \tan \phi$$

Now B is the position of sun when it has risen on equator. BD = height of sun at that time i.e. unmaṇḍala śaṅku.

Height of planet above B is called yaṣṭi.

Now A′ is the joint position of sun and moon on ecliptic, A its position on equator corresponding to arc CA in asu. Let CA = K

Height of A = CA cos Φ , where Φ is latitude) = K cos Φ

Height of A′ above B i..e yaṣṭi of A′ is

$= A'B \cos \Phi = K \cos \Phi \cos \delta$

Its rate of increase with respect to angular distance from equator is

$- K \cos \Phi \sin \delta$

Hence for change in distance corresponding to śara s of moon,

Increase in yaṣṭi = $s \cdot k \cos \Phi \sin \delta$

$$\text{Proportionate increase} = \frac{s\, k \cos \phi \sin \delta}{k \cos \phi}$$

$= s \sin \delta$ - (1)

This yaṣṭi is the proportionate increase in time units of yaṣṭi and not the iṣṭa yaṣṭi meant in chapter 7.

Thus increase in yaṣṭi is equivalent to decrease in lambana, hence moon will reach the sparśa time after corresponding interval. Thus increase in sparśa time = yasti/hāra or para where hāra is

relative speed of moon. When yaṣṭi is in vikalā/60 and hāra is in kalā/day, the result is in day X 60 X 60 = in palas. Similar addition is to be made for the times of madhya and mokśa also. When śara is in same direction as krānti, substraction is to be made.

When times are to be deducted they are changed in ratio (lambana jya / sama maṇḍala śanku), because lambana jyā is in time units.

Yaṣṭi is correction in śara of all times, hence average mid time śara is obtained by $(\text{śara}^2+\text{yaṣṭi}^2.)^{1/2}$

Verses 73-82 : Miscellancous corrections

If among sparśika and maukśika śaras, one is equal to middle time śara and other bigger, then there is a special method.

Ecliptic times are found by above methods and the sphuṭa śara of sparśa, madhya and mokśa time are multiplied by the krānti jyā of lagna of their times and divided by trijyā. When śara of sparśa and mokśa time are in same direction, these results are substracted from their śara; added if in different directions. Result is multiplied by difference of śara and divided by 36. We get yaṣṭi in liptā.

This is multiplied by jyā of distance between sun and vitribha lagna and divided by trijyā (3438), to get the third yaṣṭi. This third yaṣṭi in parā is divided by sthiti ardha for sparśa etc and the result in pala etc is added to the times of sparśa etc. when krānti and śara are in different directions, otherwise substracted. Thus we get the true times of sparśa, madhya and mokśa. Madhya kāla is

again corrected with sphuṭa lambana to get correct value.

Then madhya kāla śara and madhya yaṣṭi - both are squared, added and of the sum, square root is taken. With this sphuṭa madhya kāla śara, we calculate the sthiti ardha for mokśa and sparśa limes.

These are separately multiplied by dyujyā of madhya kāla lagna and divided by trijyā. First result is substracted from sthiti ardha of sparśa and second is added to mokśa sthiti ardha. Then both are corrected for their lambanas.

When difference between spaṣṭa śara of madhya kāla, and sum of semi diameters of biṁba is more than 3 kalā and krānti of sun is more than lagna krānti then sūrya grahaṇa is calculated according to this method.

If madhya śara is less than both the śaras at sparśa and mokśa time, more than both or equal to both, then first method should be used.

Notes : (1) Krānti of sun is between the krānti of lagna and krānti of vitribha lagna, hence it is aproximated by either of them, which are at 90° from each other. No earlier astronomer had used kranti of lagna from whiich eclipse time can be calculated through yaṣṭi difference. Yaṣṭi differernce is same as difference of śaṅku. Both methods give same errors. In calculation with yaṣṭi one time method has been used for calculating sthiti ardhas with sphuṭa śara corrected for yaṣṭi.

(2) This method of yaṣṭi and previous methods are almost same. When grāsa is 3 kalā or more,

(difference of śara and sum of semi diameters), then the approximate distance between sparśa and mokśa places will be (sun biṁba + 3 kalā) = 36 kalā aproximately. Hence śara difference is divided by 36 and resulting yaṣṭi is added to middle time śara. Approximately same will be added to other śaras also.

Verses 83-85 Only that grahaṇa (eclipse) is meaningful, which is seen from local place. No auspicious functions are needed for the grahana not seen at a place. Thus lunar eclipse in day time or solar eclipse in night time are not considered as grahaṇa for that place.

But even at the time of part solar eclipse in day time or part lunar eclipse in night should be observed according to smṛtis. Bath, charities etc should be done; cooking sleeping etc are prohibited.

As in lunar eclipse, in solar eclipse also grāsa from time and time from grāsa is calculated. Simlar method is used for ākśa and āyana valana.

Note (1) Amount of grāsa and time in solar eclipse.

Let T be the Indian standard time of conjunction in longitude, p is latitude of the moon, P the hourly change in latitude (north latitude and motion towards the north being considered positive), M is excess of hourly motion of moon in longitude over that of sun.

L is angular radius of moon, S angular radius of Sun. Then at anytime t hours after conjunction, the distance between the sun and moon's longitude

is Mt and the moon's latitude is (p + Pt). So the distance between their centres is $[M^2t^2 + (p + Pt)^2]^{1/2}$

The eclipse begins or ends, when their rims appear to touch. This can happen, even if the distance between them is greater than L+S, for the moon's parallax may push it towards the sun. The maximum of this effect is II-II' (= II); II being the equatorial horizontal parallax of the moon, II' of sun which is negligible.

Thus the rims can appear to touch when the distance between the centres is II + L+S (=d) at the most. Then $[M^2t^2 + (p+Pt)^2\ ] = d^2$ gives the times of the beginning and end of the general eclipse. Solving for t, we get

$$t = \frac{-\,p\,P}{M^2 + p^2} \mp \left\{\frac{p^2\,P^2}{M^2 + P^2} + \frac{d^2 - p^2}{M^2 + P^2}\right\}^{1/2}$$

In this, the upper sign (-) is taken for the beginning, and lower for the end. T+t is the IST of the beginning or the end.

At any given place, the eclipse begins or ends when the rims appear to touch at that place, i.e. when the apparent distance between centres is L+S. Now at any time T near the times of conjunction in longitude, let the apparent distance in longitude between the centres be m, the apparent excess of moon's hourly motion in longitude over the sun be M, apparent difference in latitude p, apparent excess of moon's hourly motion in latitude over that of sun be P, the sum of angular radii of sun and moon be d, and its variation per hour D. By apparent is meant here '(as affected by parallax)'.

Apparent m = real m + II cos A. Cos B (1 + II cos A Sin B)

Apparent p = (real p+II Sin A) (1+II cos A Sin B)

Apparent (L+S) = S+L (1+II cos A. Sin B)

where A is the zenith distance of vitribha lagna given by

Sin A = sin ω cos ϕ sin v - cos ω sin ϕ

and B is (lagna - moon's longitude) given by

B = Tan^{-1} [tan 1/2 (90°+v) cos 1/2 (90° + ϕ -w) / cos 1/2 (90°+ ϕ +w)]

+Tan^{-1} [tan 1/2 (90°+v) Sin 1/2 (90°+ ϕ-w) Sin 1/2 (90°+ ϕ +w)]

where Φ = latitude of the place

ω = obliquity of ecliptic (parama krānti)

and ν = sidereal time in degrees at the moment given by v = 97°30′ + east longitude of place in degrees from Greenwich + mean longitude of sun + IST at that moment in degrees.

For strict accuracy, the geocentric latitude and horizontal parallax at that latitude should be used.

If T is the time for which we have found m, p and d, the apparent distance between the centres of the sun and the moon at any time t hours after T is

$[m+Mt)^2 + (p+Pt)^2]^{1/2}$

When this time is equal to d+Dt, the eclipse begins or ends. Thus eclipse begins or ends at

$$T + \frac{d\,D - m\,M - pP}{M^2 + P^2}$$

$$\mp \left[\frac{(mM + pP - dD)^2}{(M^2 + P^2)^2} + \frac{d^2 - p^2 - m^2}{M^2 + p^2} \right]^{1/2}$$

The middle of the eclipse i.e. the maximum eclipse occus at $T + \dfrac{dD - mM - pP}{M^2 + P^2}$

The total eclipse begins or ends, when the rims apparently touch, the sun being within the moon. The distance between them at such time is (L-S), so by substituting for d in the above formula another d equal to (L-S), we can find the times of the beginning and end of the total phase.

S may be greater than L, so that moon may be immersed in the sun, leaving a circle of light all around. This is called annular eclipse. Beginning or end of the annular eclipse is got by making D = S-L.

(2) Bessel's method - for calculating solar eclipses - Bessel's method for calculating the circumstances of a solar eclipse as seen from a given place on the surface of earth consists in choosing a suitable system of axes, finding coordinates of the observer with respect to these axes and putting down in terms of these coordinates, the condition that the observer lies on the boundary of the penumbral cone at the beginning or end of the eclipse. All variable quantities in this condition are written in the form $x_0+x't$, where x_0 is the value of the variable quantity at t = o and x^1 is the rate of change of the variable quantity. The origin of time is chosen near the middle of the eclipse so that t is small. The condition now

becomes a quadratic equation in t, solving which we know the beginning and the end of the eclipse.

Besselian elements - Through the centre E of the earth draw a line paralled to the line joining the centres S, M of the sun and moon. Call this Z axis, its positive direction being on the side on which sun and moon are situated.

Choose the y axis to lie in the plane determined by the z-axis and the axis EN of the earth, the positive direction of y axis making an acute angle with EN. Finally choose the x-axis to be perpendicular to the axis of y and z, its positive direction being towards the point of equator, which the earth's rotation is carrying from the positive side to the negative side.

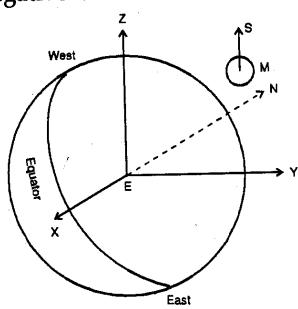



Figure7 - Bessalian elements for solar eclipoe

The plane z = O is called the fundamental plane.

These axes are not fixed with with respect to the surface of the earth. Therefore, the coordinates of a point on the surface of earth keep changing.

Certain quantities need to be calculated first which are required in the equations. These are called the Besselian elements.

(i) The elements d, x and y - Let the axes of x, y, z chosen as above meet the geocentric celestial sphere in X, Y, Z respectively. Let the right ascencion and declination of Z be (a,d)

Then, as is evident from the figure, equatorial coordinates of X and Y are, $(90^\circ + a, o)$ and $(180^\circ + a, 90^\circ - d)$.

To find a and d, we note that x and y coordinates of the sun and the moon are same (for Z axis is parallel to SM)

Let (d, δ) be the R.A. and declination of the sun and (d_1, δ_1) those of moon. If A is the sun's position on the celestial sphere, the values of Cos X A, Cos YA and Cos ZA can be easily written down. Thus, if (x, y, z) are the coordinates of the sun's centre S, and r is its distance from E, we have

$x = r \cos XA = r \cos \delta \sin (\alpha - a)$

$y = r \cos YA = r$ [Sin δ cos d - cos δ sin d. cos $(\alpha - a)$]

$z = r \cos ZA = r$ [sin δ sin d + cos δ cos d cos $(\alpha - a)$]

Similarly, coordinates (x_1, y_1, z_1) of the moon are

$x_1 = r_1 \cos \delta_1 \sin (\alpha_1 - a)$

$y_1 = r_1$ [Sin δ_1 cos d - cos δ_1 Sin d cos $(\alpha_1 - a]$

$z_1 = r_1$ [sin δ_1 sin d + cos δ cos d cos $(\alpha_1 - a]$

where r_1 is distance of moon's centre from E. Solving the equations obtained by putting $x = x_1$ and $y = y_1$, we get a and d, the later being one of the Besselian elements. Substitution of these

values in the expressions for x and y will give us x and y, the other two elements.

Values of x and y are calculated at the interval of 10 minutes for the whole duration of the eclipse. Therefore, x′ and y′, the variations in x and y per minute can also be easily determined.

The elements x and y are obviously the coordinates of the centre of the shadow on the fundamental plane.

(ii) The element μ - Let μ be the hour angle of Z from the meridian of Greenwich at the instant. The Greenwich sidereal time is g. Since the R.A. of Z is a, the value of μ is G-a. After μ has been tabulated at intervals of 10 minutes, μ' (the variation of μ per minute) can also be easily tabulated.

(iii) The elements f_1, f_2 - The semi vertical angles of the penumbral and umbral cones are denoted by f_1 and f_2 respectively. Now the radii of the sun and moon are R and b, and the distance between their centres is approximately $r-r_1$; so f_1 and f_2 are given by

$$\sin f_1 = \frac{R+b}{r-r_1}, \quad \sin f_2 = \frac{R-b}{r-r_1}$$

(iv) The elements l_1, l_2 - The radii of the circles in which the penumbral and umbral cones intersect the fundamental plane are denoted by l_1 and l_2 respectively. These also can be found by simple geometry.

$$l_1 = b \sec f_1 + z_1 \tan f_1$$

$$\text{and } l_2 = b \sec f_2 - z_1 \tan f_2$$

where z_1 is the distance of the moon's centre from the fundamental plane and has been found in paragraph (i) above.

In the Nautical Almanc, the quantities x, y, sin d, cos d, μ, l_1 and l_2 are tabulated at the intervals of 10 minutes for every solar eclipse. It is to be noted that these quantities relate to the whole of earth and not to any particular place on it.

Circumstances of solar eclipse at a given place -

Let ρ and Φ' be the geocentric distance and latitude of the place and x its longitude west of Greenwich . The hour angle of Z from the meridian of the place is $\mu-\lambda$ since the hour angle of Z from Greenwich is μ. So if (ξ, η, ζ) are the coordinates of the place at any instant, we have

$$\xi = \rho \cos \phi' \sin (\mu-\lambda)$$

$$\eta = \rho [\text{Sin } \phi' \cos d - \cos \phi' \sin d \cos (\mu-\lambda)]$$

$$\zeta = \rho [\sin \phi' \sin d + \cos \phi' \cos d \cos (\mu - \lambda)]$$

The values of $(\xi \eta, \zeta)$ can be computed for any instant. Also, since μ is the only variable in these expressions, formulae for (ξ', η', ζ') (the rates of changes of ξ, η, and, ζ per minute,) can be found by differentiation and the numerical values of (ξ', η', ζ') can be determined for the time of eclipse.

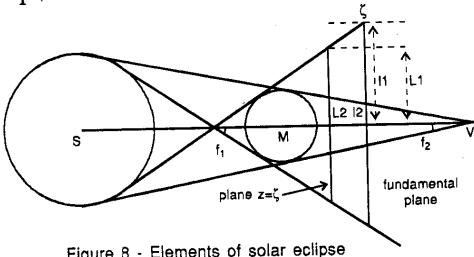



Figure 8 - Elements of solar eclipse

Consider now the sections of the penumbral and umbral cones by the plane $z = \zeta$, i.e. the plane through the observer parallel to the fundamental plane. The sections will be circles; and if their radii are L_1 (for the penumbra) and L_2 (for the umbra), we have from the figure

$$L_1 = l_1 - \zeta \tan f_1$$

$$L_2 = l_2 + \zeta \tan f_2$$

from which L_1 and L_2 can be determined.

Consider now the beginning or the end of a partial eclipse at the given place. At these two instants, the point (ξ,η,ζ) must be at the distance L_1 from the axis of the shadow, which cuts the fundamental plane in the point (x_1,y_1,o) and therefore cuts the plane $z = \zeta$ in the point x,y,ζ. The condition for this is

$$(x-\xi)^2 + (y-\eta)^2 = L_1^2 \quad \text{- - - (1)}$$

Replacing x, y, ξ and η by $x_o+x'1t$ and similar expressions, (1) becomes a quadratic in t. Solving it, we have the times for beginning and end of the partial eclipse. It we write L_2 for L_1 in (1), we can similarly determine the beginning and end of the total eclipse. In (1) it is sufficient to take the value of L_1 or (L_2) at an estimated time close to the time of occurrence of the eclipse, for L_1 and L_2 change very slowly.

To determine the point on sun's disc where the eclipse begins - Figure 9 represents the penumbra section by the fundamental plane. C is centre and CX', CY' are parallel to the axes of x and y. Then the generator of the penumbra thorugh Y' touches the sun in the most northerly point because the earth's axis lies in the plane

x= o. Also, the generator through X′ touches the disc in the most easterly point. Suppose that ($\xi\ \eta\ \zeta$) lies on the generator through T. Then, if angle Y′CT = θ

$$L_1 \text{ Sin } \theta = (x_o+x't) - (\xi_0 + \xi't)$$

$$L_1 \cos \theta = (y_o+y't) - (\eta_o+\eta't)$$

Substituting in it the values of t and the other quantities for the beginning or the end of the partial eclipse, we get the corresponding value of θ, which is the position angle of the point where eclipse begins or ends, because sun's disc is almost parallel to the fundamental plane.

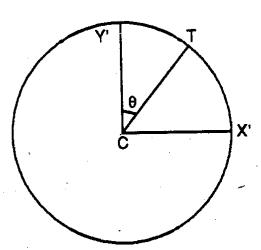



Figure 9 - Starting point of solar eclipse

Verses 86-87 : Maximum and minimum values of eclipse

Maximum duration of candra grahaṇa = 590 pala

Maximum duration of total lunar eclipse i.e. marda kāla = 273 pala

Maximum duration of solar eclipse = 632 pala

Maximum duration of annular eclipse (valaya grāsa) = 48 pala

Maximum duration of total annular eclipse (marda kāla) = 23 pala

Maximum increase in duration of a tithi = 405 pala i.e. maximum value is (60+6/45) = 66/45 daṇḍa

Maximum value of nakśatra tithi = 67/45 daṇḍa

Minimum value of nakśatra tithi = 52/12 daṇḍa

Maximum increase in yoga (beyond 60 daṇḍa) = 162 pala

Maximum decrease in yoga = 664 pala

Maximum gati phala of moon = + 7742 vikalā or - 4927 vikalā.

Maximum gati phala of sun = + 123 vikalā or - 117 vikalā

Maximum sphuṭa lambana = 5/12 ghaṭi

Notes (1) Maximum duration of lunar eclipse- The total duration of a lunar eclipse is given in hours

$$\frac{2}{\sqrt{p'^2 + m'^2}} \left[D^2 - P^2 \left(1 - \frac{p'^2}{p'^2 + m'^2}\right)\right]^{1/2}$$

where D is the distance between the centres of the moon and the shadow of first or last contact, P is the latitude of the moon at the time of opposition of the sun and the moon in longitude, p′ is increase in P per hour and m′ is the motion per hour in longitude of the moon, relative to the sun.

This is clearly 0 when $D^2 = P^2 \left(1 - \frac{p'^2}{p'^2 + m'^2}\right)$

i.e. when $D \pm P \left(1 - \frac{p'^2}{p'^2 + m'^2}\right)$

i.e. when P is numerically greater than D by $Dp'^2 / 2(p'^2+m'^2)$ approximately. This comes to about 14″ on the average. Thus even when P is greater

than D by upto 14″, at conjunction, there can be eclipse. When P=D, duration of eclipse is not O, but $2Pp'/p'^2+m'^2$, which is about 22 minutes.

The duration is maximum, when latitude of the opposition P is o. It is equal to $2D/p'^2+m'^2$

But D, m and p, are function of l and l′, mean anomalies of moon and sun respectively. Therefore, the maximum duration itself varies between limits.

Let l and l′ be anomalies at sthūla parva; time of fictitious conjunction or opposition or opposition between.

True moon = mean moon + 315′ sin l

True sun = mean sun + 127′ sin l′

Equatorial horizontal parallax of moon

$\Pi = 3447''.9 + 224''.4 \cos l$

for sun, $\Pi' = 8.8'' + 2'' \cos l$

Moon's semi diameter $r = 939''.6+61''.1 \cos l$

Sun's semi diameter $r' = 961''.2+16''.1 \cos l$

Radius of shadow $S = 2545''.4 + 228''.9 \cos l - 16''.2 \cos l'$

$\sqrt{m'^2 + p'^2} = 1875''.6 + 260''.1 \cos l - 5''.0 \cos l'$

Now the distance between the centres of the moon and the shadow of first or last contact

$D = s+r = 3485''.0+290''.0 \cos l - 16.1'' \cos l''$

and $\dfrac{2D}{\sqrt{m'^2 + p'^2}}$

$$= \frac{2\,(3485''.0 + 290''.0 \cos l - 16''.1 \cos l')}{1875.6 + 260.1 \cos l - 5''0 \cos l'}$$

This is a maximum when l = l' = 180° and not when l = l' = 0, as increase in denominator is more

Thus maximum value is

$$\frac{2(3438-290+16.1)}{1875.6-260.1+5} \text{ hours} = \text{about } 238 \text{ minutes}$$

This is correctly given as 590 pala = 5/2 X 238 min.

The lower limit occurs when l = l'= 0 and it is

$$\frac{2\ (3438 + 290-16.1)}{1875.6 + 260.1-5} = \text{about } 212 \text{ minutes}$$

If we do not neglect the function of 2l', the maximum is about 237.4 minutes.

Maximum duration of the total phase of a lunar eclipse is given by D = s-r. This also is maximum when sun andmoon are at opposition at the nodes and when l = l' = 180°. It is

$$\frac{2\ (1605.8 - 167.8 + 16.4)}{1875.6 - 260.1 + 5} \text{ hours} = \text{about } 108$$

minutes

It is given in text as 273 pala = 0.4 X 273 = 109.2 minutes

(2) Maximum duration of solar eclipse -

The formula for duration of a solar eclipse in general on any place on earth (as opposed to the duration at any particular place) is the same as for duration of a lunar eclipse. Only difference is that here

D = II-II' + r+r'

and P is the latitude of moon at conjunction of the sun and moon in longitude.

Here also the duration is not 0 when p = D, but when P is numerically greater than D by about 20″. When D = ± P, the duration is about 33 minutes

The maximum duration of a general solar eclipse occurs when P = 0, i.e. when conjunction in longitude is at a node. It is given by

$$\frac{2D}{\sqrt{p'^2 + m'^2}} \text{ hours}$$

$$= \frac{2\,(5339.9 + 285.5 \cos l + 15.9 \cos l')}{1875.6 + 260.1 \cos l - 5 \cos l} \text{ hours}$$

This is maximum when l = 180° and l′=0

Thus it is $\frac{2\,(5339.9 - 285.5 + 15.9)}{1875.6 - 260.1 - 5}$ hours

= 6 hours 18 minutes approx.

= 378 X 5/2 pala = 945 pala

Under this condition eclipse is annular. When 2 l term is not neglected maximum is about 6 hours 16 minutes.

The duration of a solar eclipse at a given place on the earth is given by $\frac{r + r'}{(P'^2 + m'^2)^{1/2}}$ corrected for parallax which changes rapidly and varies from place to place. But the maximum duration occurs when the central eclipse is at apparent noon. At this time, apparent semi diameter of moon is r + aobut 16″. Also at noon, the retardation in relative hourly motion of moon is maximum, causing increase in duration of eclipse. For an hour angle

34° on both sides of noon, the average retardaiton is (850″.3 + 55″.4 cos l) per hour. Total duration is given by

$$\frac{2\,(r + 16'' + r')}{\sqrt{p'^2 + m'^2}}$$ - hourly retardation due to parallax

=

$$\frac{2\,(1917 + 61 \cos l + 16 \cos l')}{(1875.6 + 260 \cos l - 5 \cos l') - (850.3 + 55.4 \cos l)}$$

=

$$\frac{2\,(1917 + 61 \cos l + 16 \cos l')}{1025.3 + 204.7 \cos l - 5 \cos l'}$$

When l = 180° and l′ = 0,

The maximum is about 4 hours 35 minutes = 275X5/2 pala = 687 pala (Text gives 632 pala)

This occurs when conjunction occurs at a Node, central eclipse falls at noon, l = 180° and l′= 0 .

The maximum duration of the annular or total phase at a given place is also at apparant noon for the same reason. As the period is very short, we take the motion per minute. The duration of an annular eclipse near noon is given by

$$\frac{2\,(r' - r - 16)}{(31''.3 + 4''.3 \cos l) - (15'' + 1'' \cos l)}$$

$$= \frac{2\,(5.7 - 61 \cos l + 16 \cos l')}{16.3 + 3.3 \cos l}$$

This is maximum when l = 180°, l′=0

Thus it is about 13 minutes (34 pala approx)

It is given 23 pala in the text. Minimum is clearly 0.

The total phase is given by

$$\frac{2\,(r + 16'' - r')}{(31''.3 + 4.3 \cos l) - (15'' + 1'' \cos l)}$$

$$= \frac{2\,(61.1 \cos l - 16.1 \cos l' - 5.7)}{16.3 + 3.3 \cos l}$$

This is max when $l = 0$, $l' = 180^\circ$ when it is

$$\frac{2 \times 71.5}{19.6} = \text{about 7 minutes } (= 17.5 \text{ pala})$$

This is not given in the text

(3) Other limit : Other limits depend on the maximum and minimum values of speeds of moon and sun. First we change the maximum gati phala

which is $\dfrac{\text{manda paradhi}}{360} \times$ dainika mean gati

Gati phala is + ve when manda paridhi is maximum at the end of odd quadrants. Hence maximum positive gati phala is more and negative gati phala is less.

From this we get maximum and minimum gatis of sun and moon, by

Max. gati = madhya gati + max. positive gati phala

Minimum gati = madhya gati - maximum negative gati phala

$$\text{Minimum tithi} = \frac{12^\circ}{\max(\text{moon gati} - \text{sun gati})}$$

$$\text{Maximum tithi} = \frac{12^\circ}{\text{min moon gati} - \text{max sun gati}}$$

$$\text{Max yoga} = \frac{13^\circ\ 20'}{\text{min (moon gati + sun gati)}}$$

$$\text{Minimum yoga} = \frac{13^\circ\ 20'}{\text{max (mon gati + sun gati)}}$$

$$\text{Maximum nakśatra} = \frac{13^\circ\ 20'}{\text{min moon gati}}$$

$$\text{Minimum nakśatra} = \frac{13^\circ\ 20'}{\text{maximum moon gati}}$$

Mean values of moon and sun gati are 790/35 and 59/8 Kalā. There mandaparidhi at odd quadrants is 12°6′ and 31°30′. Mandaparidhi of sun at end of second quadrant is 12°30′ and at the end of 4th quadrant is 11°54′. (Verses 95-96 of spaśtha dhikara, chapter 5). Moon's manda paridhi it is 188.5 kalā more at end of Ist quadrant and minimum is 188.5 kalā less at end of 3rd quadrant.

Verses 88-89 : Prayer and conclusion

The god is worshipped in forms of Pārvatī, Sūrya, Śiva and Ganeśa and gives fortunes to devotees. He also changes moon into rāhu (at the time of solar eclipse) and puts little knowing earthly creatures in confusion by covering sun like a flake of cloud. The same god may remove our troubles.

Thus ends the ninth chapter describing solar eclipse in Siddhānta Darpaṇa written for tally in observation and calculation and education of students by Śrī Candraśekhara born in renowned royal family of Orissa.

Chapter - 10

PARILEKHA

(Parilekha Varṇana)

Verse 1 : Scope - To show the direction of sparśa, madhya and mokśa in sūrya and candra grahaṇa clearly through diagrams, I explain the methods now.

Verse 2-3 : Valana - An oblique ray of light bends in water but doesn't bend in vertical direction. Due to that reason, the size of sun and moon and śara of moon, remaining same, it looks smaller in middle sky and bigger at horizon. Hence, earlier astronomers, changed the values of moon, sun earth's shadow in meridian depending on hāra at that time. This is being explained now.

Notes : Valana means bending. Light rays bend due to refraction, hence it is now called refraction effect. In appendix to Tripraśnādhikāra (chapter 7) this has been explained. If angle of incidence of light to a denser medium be i and angle of reflection be r then

$$\mu = \frac{\text{Sin i}}{\text{sin r}} = \text{a constant for the medium} \quad (1)$$

Hence the bending (i-r) increase with increase in angle of incidence. This angle is measured from perpendicular to the surface, hence in vertical direction there is no bending. As we move towards horizon, the bending is more. Thus, at sunrise (or

setting time) its lower end is at horizon having 90° natāmśa and upper end has slightly less (90°-32′) natāmśa. Thus the lower end will be raised more compared to upper and it will be flattened and look more elliptical.

The angle of bending or valana, R is

$R = K \tan z'$ - - - (2)

where $K = \mu - 1$ and z' is apparent zenith distance

Difference $z - z'$ is proportional to $K \sec^2 z$. $dz = 32' (K \sec^2 z)$ for sun.

Hence apparent angular diameter is difference between apparent natāmśa z and z′ of upper and lower.

Maximum refraction at horizon is about 35′. Its variation is very fast near horizon due to very high value of sec z near z = 90°.

Natāmśa	Refraction
0°	0″
5°	5″
10°	10″
15°	16″
30°	34″
45°	58″
60°	1′41″
80°	5′19″
85°	9′51″
88°	18′16″
88°40′	22′23″
90°	35′

Thus apparent reduction in vertical angular diameter at horizon is about 5′.

If D is the average of two perpendicular angular diameters observed at vertical distance z, then the real diameter is

D [1 + 1/2 K (1 + sec^2 z] - - - (4)

which is bigger than the observed. This means that observed diameter will decrease as Z increases and is minimum at horizon.

Verses 4-5 : Value of angular measure for biṁba.

Unnata kāla śaṅku of moon (for lunar eclipse) or sun (for solar eclipse) is calculated for middle time of eclipse. We add 10314 and divide the sum by trijyā (3438) to get the hāra or value of 1 aṅgula in kalā. On dividing the bimba of planets or shadow or śara of moon by this hāra, we get their diameters is aṅgula units.

Atternatively, half day is multiplied by 3 and added to unnata, kāla of moon (or sun) and divided by half day to get the same hāra. Value of biṁba and śara in angula units is obtained by dividing their values in kalā by this hāra.

Alternatively, for rough calculation, bimba Kalā is divided by 3 to get its value in angula.

Notes : (1) Sūrya siddhānta assumes (Candra grahaṇa verse 26) that the proportional angular diameter of a graha is 3 units at horizon, then it becomes 4 unit at vertical position i.e. increase in the ratio of 4/3. Bhāskarācārya and Lalla have assumed 2-1/2 : 3-1/2 increase i.e., in ratio of 7/5. Actual increase as we have seen after verse 3 is from (32′-5′) to 32′ in sun's biṁba i.e. in ratio of 32/27 = 1.2 approx. Thus the ratios 1.33 of sūrya

siddhānta and 1.4 of Bhāskara II are much higher than the true ratio.

Another approximation is that the increase has been assumed proportional to the angular rise above horizon upto value of 90° rise to top position, where it is maximum. Angle of rise $\theta°$ = 90°-z. Putting it in equation (4) above, apparent diameter is

$$D = \frac{T}{1 + \frac{1}{2} K (1 + \text{cosec}^2 \theta)}$$

For 0 = 0, lower term cosec ∞ = which is not correct approximation. However, the increase is in proportion to value of cosec θ and not proportional to θ as assumed. This is increase of average diameter. Vertical diameter will increase at double rate.

(2) 1 aṅgula = 3 Kalā at horizon

and = 4 Kalā at vertical position

Height is proportional to unnata śaṅku, as assumed.

For height of R (Trijyā = 3438′) increase is 1 kalā. Hence, for unnata śaṅku U,

increase is $\frac{U}{R}$ Kalā

Thus 1 angula = $3 + \frac{U}{R}$ Kalā

$$= \frac{3R + U}{R} \text{ Kalā} = \frac{3 \times 3438 + U}{3438} \text{ Kalā}$$

$$= \frac{U + 10314}{3438} \text{ Kalā}$$

This is the first formula

Roughly half day is of 15 ghaṭī when sun reaches at top. Actually it is still slightly away from zenith but that distance is ignored. Unnata kāla is in proportion to half day taken as 90° or 15 ghati.

Hence, $\frac{\text{Unnata kāla}}{\text{half day}} = \frac{U}{R}$

or, 1 aṅgula $= 3 + \frac{U}{R} = 3 + \frac{\text{Unnata Kāla}}{\text{half day}}$

$= \frac{3 \times \text{half day} + \text{unnata kāla}}{\text{half day}}$

This is alternative formula

If we totally ignore the variation due to refraction, except for horizon position, diameter is almost same, and 1 aṅgula = 3 kalā is uniformly assumed.

Thus $\frac{\text{biṁba in Kalā}}{\text{Kalā in 1 angula}}$ = biṁba in aṅgula

Verses 6-14 : Diagram for direction of eclipse

On a ground, plane like water level, a circle of 57/18 aṅgula semi-diameter is drawn with a compass. This is known as khagola vṛtta having two valanas.

From this centre only, another circle with radius of sum of semi diameters is also drawn which is called samāsa vṛtta.

From same centre a third circle is drawn with radius equal to the grāhya bimba (which is eclipsed)

Now according to method explained in Tripraśnādhikāra north south line and east west lines are drawn in khagola vṛtta. In lunar eclipse;

sparśa is from east and mokśa is in west direction. But in solar eclipse sparśa (beginning) is from west and mokśa is in east direction.

In khagola vṛtta we mark a point at a distance from east point for lunar eclipse equal to jyā of sphuṭa valana and in same direction as valana. A line from centre to that point is drawn. Similarly, at a distance from west point equal to and in direction of mokśa time valana, another point is chosen and a line from centre is drawn. In solar eclipse, the order of valana lines is reverse i.e. sparśa in west and mokśa in east direction. These lines are called valanāgra rekhā. Valanāgra rekhā cuts samāsa vṛtta on valana points. From these points, we mark the distance equal to sphuṭa śara jyā of moon at the time of sparśa or mokśa. These are called śarāgra vindu (in east for sparśa and nimīlana and west for unmīlana and mokśa in lunar eclipse, opposite direction in solar eclipse).

The line from centre to śarāgra point cuts grāhya and mokśa. Here śara and valana are given according to their current values.

Śara is in north south direction, some times in angle direction like agni koṇa (north east).

Notes : (1) Radius of khagola vrtta is 57/18 aṅgula because 57°18′ = 3438′ = length of radius. Hence 1/60 aṅgula on radius or circumference is equal to 1 minute or kalā. The method is same as in sūrya siddhānta, but there the radius is 49 aṅgula where 1 aṅgula was 70′.

Radius of samāsa vṛtta or grāhya vṛtta will be calculated according to value of aṅgula in kalā

calculated in verses 4-5. Roughly 1 aṅgula = 3 kalā. Similarly length of śara also is calculated in aṅgula.

However, valana is measured on khagola vṛtta where 1 pratyaṅgula (1/60 aṅgula) is equal to 1 kalā or 1 aṅgula = 1°. With this unit we measure the lengths.

(2) Method of drawing is best explained by actual diagram.

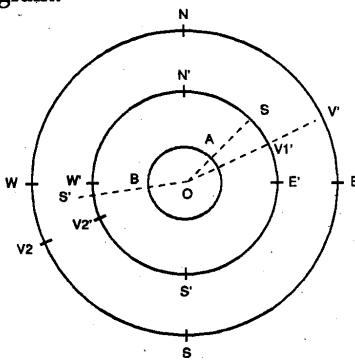



Figure 1 - Diagram for sparśa and mokśa in eclipse

ENWS - Khagola circle, 1 aṅgula = 1°, 57/18 radius, E'N'W'S' - samāsa vṛtta, E,E' east points, N, N', North points; W, W' west points S,S' South points, AB is grāhya bimba

EV_1 = Valana jyā for sparśa in solar, mokśa in lunar eclipse. WV_2 = Valana jyā for mokśa in solar eclipse, sparśa in innar eclipse $V_1'S$ = Current śara of moon

$V_2'S'$ current śara

V_1', V_2' are lines on samāsa vṛtta cut by OV_1, OV_2. OS; OS' cut grāhya on A,B which are points of contact.

Verse 15-30 : Further details for periods within sparśa and mokśa. Now, I describe the details of eclipse between the end points of sparśa and mokśa.

In lunar eclipse, when moon is near rāhu or ketu, spaṣṭa valana in khavṛtta is given in own direction from east or west point in north or south direction. From these valana end points, we give two points at distance of 5 aṅgula, in north direction from east valana, and south direction from west valana point. We draw a line through these points which also passes through centre of the circle.

In solar eclipse, we mark a point from eastern valana point at a distance equal to lagna kṛānti in the direction of krānti. This point is joined with centre and extended to make it diameter. Śara of moon is put in perpendicular direction on its line according to direction of the śara. (Śara will be at central point for middle position of the eclipse or any other point according to time of eclipse). From end point of śara a circle is drawn with radius of grāhaka bimba (eclipser) (This circle is drawn in lunar eclipse on 5° difference line).

The portion cut by grāhaka bimba will be the extent of eclipse visible to people.

Śara of sparśa, madhya and mokśa periods are put at their positions. From the three end points of śara, we draw three circles with radius equal to 1/3 of the distance between sparśa and mokśa. From intersection of adjacent triangles two fish like figures are formed. The head tail lines of these fish figures join at a point which is centre

of circle passing through these points. With this centre an arc is drawn through śara ends of sparśa, madhya and mokśa which is the grāhaka mārga (path of the eclipsing planet or shadow).

From centre of this grāhaka mārga, we draw a line in the direction of sparśa (eastern direction in lunar eclipse and west in solar eclipse), at the distance of grāhaka diameter from sparśa point, there will be nimīlana point on the grāhya circle. Similarly, unmīlana point on grāhya circle will be on the mokśa side of the grāhaka mārga.

To find the amount of grāsa at desired time we assume two parts of grāhaka mārga - from mid point to sparśa, it will be sparśa khaṇḍa and the other side will be mokśa khaṇḍa. Their length is measured in aṅgulas. The aṅgula measure is multiplied by required time (after sparśa or before mokśa) and divided by its sthiti ardha time. We give a point at a distance equal to aṅgula measure of required time from sparśa or mokśa point. From that point, we draw a circle with radius of grāhaka circle. The portion cut by this circle in the grāhya biṁba will be the required amount of grāsa at desired time.

Sum of semi diameters of grāhya and grāhaka is substracted from the required grāsa in aṅgula. A pointer equal to remaining length in aṅgula is taken. With this, we find two points on grāhaka mārga at distance of grāsa from centre of grāhya circle. One point is in sparśa khaṇḍa and the other in mokśa khaṇḍa. From these points we draw circle with radius of grāhaka biṁba. The portion covered by this circle will be the portion eclipsed.

At the distance of difference of semi-diameters of grāhya and grāhaka from centre of grāhya bimba, we get two points on grāhaka mārga - one on mokśa khaṇḍa and the other on sparśa khaṇḍa. These are the points of nimīlana (on sparśa khaṇḍa) and unmīlana.

Notes :

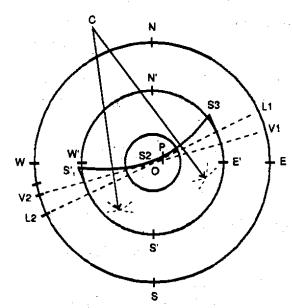



Figure 2 - Diagram for sparśa and mokśa in eclipse

ENWS are direction points on khagola circle, 1° = 1 aṅgula; Radius = 3438′ = 57°18′ = 57/18 aṅgula.

E′N′W′S′ - Samāsa circle direction points, radius equal to sum of semi diameters of grāhya and grāhaka. For bimba and śara length, 1 aṅgula

$$= 3 + \frac{\text{unnata sanku kalā}}{\text{Trijyā}}$$

V_1, V_2 are valana points. EV_1 and WV_2 are equal to magnitude of direction of valana jyā.

$V_1L_1 = V_2L_2 = 5°$ i.e. 5 aṅgula on Khagola circle which is equal to inclination of moon's orbit with ecliptic. Thus $L_1 L_2$ is path of moon for lunar eclipse.

For solar eclipse $V_1L_1 = V_2L_2$ = krānti of lagna.

On its intersection with samāsa circle and at centre, śara lengths at sparśa, madhya and mokśa points are drawn. perpendicular to it. It will be least at the centre and in direction of śara at all places. Their ends are S_1, S_2, S_3. The three circles through these points from two fish figures which intersect at point C. From C as centre with radius $CS_1 = CS_2 = CS_3$ we draw a circle. S_1 S_2 S_3 arc is the grāhaka mārga on which eclipser planet or shadow moves.

For lunar eclipse S_2 S_3 is sparśa khaṇḍa and S_1 S_2 is mokśa khaṇḍa. For solar eclipse S_1 S_2 is sparśa khaṇḍa and S_2 S_3 is mokśa khaṇḍa.

Nimīlana point P for lunar eclipse (or un-milana point for solar eclipse) is on grāhaka mārga such that S_3 P = diameter of grāhaka biṁba.

Length on grāhaka mārga is proportional to time.

Hence for any point P_c

$$\frac{\text{Length}}{S_2\ S_3} = \frac{\text{Desired time}}{\text{Sthiti ardha}}$$

This formula is used to calculate grāsa at desired time.

Verse 30-35 : Another method of diagram -

At śarāgra point on one side of valanāgara rekhā (śara is madhya śara), another line parallel to valana rekhā is drawn. From its end points on khagola circle, a point is given towards north for lunar eclipse (south for solar eclipse) at a distance of 1/60 of Jyā of local akśāṁśa.

From these two points and the point of madhya śara point (i.e. mid point of parallel line to valana rekhā) we draw a circle as explained in above verse.

Portion of this circle within samāsa circle will be grāhaka mārga. On this path, we can find nimīlana and unmīlana points from centre of grāhya circle at distance of difference of semi-diameters of grāhya-grāhaka,. as before. In this diagram sparśa and nimīlana of solar eclipse can be seen in west direction and, for lunar eclipse in opposite direction very easily. This method doesn't need śara or valana time at time of sparśa, etc.

But,. for diagram of solar eclipse, 1/3 of śara of moon (i.e. aṅgula value) is kept at two places. At one place it is multiplied by sun śaṅku of that time and divided by 4400. Quotient is added at first place.

On a single board both solar and lunar eclipses can be shown. Only difference will be that the direction of sparśa, mokśa etc will be opposite for the two types of eclipses.

Note : This is almost same procedure. In stead of marking śara at sparśa, mokśa and mid points, we mark the middle śara only. In stead of other śara, we mark the krānti of lagna on khagola at distance from middle śara. Reason is that the diurnal circle of moon will be parallel to ecliptic and at same angular distance from lagna point of ecliptic as on middle point of eclipse.

For solar eclipse śara is corrected for parallax. The correction is slightly less, which appears to

compensate effective increase of tamo-māna of moon as explained in chapter 9 verses 43-45.

Figure 1 and 2 show, that both the diagrams for solar and lunar eclipse can be combined, which has been prescribed here.

Verses 37-38 : Prayer and conclusion

I pray to lord Jagannātha, who smiles with beautiful lips, beauty of whose round eyes defeats the beauty of morning sun of spring time and full moon of winter night, who gives freedom from fear to people flocking to Nīlācala from different regions, and whose sight can emancipate the world.

Thus ends the tenth chapter on diagrams in siddhānta darpaṇa wirtten for calculation according to observation and instruction to students by Śrī Candraśekhara, born in famous royal family of Orissa.

Chapter - 11

CONJUNCTION OF PLANETS

GRAHA YUTI VARṆANA

(Conjunction of planets)

Verses 1-2-Scope - While the planets are moving in their own orbits, their position is seen same from earth. This is called graha yuti (conjunction of planets). Graha yuti and its good or bad results are described in this chapter.

According to Sūrya siddhānta, when tārā graha (maṅgala etc.) are seen joint, then their (apparent) coming together is called graha yuti or yuddha. When any tārā graha comes together with moon, it is called samāgama. When tārā graha is with sun, it is not visible due to bright rays of sun, and it is called 'asta mita' (heliacal setting of planets).

Notes (1) Planets do not really come together. They are in their own orbit which are far from each other. But due to parallax, they are seen together, as in solar eclipse, sun and moon are seen in same direction. However, the parallax is same for all positions from earth due to large distances of star like planets (tārā graha). Compared to eclipse of sun, the diameters of tārā graha are much smaller and their orbits are farther and slower, hence their conjunctions are rare. However,

their number is more causing different combinations of yuti and their śara also is small compared to moon's orbit, so we are able to see the yuti some times.

(2) Moon is considered the king of stars and the nakśatras as its wives. It lives with one nakśatra each day like a husband and wife - 'nakśate' means lives together. Thus conjunction of moon with any nakśatra or tārā graha is called śamāgama or happy union. Conjunction between tārā graha is called 'yuddhā, as it is not considered friendly. In this 'yuddha ' or war, the planet which is behind is like a chaser and takes away half the strength of the other planet which is considered defeated. This strength is considered in astrology for considering their power in giving good or bad results. The reduction or increase of strength is according to their mutual covering and depends on their aṅgular diameters. At present, we follow the method of Śrīpati for calculating the reduction or increase in strength due to planetary war.

Due to nearness with sun, the planets are invisible and called set due to sun. This has already been mentioned in chapter 6 and will be discussed in an independent chapter on it.

(3) Varāhamihira in his Bṛhat samhitā, explained in detail the various results of graha yuti. According to the degree of their seeming approachment, there are four kinds of wars (among planets) as stated by Parāśara and other sages - Bheda (occulation or cleaving), Ullekha (grazing), Aṁśu mardana (clashing of rays) and Apasavya (passing south ward).

Verses 3-5 : Principles of computation

We find rāśi, aṁśa and kalā of two planets in conjunction. When they are equal in ecliptic (kadamba prota vṛtta), their values on equator are found (dhruva prota vṛtta). From this, their śara and lambana are found. Then bimba (angular diameter) is calculated.

In sūrya siddhānta - When faster of the two planets has greater longitude (i.e. it is towards east), then conjunction has already occurred. If it is less (i.e. in west), then the conjunction is yet to occur. If both are vakrī (retrograde) then reverse will happen, i.e. planet in east indicates, conjunction is to occur, in west means conjunction has already passed. If one body alone is retrograde and its longitude is greater (in east), then the conjunction is to come, if less, it has passed.

Notes : (1) Conjunction is calculated first in longitude measured along ecliptic, when their positions are same. However, their difference in perpendicular direction (śara) and apparent deviation due to observing from earth will depend on position with respect to equator. Size of the bimba of planets will decide, at what distance they will meet.

(2) Finding conjunction, time, whether gone or yet to come is very easy to find, from diagram.

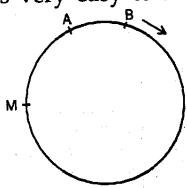



Figure 1 - Conjunction of planets

In figure 1, M is position on ecliptic, which is meṣa 0° from which position of planet is measured. When arrow direction indicates rotation in east direction, the rāśi aṁśa etc (longitude) of two points A and B are MA and MB. When longitude of B is more, it is east from A as seen from figure.

When B is faster, it will move further east from A, and at some earlier time it was with A i.e. in conjunction. If A or western planet is faster it will meet B in time needed to cover AB with relative speed. If B is retrograde in east position A and B, both approach each other with their speeds, hence it will approach with speed equal to sum of speeds. When both are moving in western direction, obviously the reverse of direct motion will happen.

Verses 6-9 : Finding the time and place of conjunction - At required time we find the bhogāṁśa (longitude) of the two graha and convert their difference into kalā. This is separately multiplied by daily speeds of graha in kalā. Each product is divided by difference of speeds if both have direct or both retrograde motion. But if one graha is mārgī and the other, vakrī, the products are divided by sum of speeds in kalā. If both planets have already joined and both are mārgī, then each quotient is deducted from the bhogaṁśa of its planet by whose speed it had been multiplied. If conjunction is yet to happen, then the quotients are added. If both are retrograde (vakrī), reverse is done. If one is vakrī and the other mārgī, then addition and substraction are done as per rules explained earlier. By this, we get the bhogāmśa of

Krānti vṛtta (position on ecliptic) where conjunction has happened. If the kalā of planets doesn't become equal in a single operation, this process is repeated again.

Notes : In figure 1,

longitude of A is MA, B is MB

Differnece in longitudes is MB-MA = AB

Speed of A is a and B is b kalā per day

Difference in speeds is a-b

if a > b, then A will catch up with B in time AB / (a-b)

if a, < b then B has gone ahead this difference AB in time AB / (b-a)

Thus in first case the longitude of conjunction for A will increase $\frac{AB}{a-b}$ X a, increase in B will be $\frac{AB}{a-b}$ x b. This increase in A will be $\frac{AB}{a-b}$ (a-b) more i.e. AB more and they will catch up.

If a ‹ b, then the conjunction time is earlier and longitude of A and B will be reduced by distances travelled by them.

For retrograde planets obviously situation will be reversed. If B is faster, it will catch up distance BA in time t = BA / (b-a) in which the longitude of B and A will be reduced by t b and t a.

Suppose A is retrograde and B is forward motion. There relative speed is at a+b and their distance is increasing. Then they are together at time AB / (a+b) = t before the present time. In this time longitude of A was more by ta because

it is retrograde and B was tb less, in earlier time of conjunction.

If A is direct and B is retrograde, then the plenets are approaching each other with velocity a+b and they will cover the distance AB in time AB / (a+b) = t when they will be together. After that time position of A will be ta more and of B will be tb less because it is moving in reverse direction.

(2) We are assuming uniform motion of planets in the interval AB. Within this the speeds will change, forward motion may become retrograde and vice verse. Thus after getting the conjunction time approximately on basis of present speeds, we again calculate the position difference at this approximate conjunction time. Then we calculate more accurately as to when conjunction had occurred or would occur.

Verses 10-11 : Śara of planets

(From Sūrya siddhānta - Spaṣṭādhikāra verse 56-57).

In pāta of maṅgala, śani and guru, correction for second śīghraphala is made in same manner, in which it is done for the planet (i.e. positive result is added and negative substracted). This will give the true postions of pāta of these three planets. But in pāta of budha and śukra, correction is made with second mandaphala (used in third step of correction) in reverse manner - i.e. positive result is substracted and negative added. By this, true pāta of budha and śukra will be known.

From true postions of maṅgala, śani and guru, true positions of their pāta are deducted to get vikśepa kendra. Vikśepa kendra of budha and śukra are found by substracting their true pāta from their śīghrocca positions.

Jyā of vikśepa kendra is multiplied by madhya vikśepa and divided by fourth śīghra karṇa to get the sphuṭa śara.

Notes : (1) Mean inclinations (vikśepa) of planetary orbits - This has been explained by Bhāskarācārya II. in his chapter on grahacchāyādhikāra (siddhānta śiromaṇi). Reasons of the method have also been explained.

The values of madhya vikśepa are given in chapter 5 - spaṣṭādhikāra verses 28-33, reproduced here

Planet	Siddhānta Darpaṇa value	Modern value
Candra	5°9′	5°8′42″
Maṅgala	1°51′	1°51′0″
Budha	2°44′	7°0′14″
Guru	1°18′	1°18′21″
Śukra	2°28′	3°23′39″
Sani	2°29′	2°29′25″

The values of superior planets are almost same as modern values. Bhāskara says that these values are for that time when śīghra anomaly is equal to $90° + 1/2\ R \sin^{-1} a$, where a is R sine of the maximum sīghra phala. This is quite correct,

Figure 2

Figure 3

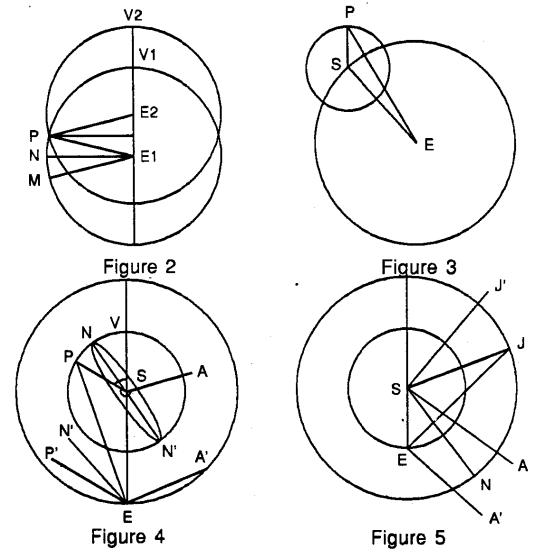



Figure 4

Figure 5

because when the śīghra anomaly has this value, the true planet is at point of intersection (P) of the deferent and eccentric circle. Then the planet is equidistant from E_1 and E_2 (figure 2) For superior planets, E_1 is taken as earth's centre and E_2 is sun, the mean latitude of the planet observed will be same, whether observed from earth or sun. Hence, maximum latitudes of the superior planets are same for geocentric and heliocentric observations. These are the mean values.

For inferior planets, mean planet in this case is taken to be sun, the linear values of the latitude observed from E and S, the centres of Earth and sun will be in ratio SP/ES (figure 3) For mercury

this ratio is 4/10 and for venus it is 7/10. Hence the modern values are reduced to $\frac{420 \times 4}{10}$ and $\frac{204 \times 7}{10}$ i.e. 168 and 142 which are approximately equal with the values given in siddhānta.

(2) Pāta is calculated for orbit round sun and converted to geocentric position -

Figure 4 shows an inferior planet indicated by P and Figure 5 an superior planet J. Position of earth and sun are E and S., position of meṣa 0° from sun and earth are A and A′. Position of node from sun and earth is N and N′.

True position of inferior planet is P and superior planet is J. U is the mandocca position (i.e. sun) for P.

Pāta of inferior planet -

Convex angle ASN is heliocentric longitude of node measured negatively, as node has a negative motion on ecliptic. Rule says that heliocentric śīghra anomaly is added to this which becomes

Convex angle ASN + $\angle USP = 360° + \angle NSP - \angle ASU = \angle NSP - \angle ASU$

Now longitude of planets is added here i.e. $\angle ASU$ (= $\angle A'ES$)

Result is $\angle NSP$.

Śara as seen from sun is $\frac{R \text{ Sin } NSP \times \beta}{R}$ where β is maximum śara (latitude).

As seen from earth this is to be reduced in ratio R/K where K is distance from earth i.e. śīghrakarṇa.

Thus śara seen from earth = $\frac{R \text{ Sin NSP } \times \beta}{K}$ which is the formula.

Pāta of superior planet - True geocentric longitude of J is $\angle A'EJ = \angle ASJ'$

Substracting śīghraphala EJS = JEJ′ from this we get $\angle ASJ$ = heliocentric longitude.

Then retrograde longitude of N i.e. $\angle ASN$ is added.

We get $\angle ASN + \angle ASJ = \angle NSJ$. From this heliocentric śara (latitude) is first calculated as in above case by multipliying with β/R and then geocentric valkue is obtained by R/K.

$\angle NSP$ or $\angle NSJ$ has been called vikśepa kendra i.e. heliocentric distance between pāta and planet in both cases.

Verses 12-26 : Further correction for śara -

The above śara has been written according to old siddhānta which is inaccurate according to author. Now accurate śara of maṅgala etc as actually seen in explained.

Sun and moon are to be corrected for parallax, when away from midday-sun (i.e. zenith), due to difference of observation from earth's centre and surface. Similarly, correction in śara is to make it sphuṭa (from heliocentric to geocentric position).

Mean positions of maṅgala, guru and śani are substracted from their sphuṭa mandocca to get the manda kendra. Jyā of manda kendra is mandaphala

approximately. By adding or substracting this from mean position we get manda sphuṭa graha.

Śīghrocca of budha and śukra is substracted from their mandocca sphuṭa. For budha, its śīghrocca is corrected by its parocca kandraphala. Result is śīghra kendra for vikśepa purpose.

For vikśepa kendra of other three planets, manda spaṣṭa graha is substracted from its pāta.

These are śara kendra of all 5 planets. From its bhuja jyā, śara is found by multiplying with parama śara and dividing with trijyā - heliocentric value. Śara is in north or south direction as explained in case of moon.

Difference of third mandakarṇa and trijyā is multiplied by difference of fourth śīghra karṇa and trijyā and divided by trijyā. We get kśepa karṇāntara.

Śara Karṇa - (1) When fourth śīghra karṇa is more than trijyā - (a) when third manda karṇa also is bigger than trijyā - Karṇāntara is substracted from trijyā (b) when third manda karṇa is less than trijyā - kśepa karṇāntara is added to trijyā.

(2) When fourth śīghra karṇa is less than trijyā- (a) manda karṇa is more - then karṇāntara is added to trijyā (b) when mandakarṇa is less - then karṇāntara is substracted from trijyā.

For budha and śukra, karṇāntara is added or substracted from mandakarṇa instead of trijyā.

Thus we get śara karṇa of all the five planets for all situations.

Sphuṭa śara : As in previous method, pāta is substracted from graha. Jyā of this vikśepa kendra is multiplied by madhyama śara and divided by śara karṇa. Quotient is multiplied by trijyā and divided by fourth śīghra karṇa to obtain sphuṭa śara of planets in kalā. Its difference with sthūla (rough) śara also can be used.

Notes : (1) First we calculate the heliocentric position by mandasphuṭa graha as explained in spaṣṭādhikāra.

(2) Śara karṇa is real distance of planet from sun due to śara in its śīghra gati. Difference of manda karṇa and trijyā is proportional change of distance due to mandaphala. It is multiplied by proportional change due to śīghra phala by multiplying with (śīghra karṇa trijyā) and dividing by trijya.

When mandakarṇa or śīghra karṇa is greater than trijya, śara karṇa i.e. true position of planet with śara, is less because śara will look smaller from larger distance. Hence śara karṇāntara is substracted from trijyā, average distance.

For budha and śukra average distance is their manda karṇa i.e. distance of sun from earth.

(3) Madhyama śara is value of śara seen from sun, it is multiplied by śara karṇa to get its true value as seen from sun. For proportionate reduction for geocentric value; it is multiplied by R/K. as explained in notes after pervious verse.

Verses 27-31 : Āyana dṛk-karma :

Śāyana graha is added with 3 rāśi (90°) - which is satribha sāyana sphuṭa graha. Its krānti

jyā is multiplied by sphuṭa śara and divided by dyujyā of satribha sāyana graha. The result will be in liptā etc. and is called āyana drkkarma kalā.

When āyana and śara of graha are in different direction, āyana is added to graha; and substracted if they are in same direction. Then graha postition or equator will be found, i.e. kadambaprota graha will become dhruva prota. This is called āyana dṛkkarma.

After doing āyana dṛkkarma, again the difference of planets involved in war (conjunction) is found. As before; the time is calculated when their rāśi, kalā etc. are equal. This will give lapsed or remaining days of conjunction. At the time of this conjunction, the planets are equal upto kalās. Then again śara is found; āyana dṛkkarma for new position will be done. By repeating the process, we get accurate time of equatorial conjunction when kalā of the two planets are equal.

Notes : In figure 6, EMQ and CMD are nāḍī maṇḍala and krānti maṇḍala respectively. P is dhruva, K kadamba and G the planet or grahabiṁha. PGA is dhruvaprota and KGB. kadambaprota. Then B is sphuṭagraha or position of the planet on krānti maṇḍala. A is called kṛta ayana dṛkkarma graha - i.e. point on ecliptic corresponding to equator position. MA may be called polar longitude of the planet in modern terms. GB is

Figure 6 - Ayana valana

vikśepa of G, which is almost equal to GA.

From GAB considered plane triangle

$$AB = \frac{\text{Jyā G} \times \text{GA}}{\text{Jyā B}} \quad \text{- - - (1)}$$

But, in GKP, ∠GKP = 90° + sāyaṇa graha = satribha graha, PK is measure of obliquity of ecliptic or parama krānti.

$$\text{Jyā G} = \frac{\text{Jyā GKP} \times \text{Jyā PK}}{\text{Jyā PG}}$$

$$= \frac{\text{Jyā (satribha graha)} \times \text{Parama Kranti Jyā}}{\text{Dyujyā}}$$

$$= \frac{\text{Krantijyā (satribha graha)} \times \text{Trijyā}}{\text{Dyujyā}} \quad \text{- - - (2)}$$

Jyā B = Trijyā, as B = 90° in (1)

Hence from (1) and (2)

$$AB = \frac{\text{GA} \times \text{Kranti jyā of satribha graha}}{\text{Dyujyā}}$$

AB = āyana dṛkkarma, i.e. shift in position of planet on ecliptic due to inclination of axis and śara.

Verse 31-37 : Ākśa dṛkkarma -

Square of āyana dṛkkarma in kalā and square of śara are added. Square root of sum is the sūkśma śara. When śūkśma śara and krānti are in same direction they are added; otherwise difference is taken for sphuṭa krānti of the planet. This will be distance from planet to the equator on polar circle. Sun is always on krānti vṛtta so its madhya krānti and sphuṭa krānti are same.

By the method explained in Tripraśnādhikāra, for both the planets (in conjunction), from sphuṭa krānti, we find their cara, dinārdha nata and

unnata kāla. Nata and unata kāla separately multiplied by 5400 and divided by their half day give jyā of nata and unnata kāla respectively. Difference of cara asu of graha for madhyama and sphuṭa krānti is taken as kalā and multiplied by nata jyā and divided by trijyā. The result in kalā is substracted from graha in forenoon (east half of sky) and added to graha in west half, if śara is north. For south śara, reverse process will be done. Then the graha will be corrected with ākśa dṛkkarma.

After that, difference of both graha is found and the time since conjunction or remaining till that is found. For conjunction time; again ākśa dṛkkarma is done. After repeated procedure, both graha will be in same samaprota vṛtta. Then, their north south difference in found on that circle.

Notes : (1) Sphuṭa śara : Śara (or madhyama śara) is GB in figure 6 which is distance of the planet from ecliptic along the circle through kadamba K. Along this circle the distance of planet from equator is GB′. But distance from equator is calculated along great circle through dhruva P. Hence the total krānti i.e. distance from equator is

$GA' = GA + AA'$. We take as spaṣṭa graha, not real planet G but its projection B on ecliptic. Hence, krānti of B is the real krānti.

First we have to calculate GA, which is given by $GA = \sqrt{GB^2 + AB^2}$ as $\angle GBA$ is 90° and ΔGBA is small and considered a plane triangle.

AA′ is almost equal to BB′ which is krānti of the sphuṭa planet i..e madhya krānti.

Calculation of GA is really not necessary by the above formula, as we have already assumed GA = GB in derivation.

(2) Bhāskarācārya has explained the dṛkkarma with difference in rising time on horizon due to śara of the planet. When the ecliptic position of the planet is rising on horizon, then due to śara, the real planet is above the horizon for north śara (down for south śara) and rises earlier (or later for south śara). The difference in rising time is known by dṛkkarma. One component of dṛkkarma depends upon āyana valana (i.e. inclination between equator and ecliptic) and the other component depends on ākśavalana (i.e. local akśāmśa - inclination of local horizon or vertical with horizon or vertical of equator). These components are called āyana dṛkkarma and ākśa dṛkarma.

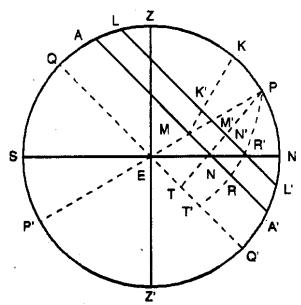



Figure 7 - Ākśa dṛkkarma

In figure 7, NZSZ′ is yāmyottara vṛtta of a place and NES is diameter of horizon in its plane. QQ′ and AA′ are diameters of equator and ahorātra vṛtta. PP′ is diameter of unmaṇḍala. EM is krānti jyā; AM is dyujyā = R cosϕ, where ϕ is akśāmśa.

Due to krānti, the planet rises earlier at position N, MN = kujyā = R sin δ tanϕ where δ is krānti. Its value on equator is ET where 1 kalā = 1 asu in time. ET = carajyā = R tanδ tan ϕ

Due to śara, the planet at M on ecliptic is seen at K′ in direction K, which is kadamba or pole of the ecliptic. (Sāra is shown north, when K is north from P). ∠KMP = ν = āyana valana. Thus due to śara, the longitude of planet is shifted by K′M′ on diurnal circle, K′M′ = s sin v where s is the śara. This is equivalent to shift of

s Sin ν / cos ϕ on equator, which is āyana drkkarma in asu. If we put R cosϕ = Dyujyā and R sin v = satribha krānti jyā (approx), we get the formula for āyana dṛkkarma given earlier.

Another component of śara MK′ is MṀ′ = s cos v, which is the śara in perpendicular direction to equator. Hence, sphuṭa śara is EM′ = EM + MM′ = R sin δ + s cos v. Thus effectively the diurnal circle of ecliptic planet of M will be shifted to LM′L′ parallel circle to equator passing through M′. Then the planet, will rise at position R′ (corresponding to R on diurnal circle and T′ on equator). Thus the rising time will be earlier by TT′.

TT′ = ET′ - ET = difference of carajyā. This is the simplest and most accurate formula given in any siddhānta text.

(3) Difference in carajyā is difference at horizon, corresponding to half day length (dyujyā or radius of equator). At other times it is proportional to natajyā i.e. distance from yāmyottara position A or L of the planet. Thus

$$\frac{\text{Ākśa valana at iṣṭa kāla}}{\text{Jyā of nata kāla}} = \frac{\text{Ākśa drkkarma at rising}}{\text{Half day}}$$

Formula for ākśa drkkarma at any other time has not been given by any other author. It is seen that ākśa drkkarma is deducted from planet in east sky as it rises earlier for north śara. Since it will set later in west, half proportionate addition will be done.

Verses 38-40 : Birhba of planets

Five tārā grahas like maṅgala have five types of biṁba - madhya vṛtta biṁba, madhya bhāsvara bimba, sphuṭa vṛtta biṃba, sphuṭa bhāsvara biṁba and dṛktulya biṁba.

Bimba of sun is very bright. Planets like moon take light from that and reflect it like water surface.

Tārā graha also have horns, due to the angle between direction of the graha and sun. But due to their distance from sun being large compared to moon, their horns are not seen. They are seen as point only.

Notes : (1) Biṁba of tārā grahas have been discussed in detail in biṁbādhikāra of siddhānta tattwa viveka, but this terminology has not been used any where. They are lighted by reflected light of sun, and bimba of śukra and budha are seen less than half when they are between earth and sun, due to their dark phase like moon. It has also discussed hole is sun due to śukra (like eclipse by moon). Due to smallness of tārāgrahas (small angular diameters) they are seen only as a point and their horns are not seen due to dark phase like moon.

(2) From the context, the classification of biṃba depends on their distance from earth, due to which they look small or big and due to phase

i.e. dark part depending on angular distance from sun. Thus the classifications are -

Distance difference - (i) Madhya vṛtta biṁba-

Average bimba size seen at average distance.

(ii) Sphuṭa vṛtta bimba - Current size of biṁba depending on the sphuṭa distance of planet.

Phase difference (iii) Madhya bhāsvara biṁba - Half lighted phase corresponding to about 90° angular separation from sun.

(iv) Sphuṭa bhāsvara biṃba - True lighted portion according to angular separation.

Actual biṁba - (v) Dṛktulya biṁba - which is actually seen according to distance and phase effects.

Verses 41-42 : Diameters of planets

Diameters of star planets in yojana are Mangala (450), Budha (930), Guru (4750), Śukra (2600) Śani (3500).

These divided by 2213 give the biṁba in kalā in sun orbit.

Notes : (1) Yojana value in sun's orbit is converted to kalā by dividing it with 2213 as explained in candragrahaṇa (chapter 8) verse 25. Angle made by 1 yojana at that distance is

$$\frac{1}{\text{sun's mean distance}} \text{ radian} = \frac{3438}{\text{mean distance}} \text{ kalā}$$

as 1 radian = 3438 kalā

$$= \frac{3438}{76,08,294} \text{ kalā} = \frac{1}{2213} \text{ kalā exact.}$$

This exact value indicates, that distance of sun has been calculated on basis of this ratio, after the diameter of sun was assumed 72,000 yojanas according to Atharvaveda.

All other text books have compared the diameters of planet in moon's orbit , but siddhānta darpaṇa has compared them in sun's orbit. The linear diameter is based on accumption that the distances are inversely proportional to angular speed i.e. proportional to period of rotation. As the comparative distance of moon and sun on that basis was rejected due to correct looking value of Atharvaveda, reference to moon's orbit also was rejected. However, the distance of other planets and sun are considered proportional to their periods of rotation. This is justified because all planets move round the sun and moon around earth both according to siddhānta darpaṇa and modern theory.

Period of rotaiton T and distance D are not directly proportional, but according to Keplar's third law

$T^2 \alpha D^3$

where D is distance (semi major axis)

Thus $T \alpha D^{3/2}$ instead of $T \alpha D$ assumed here. Thus, actual relative distance of farther planets will be lower than calculated here.

There is evidence in vedas that orbit was not meant the linear circle, but the surface of sphere on which this circle moves due to rotation of pāta. The same concept is used in Jain texts also. If time period is considered proportioanl to volume, then this relation $T^2 \alpha D^3$ holds as $T \alpha 4/3\ II\ r^3$, D $4\pi r^2$ where r is radius of orbit. Then T^2 and D^3 both αr^6. Time volume relation is only a conjecture

(2) Comparison of values

Mean angular diameters of planets (1)

Planet	Āryabhaṭa I and Lalla	Vaṭeśvara	Tycho Brahe	Siddhānta Darpaṇa	Modern (mean)
Mars	1′15″.6	1″19″.2	1′40″	8″	14″.3
Mercury	2′6″	2′12″	2′0″	25″	9″
Jupiter	3′9″	3′18″	2′45″	25″	41″
Venus	6′18″	6′36″	3′15″	70″	39″
Saturn	1′34″ .5	1′39″	1′50″	10″	17″

(2)

Planet	Old Sūrya siddhānta	Brahmagupta and Śrīpati	Sūrya siddhānta & Bhaṭṭotpala	Āryabhaṭa II	Bhāskara II
Mars	4′	4′46″	2′	4′ 45″	4′45″
Mercury	7′	6′14″	3′	6′ 15″	6′15″
Jupiter	8′	7′22″	3′ 30″	7′ 15″	7′20″
Venus	9′	9′	4′	9′	9′
Saturn	5′	5′24″	2′30″	5′ 15″	5′ 20″

Āryabhaṭa and Vaṭesvara have reduced the values of sūrya siddhānta and made them more correct. They are generally more correct then values of Tycho Brāhe, who had observed with telescope.

Old sūrya siddhānta value is 2 to 2-1/2 times the values of modern sūrya siddhānta and have been approximately followed by others in table (2).

Siddhānta Darpaṇa has evidently reduced the value of angular diameters in ratio of about 11, in which ratio the diameter and distance of sun have been increased. However, compared to sūrya siddhānta ratios, he has made increase in mercury and venus diameters and reduced the ratio of outer planets. For outer planets ratio of siddhānta darpana and modern values are Mars 1 : 1.8, Jupiter 1/1.64 Saturn 1/1.7

Ratio for inner planets is

Merucry 2.8/1, venus 1.8/1

One reason may be that, the visibility of outer planets reduces due to large distance from sun, hence they appear smaller.

Minute values of old S.S/Seconds value of siddhānta Darpaṇa for outer planets is

Mars 1:2, Jupiter 1:3 Saturn 1:2

For inner planets

Mercury 1:3.6, Venus 1:7.8

It is quite probable that Candraśekhara has calculated the angular diameters of inner planets according to their average distance from sun which is much less.

Comparison of linear diameters :

Planet	Siddhanta Darpaṇa		Sūryasiddhanta		Moderndiam	Distance
	yojana	Earth = 1	yojana	Earth=1	Earth = 1	Earth = 1
Mars	450	0.281	754.3	0.472	0.536	1.523688
Mercury	930	0.581	601.6	0.376	0.403	0.387099
Jupiter	4750	2.969	8324.5	5.203	10.925	5.202803
Venus	2600	1.625	802.1	0.501	0.990	0.723331
Saturn	3500	2.188	14,776.4	9.235	9.01	9.538843

Sūrya siddhānta relative figure are almost correct for all planets except Jupiter and venus whose value is about half the true value. These errors might have come due to incorrect ratio of time period and distance. However, sun's diameter comes to be less than, jupiter and saturn also as it is taken only 6500 yojanas.

Figure of siddhānta darpaṇa are more correct with two errors - Jupiter and saturn values reduced by about 1/4th of correct value, mars about half value. But mercury and venus vahes have been increased about 1.45 and 1.63 times the correct value. This appears to be due to error in estimating angular diameters of inner planets (more).

Other correct feature is that all planets are assumed much smaller than sun (72,000 yojana diameter) due to more correct diameter of sun.

Verses 43-44 : Madhya biṁba

The angular diameters of budha and śukra in sun orbit are their mean diameter. Angular diameters of other planets are obtained by multiplying their angular bimba in sun orbit by śīghra paridhi of the planet and dividing by 360°. The angular diameters in vikalā are

Maṅgala 8, budha 25, guru (25), Śukra 70 and śani 10

Notes : Average distance of budha and śukra is same as average distance of sun from earth as these inner planets are in small orbit round sun. For outer planets

$$\frac{\text{Distance of sun}}{\text{Distance of planet}} = \frac{\text{Śīghra paridhi of planet}}{360^\circ}$$

as sun is the śīghra kendra for outer planets. Hence the formula for angular diameters.

Verses 45-50 : Bhāsvara and sphuta biṁba - Madhyama biṁba (angular diameters) are kept at two places. At one place, it is multiplied by utkramajyā in the process of fourth śīghra phala and divided by two times trijyā (6876) Result will be substracted from the madhyama biṁba at other place. This will be bhāsvara biṁba of the planet.

For budha and śukra, when their śīghra is in 6 rāśis beginning from makara etc, then fourth phala kalā and śīghra koṭi kalā are added; jyā of the sum is added to trijyā. Then 3 rāśis are added to the śīghra kendra of budha and śukra and its

koṭi rāśi is substracted. Bhujājyā of the remainder is multiplied by biṁba diameter and divided by two times the trijyā. Result will be bhāsvara bimba of these two planets. If śīghra phala kalā and koṭi kalā is more than 3 rāśi together, then jyā of the sum is added to trijyā to find multipliers for budha and śukra.

Thus we get madhya vṛtta biṁba and madhya bhāsvara biṁba. They are separately multiplied by trijyā and divided by their śīghra karṇa to get the sphuṭa biṁba and sphuṭa bhāsvara biṁba.

Notes : (1) Sūrya siddhānta has given the following formula; spasṭa biṁba

$$= \frac{\text{Madhyama biṁba} \times 2 \times \text{Trijyā}}{\text{Trijyā} + \text{Fourth śīghra karṇa}}$$

This formula is correct if the madhyama biṁba is calculated at distance of sun, but in sūrya siddhānta it is calculated at the distance of moon. Ḥowever, this formula is correct for siddhānta dapraṇa where madhya biṁba has been calculated at sun's distance. This is the second formula given here and is based on the following ratio.

$$\frac{\text{Trijyā} + \text{4th śīghra karṇa}}{2} : \text{Trijyā}$$

= Madhya biṁba : spaṣṭa biṁba.

More accurately this should be found from true distance of planet from earth i.e. 4th śīghra karṇa instead of average of trijyā and śīghra karṇa. Thus siddhānta darpaṇa gives correct formula for spaṣṭa biṁba based on ratio —

Śīghra Karṇa : Trijyā = Madhya biṁba : spaṣṭa biṁba

(2) Earlier correction : Bhāsvara biṁba is measure of relative visibility. If depends upon distance from sun due to which brightness decreases in ratio of square of distance (inverse square law). However, due to phase also the brightness increases and is more when angular distance between planet and sun is 90° to 270°, we calculate utkramajyā which deducted from trijyā gives koṭijyā. Phase of a planet is equal to illuminated area divided by whole area of disc. The crecent GCHFG, bounded on one side by the semi ellipse GFH and on other side by semi circle GCH, is the illuminated part. GH is the line of cusps and CD the diameter perpendicular to it. Let CD = 2a. Then the phase

$$= \frac{\text{Illuminated arc}}{\text{Area of disc}}$$

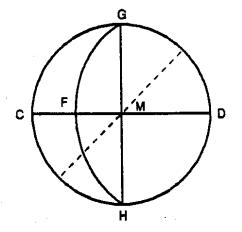
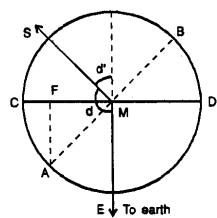



Figure 8a - Phase of the graha

Figure 8b - Lighted part dueto sun

$$= \frac{\frac{1}{2}\pi a\,(CM - FM)}{\pi a^2} = \frac{CF}{2a} \quad (1)$$

Now in figure 8b, hemisphere ACB is lighted by sun and hemesphere CAD is seen from earth.

$\angle SME = d$

Hence CF = CM - FM

= a - a cos AMC

= a (1 + cos EMS) = a (1+cos d)

Hence phase = (1 + cos d)/2 - - - (2)

If we measure the difference between planet and sun, as d′, then d′ = 180° - d

Hence phase = (1-cos d′)/2 - - - (3)

Thus in formula we find the utkrama jyā R (1 cos d′) and divide it by 2 R to get the phase according to eqn. (3). By substracting this portion from total biṁba, we get the unlighted portion which is away from sun.

For budha and śukra, the phase is calculated when they are on farther side of sun (śīghra kendra 270° to 90°) when they are more illuminated. We approximately find distance of mercury from superior conjunction (adding 3 rāśi to 270° is 0°) or inferior conjunction. One gives illuminted figure but on farther side, the other gives dark portion but on nearer side.

Verse 51 : When śīghra kendra of budha and śukra is 6 rāśi i.e. they are between earth and sun, then they are like black holes compared to bright sun in its disc.

Verses 52-55 : Now observed biṁba of bhāsvara is stated. Bhāsvara biṁba appear sthūla (i.e. round without sharp cusps) like a candle flame at far distance (which appears a round point instead of elongated figure)

When a bright object is very far, it appears 215 times its real angular diameter. Bhāsvara biṁba kalā is multiplied by 16 and square root of the product is taken. That is observed value of seen biṁba.

Notes : (1) Reasons of this arbitrary assumption are not known. However, from the discussions three variations in bimba emerge -

Sphuṭa biṁba is linear change in angular diameter which decreases with distance - like diameter of moon and sun.

Bhāsvara biṁba is the lighted portion of disc due to its phases like moon.

Observed biṁba of a point like object is seen 215 times bigger. But square root of bhāsvara biṁba is divided by 4 only for the diameter of observed biṁba in kalā.

(2) Logic of this method is not under stood. A point like object will appear bigger due to diffraction or scattering of light. That increase in angular width will be fixed and not 215 times the radius. Its angular increase will be same for sun and moon also. Possibly Candraśekhar had seen some star planets with a telescope set at 215 times magnification as mentioned by Prof. J.C. Ray in his introduction.

Verse 55 : Nakśatras are self illuminated and their distance is fixed, as it is almost infinite compared to planetary distances. Still their seen angular diameter should be found out.

Note : Though the stars are point like, two stars or star and a star planet are seen together, even when they are slightly separated. There are two reasons for that -

Due to scattering of light in atmosphere, the point object appears to have, some width.

Even when they are separated, their distance cannot be seen if it is less than limit of resolution of human eye.

Verses 56-60 : Types of conjunction -

Now types of conjunction (yuddha or samāgama) are being stated.

(1) When the observed biṁba of two planets touch each other, that is called ullekha yuddha (touching conjunction).

(2) When biṁba of a planet enters another planet, it is called vedha or bheda yuddha (piercing conjunction).

(3) When north south difference of two planets in conjunction is less then sum of semi diamters, then it is aṁśa vimarda yuddha (part eclipse conjunction).

(4) When the mutual distance is more than sum of semi-diameters, then it is called apasavya (i.e. separated),

Then the difference is upto 1° (60 kalā) i.e. Distance between centres - sum of semi diameters ≤ 60 Kalā.

(5) When the separation is more than 60 kalā then it is called samāgama.

(6) When, in an apasavya (separation less than 60 kalā), one planet is bright and the other is dark (inferior planet between earth and sun), then it is called yuddha.

When both are bright, it is called samāgama

When both are dark, it is called kūṭa yuddha.

(7) When two planets are equal in longitude (i.e. in yuddha) and northern planet has bigger diameter, then the southern planet is conquered.

When both are equal, then north biṁba is conquered, south is victor.

Śukra is victor, whether in north or south (as it has largest bimba among tarā grahas and is brightest).

Notes : These are only conventions for predicting furture events and described in Bṛhat saṁhitā etc. Here samāgama has been used twice. One is conjunction when rim distance is more than 60 kalā. Another is yuddha in which both planets are equally bright. However, conjunction of moon with a star has been called samāgama generally.

Verses 61-63 - South north distance

To know the north south distance, two dṛkkarmas have already been described. As in eclipse, nata and lambana corrections also are needed for the true north south distance. Earlier astronomers didn't observe or calculate less than 1/2 degree or 30 kalā, hence they ignored nata and lambana of tārā graha which is much smaller. Still for academic interest it is being described to explain the mathematics.

Verses 64-67 : Nati of planets

Parama nati of sun is 22 vikalā. Madhyama nati of budha and śukra also in same. Nati Kalā of budha and śukra (22/60) is multiplied by trijyā and divided by last śīghra karṇa. Quotient is again multiplied by vitribha natāṁśa (dṛkkśepa) and divided by trijyā for spaṣṭa nati of budha and śukra.

For other three planets (maṅgala, guru and śani, parama nati of ravi is multiplied by their śīghra paridhi and divided by 360. Quotient is multiplied by trijyā and divided by fourth śīghra

karṇa. Result is again multiplied by dṛkkśepa and divided by trijyā to get the spaṣṭa nati.

As in solar eclipse, viksepa of the 5 planets is corrected with spaṣṭa nati to get the sphuṭa śara.

Notes : (1) Average distance of budha and śukra is same as that of sun, hence their parama madhya nati will be same as that of sun. As the parallax reduces in proportion to distance similarly for outer planets -

$$\frac{\text{mean parallax of planet}}{\text{mean parallax of sun}} = \frac{\text{mean distance of sun}}{\text{mean distance of planet}}$$

$$= \frac{\text{śighra paridhi}}{360^\circ}$$

as sun is considered śīghra kendra of outer planets.

(2) $$\frac{\text{True parama nati}}{\text{Mean parama nati}} = \frac{\text{mean distance}}{\text{True distance}}$$

$$= \frac{\text{Trijyā}}{\text{Fourth śīghra karṇa}}$$

(3) Parama nati is for horizontal position for which dṛkksepa or jyā of vertical distance (south) is maximum or equal to trijyā (R). Since nati depends on jyā of vertical distance towards south

$$\frac{\text{spaṣṭa nati}}{\text{parama nati}} = \frac{\text{dṛkkśepa}}{\text{Trijyā}}$$

(4) Correction of śara for nati has already been explained for solar eclipse. They are added if in same direction and substracted if in different direction.

Verses 68-71 : Laṁbana correction

At the time of conjunction, parama nati of the planet is multiplied by dṛggati and divided by trijyā, and quotient is multiplied by jyā of difference between planet with vitribha lagna and divided by trijyā. Then we get sphuṭa laṁbana (parallax in east west direction).

When planet is east of vitribha lagna, sphuṭa laṁbana is added to planet, otherwise substracted.

After laṁbana correction, some difference comes in the longitudes of the planets. Then again conjunction time is corrected when the longitudes are same. For this new conjunction time, again laṁbana is calculated and, new conjunction time is found, when they will be equal in longitude. After repeated processes, we get the true conjunction time.

Notes : Parama nati of the planet is found as above section. Dṛgjyā is the distance of planet from vertical direction and nati will be proportional to it. Its value in ecliptic is proportionately known from distance of planet from vitribha. This has been explained in solar eclipse.

Verses 72 : Conmjunction of graha and nakśatra -

Since nakśatras are very far from earth, their speed and parallax both are zero. Hence, its conjunction with a planet is calculated only from the speed of graha.

Verses 73-75 : Bheda yuddha

Since laṁbana and nati are very difficult, this correction is done only for finding bheda yuddha,

when biṁba of one planet enters the bimba of another. For other conjunctions this is not necessary.

Bheda of sun by budha or śukra should be calculated like other conjunctions. When they are moving in opposite direction (budha or śukra is vakrī), then from sum of the gati and when both are mārgī, by difference of gati, we calculate the conjunction. According to the respective sizes of biṁba, times of sparśa etc can be found.

Śara of vakrī budha or śukra is very little so vedha of sun is done by them. In this case time of sparśa etc is found from sum of speeds.

Verses 76 : Moon and star planets -

Moon is corrected for nati and laṁbana and its vedha by graha bimba is calculated like sun.

Verses 77-90 - Samāgama of moon and star planets– When a tārā graha and candra have equal longitude (rāśi, amśa and kalā), then for finding their laṁbana, madhya gati of moon (790/35) is divided by 14. Quotient (56/28) is reduced by laṁbana of tārā graha found from its parama nati. This will be maximum value of nati difference of moon and that planet.

Parama nati difference is kept at two places. It is multiplied by 60 (to make it vikalā) and divided by madhyama gati difference of moon and the planets. If the planet is vakrī, then it is divided by sum of gati. This is time of parama laṁbana in ghaṭī etc; It is multiplied by dṛggati of that time (vitribha śaṅku) and divided by trijyā (3438). Result is made asu. It is assumed kalā and its jyā is called 'para'.

Bhuja and koṭi jyā of difference between moon and lagna is found. Difference of bhuja jyā and para is squared and added to square of koṭi jyā. Square root of the sum will be chāyā karṇa. Koṭijyā is multiplied by para and divided by chāyā karṇa. Result will be madhyama laṁbana.

Madhyama laṁbana is multiplied by difference of madhyama gati and divided by difference of sphuṭa gati if the tārā graha is mārgī. If tārā graha is vakrī, then madhya laṁbana is multiplied by sum of madhya gati and divided by sum of sphuṭa gati. Result is spaṣṭa laṁbana.

This laṁbana is substracted from moon, if it is east (more) of vitribha lagna, otherwise added. Then the new time of conjunction is found when moon and graha have the same liptā. The laṁbana asu is multiplied by second vitribha śaṅku and divided by Ist vitribha śaṅku (before laṁbana correction). After correction of moon by this sphuṭa laṁbana asu, we find the sphuṭa madhya kāla of conjunction.

According to method of solar eclipse, dṛkkśepa of vitribha lagna at mid conjunction time is found. Its 1/513 is added and divided by 61 to find nati of moon.

By method of solar eclipce, from nata jyā of vitribha lagna, śara and akśāṁśa valana are found. When śara of moon and graha are in same direction, difference is taken, when they are in different direction they are added. This śara will be useful for diagram (parilekha) of samāgama. When graha is south from moon, śara will be yāmya, when it is north, śara will be saumya.

For tārā graha, moon is chādaka (eclipser) because it is closest to earth. Since moon has more speed, sparśa of its bimba by the planet will be in east and mokśa will be in west.

After doing āyana dṛkkarma of graha, graha and nakśatra conjunction is calculated from nati corrected śara.

Notes : The methods are exactly similar to methods of solar eclipse. Only difference is that the tārā graha can be vakrī also, when sum of gati is used instead of their difference.

Verses 91-96 - Parilekha

Like diagram of eclipse, we draw the mānaikya vṛtta (circle with radius as sum of semi diameters) inside khagola vṛtta with radius 57°/18′ aṅgula = 3438′ radius. From same centre moon circle is drawn. For valana of khavṛtta, spārśika valana in east and maukśika valana in west is given in their own directions. From valanāgra, we draw a line to the centre of moon, called diksūtra.

From the points where diksūtra cuts mānaikya, we give śara at the time of sparśa and mokśa in their direciton (north or south). The line from śarāgra points (end points of śara) to centre of moon, cuts the moon bimba on two points indicating entry and exit points of graha or nakśatra.

In conjunction of nakśatra and moon, śaṅku of vitribha lagna is multiplied by 100 and divided by 231 to give jyā of parama lambana or 'para'.

Like moon and star/planet conjunction, vakrī budha and śukra enter the sun disc from east side

and exit from west side. Since sun has no śara, the śara of only budha or śukra is the total śara and direction of this will be the direction of śara. Disc of sun will be in centre of samāsa vṛtta (circle with radius as sum of semi diameters).

Notes : The discription in parilekha, chapter 10 is sufficient to understand this.

Verses 97-106 : Observing shadow of planets

From rays of star planets like maṅgala, we cannot see the shadow of a 12 aṅgula śaṅku. Hence, a mirror is kept on the shadow end point and śaṅku top is seen in mirror. Exactly at shadow end point, the planet and śaṅku end are seen in one direction.

On a plane level surface, we keep a vertical śaṅku of 5 hands hight. In it 12 divisions are marked, each being 1 angula. Śaṅku will be strong and straight and its surface will be cylinderical.

As explained in Tripraśnādhikāra, from the nata kāla of the planet at desired time, we find the shadow length of 12 aṅgula śaṅku. With that semi diameter a circle is drawn with śaṅku centre as the centre. Direction points are marked (earlier in day time) and from the centre, lines are drawn in east west and north south direction.

Then the krānti jyā of graha at the desired time is multiplied by chāyā karṇa and divided by lambajyā. Quotient will be karṇa vṛttāgrā in aṅgula. It will be substracted from palabhā for north krānti of the planet and added for south kranti to get bhuja of shadow in aṅgula (its distance in north or south direction from śaṅku). On north south line through centre, we mark a point at distance

of chāyāgra bhuja in the opposite direction of inclination of the planet. From shadow length (chāyā) square, we substract the square of chāyā bhuja and take the square root. Result is dharātala śaṅku which is called koṭi also. When planet is in west half of sky, koṭi is given east from the end point of chāyā bhuja. At the point of shadow circle where it cuts, shadow end will lie. At this point a tube will be kept in direction of the śaṅku top and we see from below. Or a mirror is kept and its reflection is seen.

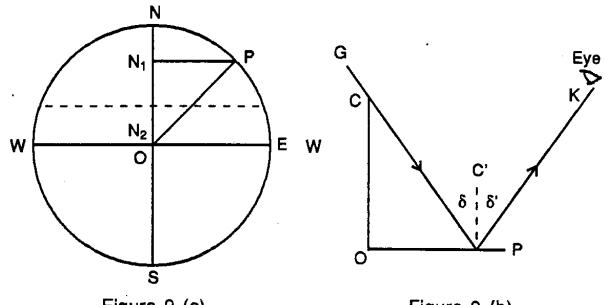



Figure 9 (a) Figure 9 (b)

Notes - In figure (a) ENWS is circle with radius of shadow length. Current direction of shadow is OP. OP is length of shaodw, ON_1 is chāyā bhuja, N_1P is its koṭi. Hence $OP^2 = ON_1^2 + N_1P^2$. ON_2' is palabhā, i.e. shaodw at the time of equinox midday. The difference with bhuja is karṇa vṛttāgrā, $N_1N_2' = ON_1 - ON_2$

$$N_1N_2 = \frac{K}{\cos \phi} \text{Sin } \delta' \text{ where } \delta' = \text{spaṣṭa krānti},$$

ϕ = akśāmśa. This is explained in Tripraśnādhikāra

In figure (b) OC is śaṅku of 12 aṅgula length. OP is chāyā and PC is chāyā karṇa. Thus PC is in direction of planet at G. If we keep a tube in PC direction, planet can be seen from P end. By a long dark tube we can see a planet in day time also as scattered day light is absorbed by inner surface of tube and only light of planet is seen which is not obstructed. Alternatively, by keeping a horizontal mirror at P we can see planet by keeping eye in direction of PK, Here PK makes same angle with vertical PC′ as $\angle CPC' = \delta^1$ in opposite direction.

Verses 107-108 : Seeing the yuti

At the time of yuti (conjunction of planets) we keep two śaṅku at the distance of śara difference and from the same point P we can see both planets through tube or a mirror. Result of different types of yuti are given in books of saṁhitā (like Bṛhat saṁhitā of Varāhamihira).

Verse 109 : Increased size of vṛtta biṁba -

Here, the biṁba of planets decribed or biṁbas of stars to be told later, are very bright, hence they are seen 16 times more lighted than moon. At the time of sunrise and sunset, their discs are as bright as moon, hence their biṁba value has been stated as 4 times = $\sqrt{16}$ larger. Thus the real angular diameter is 1/4 of the seen diameter.

Notes : (1) This explains the logic of formula for observed biṁba in verse 54. But it is not correct.

(2) Due to diffraction of light, two points at angle less than θ radians cannot be seen separately where

$$\text{Sin } \theta = \frac{1.22\lambda}{D}$$

where D is diameter of aperture through which planet is seen (it may be aperture of pupil of eye or lens of a telescope). λ is wave length of light (4000 to 8000 angstrom = 10^{-8} cm units). This is Raleigh criterion. Thus for visible light, when pupil is 1.5 mm diameter in day time, we cannot see two points which are separated by les than about 1′ kalā. In night time when pupil is bigger it will be about 20″ vikalā. Thus the angular diameters of outer planets are smaller than the limit of resolution of eye and even when they are separated, they appear together. This explains as to why separation upto 1 kalā is called samāgama and only for larger separation, they are really seen separate.

Thus at the time of conjunction, the effective diameters of planets are seen bigger.

(3) Other reasons of fluctuation are scattering of light, and fluctuations in atmosphere, which are almost same for both the nearby stars or planets. The stars are so distant, that their angular diameter is zero even after seeing through largest telescopes. Their diameter of conjunction time is seen much more than 215 times due to diffraction.

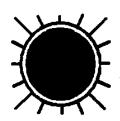
Verse 110 : Solar eclipse due to śukra

To find eclipse of sun due to venus, their biṁba and size of other tārā graha is stated. In kali year 4975 (1874 AD) there was a solar eclipse due to śukra in vṛścika rāśi (i.e. in Nov.- Dec. month). Then śukra bimba was seen as 1/32 of solar biṁba which is equal to 650 yojana. Thus it

is well proved that biṁba of śukra and planets is much smaller then sun.

Verses 111-112 : Prayer and conclusion

May Lord Jagannātha remove our ignorance, who defeats beauty of blue clouds by his blue light and lives on sea coast.

Thus ends the eleventh chapter describing conjunction of planets in Siddhānta Darpaṇa written for tallying calculation and observation and education of students by Śrī Candraśekhara, born in famous royal family of Orissa.

Chapter - 12

CONJUNCTION WITH STARS

Verse 1 - Scope – To know the conjunction of planets with nakśatras, the longitude and latitude of identifying star in each nakśatra starting with aśvinī, shape of nakśatras and number of stars in it and biṁba of yogatārā (identifying star) is stated first.

Verses 2-11 : Longitudes and latitudes of identifying stars (yogtārā)

S.No. of Nakśatra	Name of nakśatra	Beginning point longitude	Name of yogatārā	Longitude of yogatārā	Latitude of yogatārā	Position of yogatarā
1.	Aśvinī	0°0′	βArietis	10°07′	+8°29′	10°07′
2.	Bharaṇī	13°20′	Arietis	24°21	+1027	11°01
3.	Kṛttikā	26°40′	ηTauri	3608	+403	928
4.	Rohiṇī	40°0′	αTauri	4556	-528	556
5.	Mṛgaśirā	53°20′	λOrionis	4951	-1323	631
6.	Ārdrā	66°40′	αOrionis	6454	-1602	-146
7.	Punarvasu	80°0′	βGeminorum	8922	+641	922
8.	Puṣya	93°20′	δCancri	10452	+005	1132
9.	Aśleṣā	106°40′	αCancri	10947	-505	307
10.	Maghā	120°00′	αLeonis	12558	+028	558
11.	Pūrvā Phālgunī	133°20′	δLeonis	13727	+1420	407
12.	Uttarā Phālgunī	146°40′	βLeonis	14746	+1216	106
13.	Hasta	160°0′	δCorvi	16936	-1212	936
14.	Citrā	173°20′	αVirginis	17959	-203	639
15.	Svātī	186°40′	αBootis	18023	+3046	-617
16.	Viśākhā	200°0′	αLibra	20114	+020	113
17.	Anurādhā	213°20′	δScorpii	21843	-159	523
18.	Jyeṣṭhā	226°40′	αScorpii	22554	-434	-046
19.	Mūla	240°0′	λScorpii	24044	-1347	044
20.	Pūrva Āṣādha	253°20′	δSagittarii	25043	-628	-237

21.	Uttara Āṣāḍha	266°40′	σSagittarii	25832	-327	-808
22.	Śravaṇa	280°0′	αAquilae	27755	+2918	-205
23.	Dhaniṣṭhā	293°20′	βDelphini	29229	+3155	-051
24.	Śatabhiṣaj	306°40′	λAquarii	31743	-023	1103
25.	Pūrva bhādrapada	320°0′	αPegasi	32938	+1924	938
26.	Uttara bhādrapada	333°20′	γPegasi	34518	+1236	1158
27.	Revatī	346°40′	ξPiscium	35601	-013	921

These are the modern positions and names of identifying stars. Nirayana longitude of Citrā (α-Virginis) was fixed as 180° at 285 AD to fix the nirayana position accurately in zero ayanāmśa year. Now it has become 179°59′ due to negative proper motion of citrā.

Verse 12-24 : Verses 12-14 give the number of stars in each nakśatra. Verses 15-18 give the shape of each nakṣatra.

Verses 19-22 give the direction of yogatārā within the nakśatra (this can be known from their latitude and position in nakśatra also given in previous table). Verses 23-24 give the diameter of yoga tārā in vikalā. Actually the diameters are almost zero even by talescope viewing, they are measures of visual magnitudes of brightness. The yogatārā positions of 28 nakśatras including Abhijit according to siddhānta darpaṇa in previous verses and the other details are given in chart form.

Sl. No.	Nakśatra	Owner (yajurveda)	Yogatārā Longitude	Latitude	Bimba Vikalā	Shape	No. of Stars
1	Aśvinī	Aśvina	9°45′	+10°30′	6	Horse mouth	3
2	Bharaṇī	Yama	2100	+1100	2	Triangle	3
3	Kṛttikā	Agni	3515	+415	3	Flame	6

4	Rohiṇī	Prajāpati	4630	-537	7	Cart (Śakaṭa)	5
5	Mṛgaśirā	Soma	6015	-1330	2	Cat's paw or head of dear	3
6	Ārdrā	Rūdra	6500	-1540	7	Coral or water drop	1
7	Punarvasu	Aditi	9015	+630	8	Bow	5
8	Puṣya	Bṛhaspati	10400	+115	2	Arrow	3
9	Āśleṣā	Sarpa	10800	-1200	4	Dog tail	5
10	Maghā	Pitṛ	12600	+022	6	Plough	5
11	Purvā Phālgunī	Bhaga	14300	+1200	12	weight on two ends of beam	2
12	Uttarā Phālgunī	Aryamā	15300	+1300	13	-do-	2
13	Hasta	Savitṛ	16500	-1100	4	Hand	5
14	Citrā	Tvaṣṭā	17900	-210	7	Pearl	1
15	Svātī	Vāyu	19300	+3300	13	Coral orjewel	1
16	Viśākhā	Indrāgni	20700	-200	2	Shed or tent	5
17	Anurādhā	Mitra	21830	-200	4	Snake hood	7
18	Jyeṣṭhā	Indra	22530	-415	7	Teeth of Boer	3
19	Mūla	Nirṛti	24040	-1330	5	Cronch or lion's tail	9
20	Pūrva Aṣāḍha	Āpah	25000	-630	4	tusk	4
21	Uttara Aṣāḍha	Viśvedavah	25630	-340	4	Chute (Sūpa)	4
	Abhijit	Brahmā	25630	+6200	14	Triangle or fire bail	3

22	Śravaṇa	Visṇu	27300	+3000	7	Arrow or short men	3
23	Dhaniṣṭhā	Vasava	28530	+3600	3	Long drum	5
24	Śatabhiṣaj	Varuṇa	31745	-020	3	Canopy	100
25	Pūrva bhādrapada	Aja-Ekapāda	32200	+3200	4	Cot or weights from beam	2
26	uttara bhādrapada	Ahirbudhnya	33800	+2800	4	do	2
27.	Revati	Pūṣā	0°00′	+500	3	drum or fish	32

Notes : (1) Yoga tārā in north position of nakśatra - (1) Aśvinī 5. Mṛgaśirā 11. Pūrvāphālgunī, 16. Viśākhā 20 Pūrvāṣāḍha 21. Uttarāśāḍha, 25 Pūrva Bhādrapada, 26 - Uttara bhādrapada

Yogatārā in centre - 19. Jyeṣṭha, 22 Śravaṇa, 17 Anurādhā, 3. Kṛttikā, 8 Puṣya

Yogatārā in īśāna (north east) - 7 Punarvasu, 13- Hasta, 19-Mūla.

Yogatārā in west - 23. Dhaniśṭhā, 0 - Abhijit

Yogatāra in east - 4. Rohini, 9. Aśleṣā

Yogatārā in south - 10. Maghā (very bright), 27. Revatī 12. Uttarā phālgunī

Yogatārā in agni koṇa (south east) - 24. Śatabhiṣaj. Single stars are in 6. Ardra, 14. Citrā and 15. Svātī, hence there is no difference between the nakśatra and yogatārā.

(2) Shape of nakśatras have been decribed differently by different authorities. Actually, it is only imagination and convention.

(3) Longitudes and latitudes also differ slightly according to different authorities.

(4) It may be seen that many yagatārāa do not come within extent of their nakśatra. Hence three nakśatras are divided into pūrva and uttara part. In unequal division of nakśatras, mȯst of the nakśatras have yogatārā in their extent.

(5) It has already been stated that diffraction and partly scattering of light in atmlsphere spreads the point like stars. Bright star has bigger spread as, greater spread of diffraction ring remains visible.

Verses 25-40 - Other stars -

Now many other stars are described.

(1) Lubdhaka (Sirius) - It is brightest star south of punarvasu with biṁba of 20 vikalā, dhruva 77° and dhruva prota krānti 40°. Sūrya siddhānta name of this star is lubdhaka. Bhāskara II has given its longitude (polar) as 86°. It is 8.6 light years away and brightest star.

(2) Mṛgavyādha - There is another small star south of punarvasu. Sūrya siddhānta and Lalla have called this same as lubdhaka, but it is different star. Its dhruva is 56°, south śara 32° and biṁba is 10 vikalā It may be identified with Orion, which is also called hunter is greek stories borrowed from Egypt.

Its south latitude is same as south latitude of Magadaskara (now Malagasi) an island in south east direction of Africa - hence this island was called Mṛga or Hariṇa dvīpa

(3) Ilvala - This is a group of three stars between mṛgavyādha and ardrā. Its middle star is

yogatārā, whose dhruva is 61° and south śara 23°30′.

(4) Hutabhuk - According to sūrya siddhānta, its dhruva is 52° and north śara is 8°.

(5) Brahmahṛdaya - According to sūrya siddhānta, its dhruva is 52° and north śara 30°.

(6) Prajāpati - It is 5° east brahmahṛdaya whose dhruva is 57° and north śara is 38°. (Sūrya siddhānta)

Modern observations have indicated the following positions (by author).

(4) Hutabhuk - Dhruva 58°15′, śara 5°15′ north biṁba 6″ vikalā

(5) Brahmahṛdaya - Dhruva 56°, north śara 23°, biṁba 16″

(1) Lubdhaka is now called prajāpati.

(7) Apāmvatsa - This is 5° north from citrā.

(8) Āpa - This is 6° north of Apāmvatsa. It is also called āpyavasu.

Dhruva of both (7) and (8) above are equal to citrā. North śara of (7) is 2°50′ and (8) is 8°50′.

(9) Agastya - Its dhruva is 95° and south śara is 75°. Its dhruva becomes sphuṭa after doing ayanāmśa correction. Its bimba is 18″ vikalā.

(10) Yama - Its dhruva is 22°, śara is 66° south and biṁba is 8″.

Sūrya siddhānta has stated dhruva of agastya as 90°. This was the value at the time of writing that book when 121 years were remaining in satya yuga. In Kali era 4251, Bhāskara II has stated its

dhruva to be 87°. He has stated dhruva of punarvasu as 93° and Agastya 6° less i.e. 87°. At the time of siddhānta darpaṇa, it is 17°30′ west from punarvasu i.e. 90°15′ - 17°30′ = 72°45′. From agastya dhruva 95°, on substracting ayanāmśa 22°, we get the same value 73° approximately. The change of agastya dhruva from 87° at the time of siddhānta śiromaṇi when ayanāmśa was 11°30′ to mithun 13° (73°) is the change in 719 years (1869 AD).

Notes : Ayanāṁśa correction is not needed when the distances have always been measured with respect to fixed stars. There may be some error in identification of stars. Otherwise relative motion of stars is very little and negligible compared to ayana movement. Opinions differ regarding correct identifications of these stars with current greek names used. Modern names of yogatārā have already been given. Agastya is canopus, apāmvatsa is θ-virginis and Āpa - δ virginis, Agni or hutabhuk in β tauri. Prajāpati is β aurigae, Brahmā is α aurigae.

Verses 41-56 - Saptarṣi maṇḍala

Since saptarṣi maṇḍala (great bear) is moving, its dhruva has not been stated by earlier astronomers. Still, I state their position, based on my experience.

In north direction saptarṣi maṇḍala spread in east west direction like a bullock cart is very prominent in the sky. It has been most reverred in saṁhitā and purāṇa.

Within this group, there is an upward raised line towards east. Marīci is in its front. Behind it Vaśiṣṭha is with Arundhati. Still west from Vaśiṣṭha is Aṅgirā.

After that, is a quadrilateral. In its īśāna koṇa (north east), lies Atri. South from it is Pulastya and west from Pulastya in Pulaha. North of Pulaha is Kratu. The great circle joining Pulaha and Kratu, cuts ecliptic in some point, the nakśatra or rāśi of that point is considered the rāśi of saptarṣi.

At present Pulaha and Kratu are in 21° of siṁha i.e. 3rd quarter or pūrvā-phālgunī. 13 kālāṁśa east from them is Pulastya.

Atri is 5 kālāṁśa east from Pulaha, 9 kālāṃśa east from Atri is Aṅgirā, 8 kālāṁśa east from Aṅgirā lies Vaśiṣṭha and 8 kālāṁśa east from Vaśiṣṭha is Marīci.

Arundhati is a very small star, east from Vaśiṣṭha which is barely visible and can be seen with telescope. This is not giver of good or bad omen, like the seven main stars. Its biṁba is 1 vikalā. Biṁba of Atri is 3 vikalā, and all others are 8 vikalā. Mutual distance between these stars is same and equal to 10 pala kālām'sa.

This 10 pala is multiplied by 1800 and divided by rising time of that rāśi at equator. The quotient is added to the dhruva of Pulaha or Kratu (siṁha 21° = 141°). We get the dhruva in rāśi etc for other stars. East west angular distance (along ecliptic) of saptarṣi is 43°, but due to its position in sāyana kanyā and tulā, it appears 46° (in rising time at equator).

Distance from ecliptic along dhruva prota vṛtta (great circle through dhruva, not kadamba - pole of ecliptic) in north direction are -

Kratu 56°, Pulaha 51°, Pulastya 53°, Atri 59° Aṅgirā 60°, Vaśiṣṭha 62° and Marīci 60°.

If śara of Vaśiṣṭtha from krānti is fixed, then in the end of even quadrant, it wil be 4° from dhruva (North śara 62° + krānti at end of even quadrant 24° = 86° i.e. 4° from dhruva at 90°). Even if sphuṭa krānti of saptarṣi remains same, their śara changes with change in rāśi.

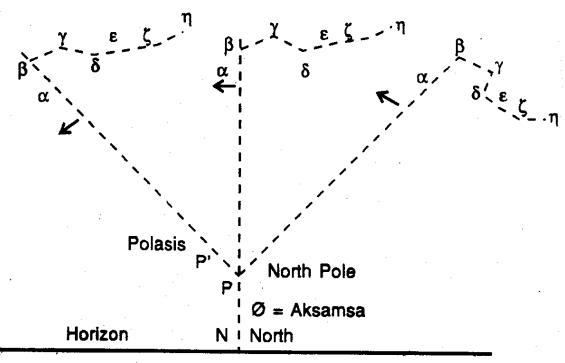



Figure 1 Position of saptarṣi and pole star

Notes (1) Due to earth's rotation, saptarṣi makes a revolution around north pole in direction of line $\beta\alpha$ of its western stars. Three positions at 3 hour intervals are shown from east to west. Polariṣ P' is very close to north pole (58' Kalā distance) and is called pole star. P is ϕ angle above north horizon, where ϕ is local north aksāmśa. The stars are indicated by greek letters starting from

western lower star. Siddhānta counts them from eastern end. Modern names, distances and visual magnitudes are given below -

Stars	Greek names	Visual magnitude	Distance in light years
1 Marīci	η Alkaid	1.87	210
2 Vaśiṣṭha	ζ Mizar	2.06	88
3 Angirā	ε Alioth	1.79	68
4 Atri	δ Megrez	3.3	-
5 Pulastya	γ Phad	2.44	90
6 Pulaha	β Merak	2.37	78
7 Kratu	α Dubhe	1.81	107

More visual magnitude indicates lesser brightness, thus Atri is least bright and farthest, hence its bimba vikalā has been indicated small. Arundhati is a small star, below Vaśiṣṭha called Alcor (magnitude 5). Mizar (Vaśiṣṭha) itself is a double star when seen from telescope. It appears that Atri has faded now but earlier, it was equally bright.

(2) Mythology : Callisto was attendent of goddess Juno but was more beautiful. To protect her from jealosy, Callisto was turned into bear by god Jupiter. When her son Arcas, thought her a bear and wanted to kill her, he was also turned into bear (ursa minor)

According to Purāṇas, Saptarṣis are mental sons of Brahmā. There is a separate set of saptarṣi for each of 14 manu periods of which 7 are yet to come. Ṛṣi and Ṛkśa have been used for star, sage or bear also. Hence ursa in Persian means saint, in Greek it means bear. Like this bear around north

pole, Russian bear exists. Russian was Ṛṣīka and it is land of bear. Proverbially Russia is called Russian bear. Ṛṣi denoted sage and bear as both had long hairs. Hence the name great bear came.

(3) Motion of Saptarṣis : Only Vateśvara siddhānta chapter 1 verse 15 has given the number of revolutions of saptarṣi which is 1692 in a yuga.

On that basis, Karaṇa sāra of Vaṭeśvara has given a method to calculate movement of saptarṣis, as quoted by Albirunī (India I, page 392) -

Multiply the basis (i.e. years elapsed since beginning of śaka 821) by 47 and add 68000 to the product. Divide the sum by 10,000. Quotient is position of saptaraṣi in rāśis etc.

According to this formula, saptarṣi has a motion of 47 signs per 10,000 years which is equivalent to 1692 revolution in 43,20,000 years, as stated above.

The position in śaka year 821 (Kali year 4000 was)

$$\frac{1692 \times 12 \times 4000}{43,20,000} \text{ signs} = 1 \text{ revolution} + \frac{68,000}{10,000} \text{ signs}$$

This accounts for the addition of 68000 in formula.

(4) The stars of the constallation of the saptarṣi do not have a motion relative to nakśatras. So the statement of revolution is not correct. This appears to be the reason why many standard astromers like Āryabhata, Brahmagupta, Śrīpati, Bhāskaras I and II, Surya siddhānta etc do not deal with the subject at all, as being outside the pale of astronomy. Therefore, Kamalākara was constrained

to say in his Siddhānta Tattva Viveka, Bhagraha yutyadhikāra, verses 25-36 -

"Sage Śākalya has given the motion of the sages with their positions in his time. Sūrya and others who explain the nature of the celestial sphere in their works do not give it, and therefore, the theory cannot be sustained astronomically. Even today, this motion mentioned in the saṁhitās is not observed by astronomers. Therefore, the seven real sages who are the presiding deities (of these stars) are only to be supposed to be moving unobserved by men, for the prediction of the fruits, thereof."

But the motion has been accepted as a fact by certain common people and authors of the Purāṇas, and an era called Laukika era by the people of Kashmir region and saptarṣi era by the purāṇas have been founded on this theory.

Mahābhārata mentions, that when Yudhiṣṭhira ascended the throne, Saptarṣis were in maghā nakśatra.

Vāyu purāṇa chapter 99, tells that saptarṣi's remain for hundred years in one nakśatra. Hence they complete the round of 27 nakśatras in 2700 divya years. However, same purāṇa chapter 57, tells that saptarṣi nakśatra is of 3030 human years. Hence human year appears to be taken as 12 sideral revolution of moon. Divya year here means 1 solar year.

2700 solar years = 2700 X 365.256263 days for sidereal years

$$= \frac{2700 \times 365.256263}{12 \times 29.321661} \text{ lunar years sidereal}$$

$$= 3007.968 \text{ years}$$

Varāha Mihira has written in Bṛhatsamhitā 13/3, that according to Vṛddha Garga, Saptarṣis were in Maghā in the rule of Yudhiṣṭhira. Rāja Taranginī of Kalahaṇa has followed this era only in writing ancient history.

(5) Explaination : Kamalāākara has explained that it has no relation with astronomy and it is only for astrological predictions. Siddhānta Darpaṇa has tried to justify the movement of saptarsis on basis of their measurement of position on dhruva prota vṛtta on ecliptic. Normally kadambaprota vṛtta is used for ecliptic and dhruva prota for equator. This causes difference and has explained the difference in terms of ayanāmśa. His calculation of difference from Bhāskarācārya time is based on ayana - movement. However, this will have a cycle of 26000 years and not of 2700 years of saptarṣi era. Hence Candraśekhara has not mentioned the saptarṣi era but has vaguely tried to justify its movement.

My explaination is based on basis of vedāṅga jyotiṣta which was current in Mahābhārata period, Rk jyotiṣa has a cycle of 19 years in which 5 years are of samvatsara type (starting between Māgha śukla 1 to Māgha śukla 6.) Yajuṣ jyotiṣ starts with 5 year cycle of 366 days each, but this also becomes equivalent to 19 year cycle with 6 kśaya saṁvatsara in 5 cycles of 5 years each. P.V. Holey has assumed a bigger yuga of 19 X 8 + 8 = 160 years because

it gives very little error, but has not explained the mechanism of arranging the last eight years. This is not corroborated any internal evidence in the text.

However, saptarṣi era was very much in use and was accepted in the calendar system. This appears to be based on system of naming a century (100 solar years) on a nakśatra in same way as we name every guru varṣa or other solar years on basis of Ist week day of the civil year. Thus, we can name the century on basis of first nakśatra of moon (or may be sun) in a century, if the vedic yuga system is followed. Then bigger yuga should be 19 X 5 + 5 years = 100 years instead of 160 = 19X8+8 years. Thus in a century, we can take last 5 years as the first 5 years of the 19 years Ṛk cycle or 5 years first cycle of yajuṣa jyotiṣa. In taking yajuṣa cycle, the 19 year cycle doesn't break and in sixth cycle first 5 years are subcycle which make a century. It can also be seen that all cycles of 19 year start with Śraviṣṭha but after five years of yajus cycle, ṣixth year starts with śatabhiṣaj which is the next star. Thus on completion of 100 years, in this calender, moon gains one nakśatra in this calender. Thus each sucessive century will start with one nakśatra later, which will be saptarṣi nakśatra or nakśatra of the century.

Sri Holey has opined that Rk jyotiṣa was definitely written before 2884 B.C. According to our traditions vedic texts or purāṇas were written within 300 years of Mahabhārata war. Thus it roughly indicates the calender system fixed in Yudhisthira time who had really started an era.

That time year started with Māgha śukla pakśa, hence saptarṣis were assumed to be in maghā to start with.

(6) Mutual distance between stars of saptarṣi maṇḍala is not equal as stated here.

Verses 57-59 : Krānti of circumpolar stars

Sphuṭa krānti of yama and Agastya is always same. Krānti of nakśatras starting with Asvinī keeps changing. Krānti of saptarṣi maṇḍala is fixed according to some, but changes according to others.

Earlier astronomers have assumed motion of saptarṣi maṇḍala as 8 kalā per year from east to west. But I (author) have not seen such gati. Hence I do not agree to it.

Ayana sanskāra of yama, Agastya and saptarṣi maṇḍala has been instructed to be done in opposite direction. This is valid for author's time only (1869 AD).

Notes (1) Ayana saṁskāra is needed because, here dhruvāmśa have been given for yogatārās and other stars. As stated earlier, Candrasekhara has assumed oscillatory motion of ayana, and according to this, the present backward movement will change after 2200 AD.

(2) 8 Kalā movement of saptarṣi is 800 Kalā or 1 nakśatra in a century whcih has been stated by Vateśvara. This has not been accepted by author correctly.

(3) Circumpolar stars are near dhruva (pole star) and appear to move round it. This is true for south polar region also. This depends on local latitude of the place. Day length of a planet or

star increases by carajyā which is increase in half day length for north krānti. It is decrease for southern krānti.

Increase in half day = Carajyā

= R $\tan \delta \tan \phi$ (δ =spaṣṭa krānti, ϕ = latitude)

If this is equal or greater than R, then the increase is equal to half day of equator itself and there will be no night i.e. the star will never set.

For this $\tan \phi \tan \delta > 1$

Thus for any star with north krānti,

it is circum polar, if

$\tan \delta > \cot \phi$

Similarly for south krānti, also, if

$\text{Tan}\ \delta > \cot \phi$

then the star will never rise.

Verses 60-62 : North pole star (dhruva tārā) has bimba of 4 vikalā. This is not the real position of dhruva i.e. pole of equator and dhruva prota is not drawn through it. The seen dhruva tārā is 1°24′ away from surface centre of equator. Hence, when revatī nakśatra comes on meridian i.e. dhruvatāra at beginning of meṣa, appears 84 Kalā above the pole.

When śravaṇa, punarvasu nakśatra are on yāmyottara, dhruva rises above horizon equal to local akśāmśa. Again, when citrā nakśatra comes on yāmyottara, dhruva in 84 kalā nata from its kendra.

Notes : Polaris (Ursae minoris) is a star of second magnitude and is 58′ Kalā away from celestial pole in west direction in 1950 AD. Celestial

pole is moving towards polaris upto 2105 AD when it will be only 30′ away, then will begin to recede from it.

It may be seen that Draco or dragon group is pole of ecliptic i.e. pole of solar system. Thus

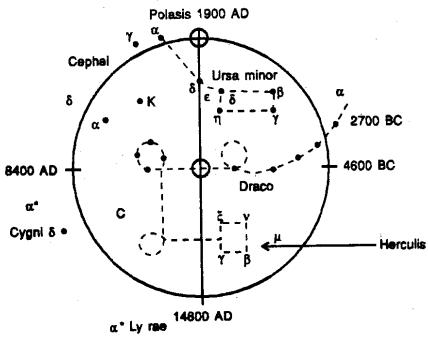



Figure 2 - centre of this circle is ecliptic pole

proverbially sun as viṣṇu is under draco or śeṣanāga with 3-1/2 turns. Base of human body cakras is also called serpent of 3-1/2 turns (Kuṇḍalinī). Thus it was called draco is Chaldia and dragon of 3-1/2 turns in China also.

Since pole star is 84 kalā from pole to meṣa 0°, or revatī nakśatra, it appears above north pole when revatī is on meridian and below 84 kalā when nakśatra 180° opposite citrā is on meridian. When nakśatras 90° from these are on meridian, (Śravaṇa or punarvasu), altitude of north star will be same as pole (though east or west by 84 kalā).

Verse 63 - Similarly south pole star also appears to move around south pole like a bullock rotating the oilseed crusher in a circle.

Note - There is no conspicous star near south pole. Octans group contains south pole, but its brightest star ν is of 3.7 magnitude and official pole star σ (sigma octanis) has 5.5 magnitude. It is in line with bigger arm of south cross group.

Verses 64-66 : Three measurements - Three angular measurements (west to east) are used - Mānāṁśa, Kālāmśa and Kśetrāṁśa. Rising times of rāśis being different, Kśetramśa and Kālāmśa are different. After one revolution, both complete 360°. Mānāmsa and Kālamśa are same on equator, but difference between them increases as we go further from equator, in north or south direction.

Notes : The different measures depend on different system of coordinates shown in figure 3.

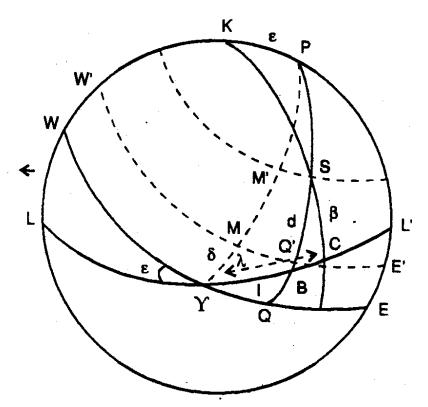



Figure 3 - System of Coordinates

P = Celestial Pole (Dhruva)

QE = Celestial equator

K = Pole of equator (Kadamba)

♈L′ = Plane of the ecliptic

♈= First point of sāyana meṣa (vernal equinox)

L,L′ = First points of sāyana makara and karka/winter and summer solstice).

S = a heavenly body

PS = A great circle through P,S, cutting equator at Q and ecliptic at B.

♈ Q = Right ascension = a = Kālāṁśa (time is measured along equator rotation) 1 Kalā at equator = 1 asu, R.A. of 1 hour = 15° at equator.

QS = Declination = Krānti δ

KS = Great circle through K, S, cutting ecliptic at C.

♈ C = Celestial longitude = λ = Kśetrāmśa or bhogāṁśa

CS = Celestial latitude = β = Śara or vikśepa

♈ B = Polar longitude or dhruvaka = l

BS = Polar latitude = vikśepa (dhruva) = d

Polar longitude (dhruvāṁśa) and latitude (vikśepa) have been used only in this chapter to indicate position of stars as we observe them with reference to fix position of pole.

W′E′ is a circle parallel to equator in north at ange δ, latitude of the circle ♈. M = ϕ where M is point on P corresponding to meṣa 0°. Absolute length of arc between M and corresponding position Q′ is almost same as great circle between them. The great circle between M and Q′ is mānāṁśa

Arc MD = Arc ♈ Q. cos δ

Though the angular difference between MQ′ and YQ is same, mānāṁśa is less. It becomes lesser, if δ increases i.e. we go farther from equator.

Kālāmśa is the distance along equator, hence it is equal to rising time of rāśis (at equator).

Verses 66-68 : Saptarṣi measures

Stars in saptarśi are taken from east to west along declining longitude (deśāntara), not in north south direction (akśāmsa) (Second half of verse 66)

Here difference between dhruva vikśepa of pulaha and kratu is only 5°. Similarly, east west difference mānāṁśa between kratu and marīci is 25° and difference in vikśepa is only 0°30′. Sphuṭa krānti can be calculated.

Verses 69-70 : Conversion of three measures

Sphuṭa krānti and dyujyā are calculated. Sum of two dyujyā for kranti vṛtta and viṣuva vṛtta is made half - it is called hāra.

Mānāṁśa multiplied by trijyā and divided by hāra gives kśetrāmśa (degree on ecliptic)

Kśetramsa multiplied by rising time of its rāśi and divided by 180° gives kālāmśa. This way kālāṁśa of marici and kratu can be found. By reverse process, mānāṁśa can be found from this kālāṁśa.

Notes : (1) In notes of previous section, vide figure (3) mānāṁśa is measured along W′E′ parallel to equator which is diurnal circle of a star of this declination. It is easier to convert it to kālāmśa as explained in the note, or in spaṣṭādhikāra for calculation of day length.

Mānāṁśa $M = H \cos \delta$

(where H is kālāṁśa, δ is krānti)

$$= \frac{H.\ R \cos \delta}{R} = \frac{H\ \text{Dyujyā}}{\text{Trijyā}}$$

However, δ here is measured along dhruva prota SQ instead of SC line. Thus length along ecliptic is reduced due to lesser rising time of B compared to C, and increases due to oblique length of B. Thus instead of dyujyā we take average dyujyā and trijyā.

$$\text{Hence Kālāmśa}\quad H = \frac{M\ \text{Trijyā}}{\frac{1}{2}(\text{dyujyā} + \text{trijyā})}$$

(2) Kśetrāmśa is converted to kālamśa as per the following approximate ratio used for calculation of lagna -

$$\frac{\text{Rising time of rāsi}}{\text{Rāsi (1800 kalā)}} = \frac{\text{Rising time for kśetrāṁśa}}{\text{Kālāmśa in kalā}}$$

Rising time of kśetrāmśa is measured along equator, hence its asu is equal to kalā of kālāṁśa.

Verses 71-75 : Variation of kālāṁśa and mānāṁśa- Dhruva star moves in a circle of 360°, hence its mānāṁśa

$$= \frac{360 \times 84}{3438} = 8'48''$$

i.e. 1° of this circle is equal to length of 8′48″ on equator.

Due to change in krānti, if shape of sapktarṣis remains fixed, then with change in krānti, their rising time will also change with change in dyujyā.

Since kālāntara of saptarṣis is fixed with change in krānti then, east west distance will

change with change in akśāmsa. If krānti is fixed, then kālāmśa will be constant.

If kśetrāmśa is constant, then kālāmśa and mānāmśa wil vary. Like kālāmśa and bhāgāmśa, relation between kālāmśa and mānāmśa also can be found.

Verses 76-79 : Śara of naksatras

The dhruvāmśa of nakśatras given here are already with āyana dṛkkarma. Their śara also is sphuṭa i.e. in dhruva prota vṛtta.

But śara of graha is asphuṭa, i.e. in kadamba prota vṛtta. After dṛkkarma, it will become śara in direction of dhruva prota vṛtta of stars.

Even when the kadmba prota śara of nakśatras is same, their krānti in dhruva prota circle will be different due to east west difference. Hence the length of their day and night will be different (as the semi diameter of diurnal circle - dyujyā, depends on distance from equator in dhruva prota direction).

If the sphuṭa krānti of a nakśatra is more than the co-latitude of a place, the nakśatra will be always rising at that place (it will be always seen there above horizon). If the south sphuṭa krānti is more than the co-latitude of the place, it will never rise above horizon, i.e. always set.

Notes : Dhruvāṁśa has already been explained. This has been used for indicating position of stars because it is easier to observe them with reference to north pole.

Circumpolar stars have already been explained. For them, carajyā = R tan δ tan Φ is

bigger than R, radius of equator, hence day length will be more than day night value.

Thus $\tan \delta \ \tan \phi > 1$

or $\tan \delta > \cot \phi = \tan \ (90^\circ - \phi)$

Here, δ is krānti, Φ is akśāmśa, hence $90^\circ - \Phi$ is lambāṁśa.

Thus $\delta > 90^\circ - \Phi$

Then the star will be always rising if krānti is bigger than lambāṁśa.

Similarly for south krānti, if carajyā is bigger than R, day length wil be less than 0, i.e. the star will not rise. This means the same condition.

Physically, we can understand it, because north pole is above horizon at angle equal to local akśamsa. Distance from north pole to the star is $90^\circ - \delta$ which should be always less than Φ if the star is to remain above horizon. Thus $\phi > 90^\circ - \delta$ or $\delta > 90^\circ - \Phi$

i.e. Krānti > lambāmśa

Similarly, south pole is Φ° below south horizon, A star with south krānti will be $90^\circ - \delta$ away from south pole. If this distance $90^\circ - \delta$ is less than Φ, then the star will never rise.

Verses 80-84 - Conjunction of graha and nakśatra

Āyana dṛkkarma is done for the involved graha and difference of dhruvāṁśa of graha and nakśatra is found. The difference in kalā is divided by sphuṭa gati of the graha in kalā to get result in day, ghaṭi etc. If dhruva of graha is less than nakśatra, the conjunction will occur after that interval, if it is more, then the yoga has already

occurred, that period before. When graha is vakrī (retrograde) then opposite order will happen (i.e. if graha dhruva is less, conjunction has happened earlier). For this conjunction time, we again find difference in sphuṭa dhruva of graha and nakśatra and get the more accurate value of conjunction time. After successive approximations, we get the correct conjunction time.

After that, sphuṭa krānti and cara of graha and nakśatra are found and cara is calculated. That will give periods of their day and night. From that we get the values of udaya and asta lagna of graha and nakśatra for their rising and setting times. As explained before, the rising and setting times will be when sphuṭa sun reaches those positions (of udaya and asta lagna). By finding difference of aṁśa at rising setting times, we get the proportional difference between graha and nakśatra according to the natakāla (ākśa dṛkkarma explained earlier) and again we revise the conjunction time, when longitude of graha and nakśatra are same after akśa dṛkkarma.

As explained in conjunction of planets, we find the north south difference between graha and nakśatra from difference of their dhruva prota śara. Distance between discs is obtained by substracting the semi diameter of biṁbas from this distance.

Notes : The methods of āyana and ākśa dṛkkarma have already been explained in conjunction of planets. Here, the problem is simpler, because the position of nakśatra is alrady stated corrected by āyana dṛkkarma. Further, we need not calculate motion of nakśatra, because they are fixed. Here also conjunctions will be different according to distance between discs.

Verses 85-87 : Bheda of nakśatras

Planets can enter the following 13 nakśatras (or do 'bheda' in their extent) -

Rohinī, puṣya, kṛttikā, citra, maghā, punarvasu, anurādhā, jyeṣṭha, viśākhā, revatī, śatabhiṣaj, pūrvāṣāḍha, and uttarāṣāḍha.

Other fifteen nakśatras are never crossed by planets (no bheda) -

Aśvinī, bharaṇī, mṛgaśirā, ārdrā, aśleṣā, pūrvā phālgunī, uttarāphalgunī, hasta, śvāti, mūla, abhijit, śravaṇa, dhaniṣṭhā, pūrva bhādra pada and uttara bhādrapada.

Among crossed (bhedya) nakśatras, punarvasu is crossed by every planet. Pūrvāṣaḍha, revatī, and kṛttikā are sometimes crossed. Others are less frequently crossed according to krānti of the graha.

The planet whose south krānti in 14th degree of vṛṣa (44°) is more than 2°20′, can cross the rohiṇī (in shape of śakaṭa - cart).

When other nakśatras are pierced or entered by graha, it is confirmed by seeing with instrument.

Shapes of nakśatras and planets moving north or south (beyond nakśatra) can be seen in Bṛhatsamhitā by Varāhamihira.

Thus positions of many stars have been told which are famous since ancient times. There are many other stars in unlimited number. Nothing has been told here about nakśatras expect aśvinī etc.

Notes (1) The graha move in ecliptic with little deviation according to their small śara. Many nakśatras have large deviations, where the graha

will never reach. Punarvasu is lying on ecliptic, hence it is crossed by all planets. This was the nakśatra which determined start of solar year and malamāsa in lunar year in vedic era. Hence its name is punarvasu, i.e. resettle or restart of year. 13 nakśatras with less deviations can be crossed by planets.

Śakaṭa bheda - Rohiṇī is in shape of cart i.e. śakata, hence its bheda is called śakaṭa bheda. Its yogatārā has 5°32′ south śara, but northern most star has 2°35′ south śara. According to siddhānta darpaṇa it is 2°20′. Moon has śara upto 5° hence it can easily cross rohiṇī, but except budha and śukra, no other graha has parama śara of this value. Parama śara of śani is 2°29′39″ hence śaniś śakata bheda also appears impossible. But Varāhamihira and Grahalāghava author have stated that śakaṭa bheda by śani or maṅgala is very inauspicious.

For siddhānta darpaṇa value of śaniś śara, its śakaṭa bheda is just possible (at 2°20′ south śara).

Maṅgala parama śara is only 1°51′ according to siddhānta darpaṇa and modern value but still less according to earlier texts. Vedha of rohiṇī is possible only when south śara is assumed less, which is not given in the texts.

According to star catelogues 3000 to 6000 stars only can be seen with naked eye. There are about 10^{11} stars in our galaxy (of average size of sun) and there are about 10^{9}) galaxies in universe.

Verses 88-92 : MIlky way -

A fine circular way of dense fine stars in seen in the sky. This is called chāyā patha, vaiśvānara

patha or abhijit mārga (ākāśa gangā also). It is proposed to describe it.

This chāyā patha is circular. It crosses ecliptic in śāyana karka and sāyana makara beginning. Again it extends 60° north from sāyana meṣa to 63° south from sāyana tulā. This crosses south part of punarvasu and goes southwards. Then it crosses mūla and śravaṇa nakśatras and goes upto centre of abhijit and śravaṇa. From there, it goes northwards. From beginning point of karka, it goes north in two branches. This can be easily shown by diagram on a sphere. But in sky, it is seen half only at a time, hence it is impossible to show it.

We can easily see the stars (separately) of milky way with telescope. We can also see puṣya nakśatra, black spots on sun, water, mountains and trees on moon. Telescope can show phases of budha and śukra also like moon. Ring around śani and many new planets and satelites can be seen with it.

Notes : (1) The galaxy is called ākāśa gangā, chāyāpatha, viṣṇupada etc. However vaiśvānara patha is name of ecliptic according to many. The ākāśa gangā, is the disc portion of galaxy which is dense area with more number of stars, hence it looks like a way. The main portion of the galaxy is a disc of about 30 kilo persec width. It has two spiral arms and sun is located in inner arm as shown in figure 4(a) and 4(b). Sun is 10 Kpc away from centre i.e. about 2/3rd of the radius. 1 persec = 3.26 light years approximately, kilo = 1000. The galaxy rotates along the central plane of disc, which is almost parallel to orbits of solar system, central

portion rotates with uniform velocity almost like a rigid body. Stars in vicinity of sun in the disc are rotating with speed of about 220-250 kms/sec around galactic centre. Mass of galaxy inside sun's orbit is 1.4 X 10^{11} sun masses. Total energy of galaxy also is about 0.8 X 10^{11} of sun. Mass of stars is 2X10^{44} gram.

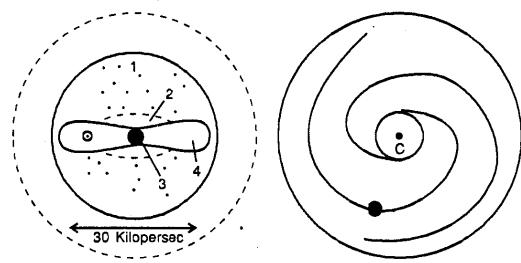



Figure 4a - Structure of galaxy Figure 4b - Spiral arms

The points in figure 4a represent some of the globular clustures. The position of sun is marked with the sign. Regions are marked 1 to 5 – 1. The spherical subsystem, 2 - the disk, 3 - the nucleus, 4 - the layer of gas dust clouds, 5 - the corona. Radius of corona is at least a dozen time the radius of galaxy.

Figure 4(b) is disc of the galaxy. The nucleus is at centre C. Two spiral arms are spreading from it. Sun is in one of the arms.

Spread of galaxy can be seen from figure 4c. C is centre and E is edge of disc. Sun is S. So SC = 10, CE = 15 kpc. $\angle ESC=\theta$ is spread of disc. $\tan\theta$ = 3/4 hence θ = 60° approximately.

Figure 4C - spread of galaxy disc

Central portion and the disc are dense and obscured by gases. It can be observed only by radio telescopes. It is believed that nucleus of galaxy contains huge black hole. Spherical subsystem contains old stars and globular clustures. They rotate with about 1/5th velocity of disc stars. Mass of corona is many times the mass of galaxy, but its density is much less and it does not emit any light. It is felt only by its gravitation.

Centre of galaxy is in mūla nakśatra. its old name was mūla barhaṇi - i.e. the root from which cosmic egg has spread. Probably its position as galectic centre was known. Linga purāṇa also states that brahmā travelled for 30,000 years in cosmic śiva linga - this is the distance in light years from centre to sun.

(2) A note about magnitude of stars - The visual magnitude of the stars has been made in a logarithmic scale. Star of Ist magnitude is 100 times bright than 6th magnitude i.e. increase of 5 magnitude reduces the brightness by 1/100. Change in magnitude of 1 reduces the brightness by $(100)^{-1/5}$ = 1/2.5 approx. Brightness in this scale is

Sun - 26.5 i.e. 6,31,000 times moon

Moon - 12.0

Venus - 3.0

Sirius -1.4

Rohiṇī + 1.0 Brahmahṛdaya 0.1

Absolute magnitude is measured by emitting power compared to sun in similar logarithmic scale.

Verses 93-94 : Prayer and conclusion

May supreme lord Jagannātha destroy our forest of mishaps, who puts the golden ornaments to shame with his yellow dress, who is closely watching the creation and events in the cosmic egg, who is expert in dancing on hoods of Kaliya nāga and who is radiant near tree of desires.

Thus ends the twelfth chapter describing conjunction of graha and nakśatra in siddhānta darpaṇa written as text book and correction of calculation by Śrī Candraśekhara born in famous royal family of Orissa.

Chapter - 13

RISING SETTING OF PLANETS, STARS

Graharkśodayāsta samaya varṇana

Verse 1 - Scope - Now I describe the rising and setting of planets and stars. In sphuṭādhikāra, udaya and asta have been roughly described on the basis of difference of their kendrāmśa.

Verses 2-3 : Types of rising and setting -

Udaya and aṣṭa are of two types - Nitya (daily) and naimittika (occasional or seen).

In first nitya type, due to rotation of pravaha. (daily rotation of earth), planets and stars rise daily in the east and set in the west. Hence, it is called nitya (regular) or pratyaha (daily).

The planets rise when they are far from sun and are visible. They set when they become invisible due to closeness of sun. This is called naimittika (i.e. casual) udayāsta.

Verses 4-6 : Rising setting of planets (Sūrya siddhānta) - Maṅgala, guru and śani, set in west when their longitude (rāśi etc.) is more than sun, when it is less then sun, they rise in the east. Vakrī budha and śukra also set in west and rise in east, when their longitude is more than sun or less than it.

Here more and less do not mean numerically bigger rāśi. If the planet is ahead of sun in nearer portion of arc, then it is more in rāśi and if behind,

it is west. For example, sun in meṣa will be considered more than a planet in mīna, because meṣa is unmedately after mīna. From meṣa to mina direction, mīna is greater in numbers but it comes at the end of circle.

(Surya siddhānta) - When budha and śukra, moon are less than sūrya, they set in east. When they are more than sūrya, they rise in west. This is because budha and śukra are faster than sun.

Notes : In general the rising and setting of planets etc is due to daily rotation of earth, due to which each star rises in east and sets in west. This is called daily rising and setting. Siddhānta darpaṇa assumes that earth is fixed and the sky is rotated east to west by a wind pravaha, which is equivalent to daily rotation of earth.

This chapters deals with the other type of rising and setting caused by brightness of sun. In western astronomy, it is called heliacal rising and setting (heliacal = caused by sun, Helios = sun in Greek). In this rising, when the planets are very close to sun and they rise around sunrise in east and set with sun, they cannot be seen due to closeness of sun. They are said asta (naimitika) or heliacally set. When they are slightly away from sun and are seen slightly before sun rise (in east or west) or after sunset, they are considered heliacally risen or naimittika udaya.

First part of the discussion is about maṅgala, guru and śani which are slower than sun. When sun is behind them sun appears to be moving towards them. When they become very close, these planets become invisible. Before that closeness, they

are seen after sunset in west. After some days, they become invisible due to closeness of sun, hence they are said to set (heliacally) in west. After the time of closeness, sun goes ahead, then the planets are seen in east before sun rise. Hence the three planets are said to rise in east (heliacally).

When vakrī budha and śukra are ahead of sun, then they are seen in west after sunset and set there itself. After some days, they go to the other side of sun (less longitude, or west) and they are seen in east before sunrise. Hence vakrī budha and śukra set in west and rise in east.

When mārgī budha, śukra (and candra) are behind sun, they become nearer due to more speed and become invisible due to closeness. Then they are behind sun and are invisible in east before sun rise. Hence they heliacally set in east when they go ahead of sun, they are visible in west after sun set and are said to rise in west.

Verse 7-11 : Dṛkkarma for rising and setting -

For knowing the rising or setting time of a graha in west, on the approximate day of rising or setting, spaṣṭa sūrya and graha are found at sunset time. If the rising or setting is to be calculated in the east, then on apporoximate day they are calculated at sunrise time. (The approximate time of rising or setting is known from rough kendrāṁśa as explained in spaṣṭa-dhikāra). After that dṛkkrama of both the planets is done. (Sūrya siddhānta quotation). Āyana dṛkkarma is done first, then ākśa drkkarma is done.

Method for ākśa dṛkkrama - Sphuṭa śara of graha in kalā is multiplied by palabhā and divided

by 12. Result is multiplied by 1800 and divided by lagna asu of that time. Result will be in kalā etc. This will added for south śara of graha and substracted for north śara when sun is in east horizon. When sun is setting in west, reverse order will be followed.

(Sūrya siddhānta) Difference of ākśa karma corrected graha and sun in asu is divided by 60 to find kālāmśa. For rising and setting in west, we find the difference between (6 rāśis added to graha) and the sun. By correcting graha with that, we again find kālāmśa difference.

Nati correction in sphuṭa śara of moon is done by the method explained in sūrya grahaṇa (chapter 9). To see the setting of moon in east, it is added to accurate moon and substracted from it to see the rise in west.

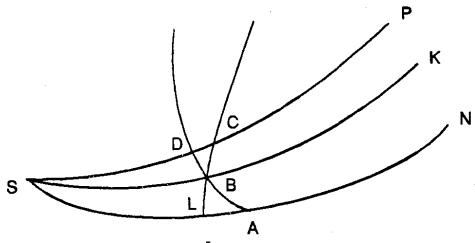



Figure 1 - Ākśakarma at Kśitija

Notes - (1) Ākśa dṛkkrama at Kśitija - In Fig.1

NALS - Eastern horizon

N = North point

S = position of rising graha

L = Udaya lagna

K = Kadamba, P = pole

B = position of rising planet S on krānti vṛtta

C = Planet S on krānti vṛtta on dhruva prota circle.

ABD = Diurnal circle of B

CL = Ākśa drkkarma of C.

Here the planet with south śara, rises after its ecliptic position B has risen, or dhruvaprota position of C has gone further above.

Diurnal circle of B cuts, dhruva prota of S on D.

ADS is a spherical right angled triangle, because AD is parallel to equator, hence perpendicular to dhruvaprota line. Hence ∠ DAS = lambāṁśa = $90°-\phi$, where ϕ is akśāmśa, $\angle DSA=\phi$

Small triangle DSA can be considered a plane figure

Hence

$$\frac{DA}{DS} = \frac{\text{Sin} \angle DSA}{\text{Sin} \angle DAS} = \frac{\text{Sin}\,\Phi}{\text{Sin}\,(90° - \Phi)} = \frac{\text{Pabbhā}}{12}$$

Now, approximately SD = SB and DA = CL

$$\frac{CL}{BS} = \frac{DA}{DS} = \frac{\text{palabhā}}{12}$$

$$\text{or } CL = \frac{\text{Śara} \times \text{palabhā}}{12}, \quad \text{as } BS = \text{Śara}$$

Since in south śara the planet is above horizon at sun rise time, its ākśa correction is added to the planet.

(2) Kālāmśa is the time before sunrise when a planet rises. It is equal to 1 asu for 1 kalā difference on equator. The difference between sun

and planet corrected for ākśa karma wil be rising time difference in asu = Kālāntara in kalā

$$\text{Hence, kalāntara in aṁśa} = \frac{\text{Kālāntara in kalā}}{60}$$

$$= \frac{\text{Rising time diff in asu}}{60}$$

(3) Nati saṁskāra is needed only for moon as it is very little for other planets.

Verse 12 - When the laṁbana corrected moon is at 11° kālāṁśa from sun, then it is seen on horizon. When its distance is less than 11 kālāmśa it cannot be seen.

Verses 13-16 - Rising of stars

Śara of nakśatra are bigger. Hence sphuṭa krānti is found from their śara. For this sphuṭa krānti, carajyā and day length in asu is found. That will give daily rising or setting time and lagna. At the time of rising (or setting), we get the difference of lagna and sun. The rising time for that difference in asu divided by 60 will give kālāmśa.

The kālāṁśa at the time of rise in east or setting in west is only dependent on sun motion, because stars don't move. Hence, they rise or set at distance of kālāṁśa from sun in east or west like farther planets maṅgala etc.

Before setting in west the stars rise in east, due to daily motion. It is not connected to distance from sun.

Notes : The method explained earlier for grahas was approximate for small śara. But nakśatras have bigger śara and accurate method as

explained in chapter 11 - for conjunction of planets is to be used. The rising time difference is found by sphuṭa krānti of star and sun i.e. it will be difference in their carajyā only. Since it is at times of sun set or sun rise, proportional difference for natāṁśa of sun need not be made.

$$\text{Thus, kalāmsa} = \frac{\text{Carjyā difference in asu}}{60}$$

Verses 17-24 - Kalāṁśa of stars

Kālāṁśa of nakśatras in degrees depends on their biṁba diameters in vikalā. The observed values of kālāṁśa for successive rise in biṁba vikalā is given below -

Biṁba Vīkalā	Kālāṁśa
1	24
1/15	23
1/30	22
1/45	21
2	20
2/30	19
3	18
4	17
5	17
6	16
7	16
8	15
9	15
10	15
11, 12, 13	14
15, 16	13
17, 18, 19	12
20, 21, 22	11

23 to 28	10
29 to 40	9

Notes : Biṁba value is not the diameter of stars because it is so small, it cannot be measured even with telescope. It is only a measure of brightness estimated empirically. Actually the visibility distance (kālāmśa) from sun is one of the measures of brightness - expressed as biṁba diameter.

Verses 25 - Kālāṁśa of tārā grahas

Kālāṁsa of tārāgrahas are

Ŝukra 9, vṛhaspati - 11, budha 13, śani 15, maṅgala 17.

For śukra and budha, the above values are average. Their kālāṁśa at cakra or cakrardha (0° from sun - farther side is cakra, 180° from sun i.e. near side is cakrārdha) are

	Cakra	Cakrārdha
Budha	14	12
Sukra	10	8

Note : At the end of cakra, on farther side of sun, the planets are farther hence light intensity is smaller. Hence, they become invisible at greater distance. Then they rise in east and set in west. At cakrārdha, budha and śūkra are between earth and sun and vakrī, then they set in east and rise in west.

Verse 27-29 : Rules for heliacal rising

When difference between sun and the planet or star is more than the kālāṁśa, it will not be visible (due to light of sun).

Difference of graha or nakśatra with sun being less than its kālāmśa in west, means it has already set. If difference in more than kālāmśa, then it will set in near future.

If in east direction, rising will be in reverse order. If difference (kālāntara) is more than kālāṁśa, then planet has already risen, if less than kālāṁśa, it is yet to rise.

Notes - (1) Condition of rising are

(1) Planet should be above horiźon.

(2) It should be night time for its being visible. Even in night, slightly before sunrise or after sunset, it becomes invisible due to twilight. In the limiting case of rising in east, its difference from sun should be more than kālāṁśa. Bigger or brighter planet will be visible at lesser distance from sun.

(2) Setting in west - slow planets maṅgala, guru or śani or stars are overtaken by faster sun. In west, they rise after sunset at the minimum distance of kālāmśa, when it has become sufficiently dark. It the limiting case, they are east from sun at kālāmśa distance, when distance is more, it will be reduced in future, when the planets or star will set. Same happens with vakrī budha or śukra.

For rising at sunrise time, they should rise before sunrise, i.e. west from sun at kālāmśa. Distance of sun increases due to its faster speed in east direction. Hence if it is more than kālāmśa, it was equal to kālāmśa earlier, when planet or star has risen. Vakrī budha and śukra also are separated further as they are in west and moving further west.

Verses 30-33 - Day of rising or setting of planets - Graha and sun are added with ayanāmśa. For setting time, six rāśi is added to both. For graha and sāyana sun at rising time (6 rāśi added to each for setting time) rising time of their rāśis are multiplied by daily speed and divided by (1800). Result will be kāla gati at the time of rising or setting.

We calculate the difference between rising times of the planet and sun at sunset or sun rise times before ākśakarma. This is iṣṭa kālāmśa from which kalāmśa of rising is substracted. This kālāntara is divided by difference of kālagati of sun and forward moving graha. If graha is retrograde kālāntara is divided by sum of kālagatis. Result will be past days or coming days of rising or setting, as per rules explained earlier.

Notes : (1) Since inclination of planetary orbits with ecliptic is very small, it can be assumed to move on ecliptic.

1800 kalā on ecliptic = rising time of that rāśi on equator is asu

$$\text{or Kāla gati in 1 day} = \frac{\text{gati kalā} \times \text{rising time}}{1800} \quad - (1)$$

(Kāla gati in kalā.)

(2) Kālāṁśa antara or kālāntara

= Kālāmśa of graha on iṣṭa day - parama kālāmsa of graha

Past or remaining days

$$= \frac{\text{Kālāntara}}{\text{Sun kāla gati} \pm \text{graha kālagati}} \quad - - - (2)$$

Here + sign is for vakrī graha and - ve sign is for forward graha. - - - (3)

Verses 34-37 - Ākśa drkkarma for stars.

Difference of iṣṭa kālāmśa between nakśatra and sphuṭa ravi and the parama kālāmśa is divided by kāla gati of sun at the place of nakśatra dhruva. This will give past time or remaining time for udaya kāla (if sphuṭa sun at rising time is taken) or asta kāla.

Half day of nakśatra is calculated for sphuṭa krānti and asphuṭa krānti. Their difference in asu is multiplied by 1800 and divided by rising time of the rāśi for rising in east (or rising time of 6 rāśi + nakśatra for setting in west).

Result will be ākśa dṛkkrama in kalā. When nakśatra has north śara, ākśa kalā will be added to nakśatra in west and substracted from nakśatra in east. For south śara of nakśatra, ākśa kalā is added to nakśatra in east and substracted for nakśatra in west. Result is dṛkkarma corrected dhruva and kśetrāṁśa is found from that.

Notes : (1) Nakśatras have no proper motion, hence their rising time is calculated only from sun's motions. Here, in place of sun gati ± nakśatra gati = sun gati – 0 = sun gati only.

Similarly, krānti of nakśatra is fixed, hence āyana dṛkkarma is not done, only ākśa karma is done.

(2) Udaya lagna or udaya vilagna of a star is that point of the ecliptic which rises in the eastern horizon simultaneously with the star and the asta vilgana or asta lagna of a star is the point of ecliptic which rises on the east horizon when the star sets on western horizon.

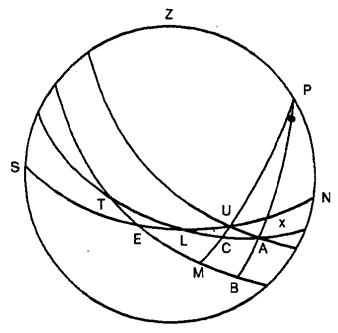



Figure 2 - Ākśa dṛkkarma of a nakśatra

Figure 2 is cēlestial sphere for a place of latitude Φ., SEN is horizon, S, E, N being south, east and north cardinal points. Z is zenith. X is position of star when it rises on the horizon (eastern). TEB is the equator and P its north pole. TLA is the ecliptic and L is the point of the ecliptic which rises with star X i.e. star's udaya lagna. The point T where the ecliptic intersects the equator is the first point of Aries (sāyana meṣa). PXAB is the hour circle (dhruvaprota circle) of star X and A the point where it intersects the ecliptic. U is the point where diurnal circle of A intersects the horizon.

Now arc EB is the ascensional difference (carajyā) due to true declination (arc XB) i.e. spaṣṭa krānti of star. Arc EM is carjyā due to madhya krānti (of the star's position on ecliptic) i.e. arc AB of the star. The difference of these carajyā is arc MB in asu. In asu of arc MB, portion CA of ecliptic rises. CA has been approximately considered equal to LA, ākśa dṛkkrama of star.

Thus ākśa dṛkkrama = Carajyā for spaṣṭa krānti – carajyā for madhya krānti.

Since dinārdha = 15 ghaṭī + carajyā

difference of dinārdha = diff. of carājyā - (1)

= ākśa dṛkkarma

Longitude of the star's udaya lagna L i.e. arc TL

= arc TA - arc LA = arc TA - arc CA approx

= Polar longitude - ākśa drkkarma. - - - (2)

This explains when the star is north of the ecliptic, then ākśa dṛkkarma is substracted from star to find udaya lagna in east.

For asta lagna, it will be added to polar longitude of star (dhumvāṁśa) added to six rāśis.

(3) Rules for rising and setting can be stated as star rises heliacally when

sun's longitude = udaya lagna of star + kālāṁśa. It sets heliacally when

sun's longitude = astalagna - kālāmśa - 6 signs.

Star is invisible if,

Sun's longitude - udaya lagna < kālāmśa

or, asta lagna - sun's longitude < kalāmśa + 6 rāśis

This can be stated in terms of udayārka and astārka. Udayārka (or udaya sūrya) is position of sun when a star rises heliacally.

Astārka is position of sun when a star sets heliacally.

Calculation of udayārka—Star's udaya lagna is taken as sun's longitude and it is assumed that

time elapsed since sunrise is equal to star's kālāmśa ghatīs. Lagna at that time is itself udayārka.

Calculation of astārka - Star's asta lagna is taken as sun's longitude and kālāṁśa ghaṭīs of time before sunrise, we calculate the lagna. By adding 6 rāśis to this lagna we get astārka.

Theorem (1) - If astārka > udayārka, star will never set.

When sun = udayārka, the star rises heliacally. Thereafter, as the sun moves, distance of sun from udayalagna increases and star remains visible. Since astārka > udayārka, the same happens when sun = astārka.

The star therefore, does not set when sun = astārka. Thus setting is impossible in this case.

This happens, when star has sufficiently big north latitude (for places of north latitude), such that star's ākśa dṛkkarma > star's kālāṁśa (on ecliptic). For, in the case.

Udayārka = star's polar longitude – ākśa dṛkkarma + kālāṁśa

< Star's polar longitude

and, astārka = star's polar longitude + ākśa dṛkkarma - kālāṁśa

> star's polar longitude

So that, Astārka > star's polar longitude > udayārka

Theorem (2) - If, for a star, astārka < udayārka, then the star will rise and also set. The star will remain set when, astārka < sun < udayārka and will remain visible when sun <astārka but > udayārka.

Proof - When sun = astārka, the star sets heliacally. As the sun's longitude increases, the distance between asta lagna and sun diminishes and star remains heliacally set. This happens until sun = udayārka, when the star rises helically. Thus sun remains set until, astārka < sun < udayārka.

When sun goes beyond this limit, it is helically visible.

This case happens when the star's latitude is north and its ākśa dṛkkarma < kālāmśa of star.

For, udayārka = polar longitude of star - ākśa dṛkkarma + kālāmśa on ecliptic

> star's polar longitude

and, Astārka = Star's polar longitude + ākśa dṛkkarma - kālāmśa

< star's polar longitude

so that, astārka < star's polar longitude < udayārka

This also happens, when star's latitude is south.

For, udayārka = star's polar longitude + ākśa dṛkkarma + kālāmśa

> star's polar longitude

Astārka = star's polar longitude - ākśa dṛkkarma - kālāmśa

< star's polar longitude

so that, Astārka < star's polar longitude < udayārka.

Rule for set period : A star remains heliacally set until astārka < sun < udayārka

Between this period we take sun's speed as the speed at position of star's polar longitude which is in between these two values, hence can be taken as average speed. Hence this period for setting

$$= \frac{\text{Udayārka} - \text{Astārka}}{\text{Average speed of sun}}$$

Verses 38 - 40 : Kālāmśa of the yogatārā of a nakśatra is expressed in kalā, multiplied by 1800 and divided by rising time of its sāyana rāśi (for rising) and by rising time of (sāyana rāśi + 6 rāśis) for setting. Result will be kśetrāmśa in krānti vṛtta. This will be added to dṛkkarma dhruva of nakśatra for rising or substracted for setting. This will be udaya or asta dhruva of yogatārā. When sun's dhruva (polar longitude) is equal to udaya or asta dhruva of the yogatārā, it will helically rise or set.

Notes : It has been explained is previous note. Udaya dhruva is udayārka, āsta dhruva is astārka.

Verses 41-44 : Extreme cases - Many nakśatras in north rise again in east before they set in west. Hence their setting is not necessary. Their setting has been discussed only to know the setting time in west. Udaya and asta of many nakśatras like Kratu should be calculated. Agastya and yama are in far south, hence they remain set for long.

Day length of any graha or star can be known from its carajyā calculated from krānti (and local akśāṁśa). Hence, their daily rising and setting times can be known. Still, detailed methods will be explained here for their times of udaya and asta.

The discussion so far has been done according to the views of earlier astronomers. Now I describe more accurate methods thought by me.

Notes (1) Permanent rising and setling of stars has been explained earlier. If krānti of the star is more than colatitude of the place, the star will never set for north krānti. For south krānti, greater than colatitude of the place, it will never rise.

Equivalent condition is that, astārka > udayārka; i.e. star will rise again before it sets in west, explained in theorem (1) of previous note (3) after verse 37.

(2) Rising of agastya (canopus) has been discussed extensively. According to Āryabhaṭa I, Varāhamihira and Sumati, agastya rises heliacally when

sun's longitude = $120° + \Phi$

and sets heliacally when

sun's longitude = $180°-(120° + \Phi) = 60°-\Phi$

where Φ is the latitude of the place.

According to Vateśvara; agastya rises heliacally when

sun's longitude = $98° + 42\ P/5$ degrees

and sets heliacally when it is $76° - 42P/5$ degrees.

where P is the equinoctical mid day shadow in angulas.

Mañjula gives the formulas as $97° + 8P$ and $77-8P$.

Bhāskara II and Gaṇeśa daivajña give

$98°+8P$ and $78° - 8P$

The above rules have been derived by substitution from the following formula

Udayārka = star's polar longitude + ākśa dṛkkarma + kālāmśa.

Astārka = star's polar longitude - ākśa dṛkkarma - kālāmśa.

Verses 45-50 - Sphuṭa kālāṁśa –

Planets and stars rise on horizon, when sun is still below horizon. Even in such situation they are invisible because sun's light reaches on horizon (twilight) due to reflection from atmosphere.

Natāṁśa of sun from dṛk - maṇḍala (when it is start of twilight) is multiplied by trijyā and divided by lambajyā. Result is again multiplied by trijyā and divided by dyujyā. Result will be sphuṭa kālāmśa of stars from sun.

This means that, there is big difference between dṛk-maṇḍala aṁśa and kālāmśa. On equator also, it is equal to difference between dyujyā and trijyā. At other places it is still more.

For example at a place of 66° north akśāmśa, in meṣa month (when sun is in meṣa rāśi) guru and śukra in mīna rāśi rise alongwith sun. Both these planets are seen only when away from sun. Hence, it is not necessary to calculate these kālāmśa difference in rising times. From the kālāmśa written for these planets, kśetrāmśa is more, though kālāmśa is below the visibility limit. Hence, they are seen.

Notes : Due to reflection from atmosphere, twilight starts when sun is still 18° below horizon. In India, it is assumed 15° below horizon, as it is north of equator. This is called uṣā in morning

and sandhyā is evening. Sandhyā is used for both twilight periods.

Then sun rises when it is still about 35′ below horizon due to refraction of rays in atmosphere. Hence twilight period extends from 18° below horizon to 35′ below horizon position of sun.

Thus the natāmśa of sun below horizon (18°) or natamśa of 108° from meridian is the time when sun light starts. Thus, it is increase in carajyā which is equal to increase in half day length.

Like carajyā, the natāmśa jyā is divided by cos ϕ = lamhajyā/trijyā to find rising difference on diurnal circle. It is divided by cos δ = R cos δ /R

= dyujyā/trijyā to get the degrees on equator whose kalā is equal to asu time. Hence the formula.

Here, dyujyā on equator means koṭijyā of natāmśa, instead of koṭijyā of krānti. For 66° north akśāmśa, the difference is Sin 18°/cos 66° = Sin 30° approx. Hence guru and śukra rise with 30° or 1 rāśi difference.

Verses 50-58 - Sphuṭa dhruva of udayāsta of graha - From the udaya and asta kendrāṁśa, we find the udaya and asta kāla of planets. For that time, mandaphala of sun is calculated. This mandaphala is substracted from fourth śīghra kendra of guru, maṅgala, śani at the time of udaya or asta, or added to it in same manner, it is substracted or added to sun. In budha or śukra, this correction will be in reverse order.

If graha is less then the true ṣāyana sun at that time, half the kālāmśa of graha is substracted from sun. If graha is more than sāyana sun, then half kālāmśa is added to sāyana sun. If sun is in

east, that will the lagna at that time. When sun is in west, 6 rāśi is added to sun ± half kālāmśa. That will be the lagna for setting time.

3 rāśi is substracted from this lagna. Krānti of that point of ecliptic (tribhona lagna) is calculated. By adding or substracting akśāmśa to krānti, natāṁśa and unnatāmśa is found (for tribhona lagna).

The natāmśa of śukra, guru, budha, śani are divided by 4,5,6,7,8 and result is added to unnatāmśa. Jyā of the resulting angle is found. Kālāmśa of the planets for udaya or asta is multiplied by trijyā and divided by jyā of the corrected unnatāmśa.

Result is degrees etc. will be kālāmśa of graha in ecliptic. This substracted from sun will be dhruva for asta or udaya. Half of this asta or udaya dhruva, is added or substracted from sāyana sun as before - That will give corrected udaya lagna or asta lagna.

Notes : No logic has been given for such a long and arbitrary process. Probable justification is given below -

(1) Kālāmśa difference from sun is measure of decrease in intensity of sun light. Since it decreases according to inverse square of distance, kālāmśa proportinate to biṁba diameter (measure of intensity) is reduced by half.

(2) Mandaphala substracted from śīghra kendra, is distance of planet from sun, on which the brightness of graha depends.

(3) Natāmśa of tribhona lagna is proportional to inclination of diurnal circle with vertical. The kālāmśa will increase in proportion to this obliquity.

It is divided by half the values of kālāmśa of graha. For bright planet, kālāmśa is less, fraction of natāmśa is more, then corrected unnatāmśa and its jyā will be more, hence sphuṭa kālāmśa will be less as it is divided by jyā.

Still the method appears to be based on trial and error and probably gave better results.

Verses 59-68 : Kśetrāmśa of planets for mid Orissa Kśetrāmśa of planets is being given for mid Orissa according to rāśi of sāyana sun.

Planet	Rāśi of sāyana sun	kśetrāmśa
Śukra	1, 12	10/26
	2, 11	9/51
	3, 10	9/40
	4, 9	9/20
	5,8	9/0
	6-7	9/1

When sun is in west, 6 rāśi is deducted from it and then kśetrāmśa is found. Then the degrees of sāyāna sun are multiplied by difference of dhruvāmśa and added to kśetrāmśa if increasing, or substracted if decreasing.

Guru	1, 12	13°/27′
	2-11	13°/2′
	3, 10	12°11′
	4, 9	11/28
	5, 9	11°/5
	6, 7	11/0
Budha	1, 12	16/11
	2, 11	15/39
	3, 10	14/32
	4, 9	13/36

	5, 8	13/7
	6, 7	13/0 and 13/1
Śani	1, 12	18/56
	2, 11	18/26
	3, 10	16/53
	4, 9	15/44
	5, 8	15/8
	6, 7	15/0, 15/1
Mangala	1, 12	21/42
	2, 11	20/53
	3, 10	19/14
	4, 9	17/52
	5, 8	17/10
	6, 7	17/0, 17/1

In fifth rāśi, astodaya dhruva will be equal to madhyama kśetrāmśa. Difference of 6th and 7th rāśi has been written as 1′ kala.

Verse 69 : For other places also, from unnatajyā, kśetrāmśa between sun and the planet for udaya or asta can be found out.

Verses 70-76 - Sphuṭa udayāsta time.

When difference of sun and graha is equal to dhruvāmśa, then that will be the sphuṭa time. It will be made more correct by successive approximation.

From śara of graha or nakśatra, both āyana and ākśa dṛkkrama correction are done. Both corrections are added together or difference is taken according to sign. Resultant correction (positive or negative) in vikalā is divided by difference of sun gati and graha gati in kalā. When budha or śukra are vakrī, it will be divided by sum of gati kalā.

Result in daṇḍa etc. is added to rising and setting time, if rising is in east and setting in west - and when dṛkkarma was positive. If rising is in west and setting in east, then it is substracted from rising or setting times.

For negative dṛkkarma, reverse in done. The correction time is substracted from rising time in east or setting time in west. It is added to rising time in west or setting time in east.

Then we get more correct time for udaya or asta. This correction is due to motion of planet at the cakra or cakrādha.

When sphuṭa sun and sphuṭa graha are in same rāśi then it is cakra time for all tārā graha, for budha and śukra it can be cakrārdha also.

Note - Āyaṇa and ākśa dṛkkarma have already been explained in conjunction of planets. Positive dṛkkarma means rising of planet is later and setting time is earlier, i.e. difference with sun is reduced. (rising in east and setting in west). Then their difference will again increase to dhruvāmśa distance after sometime depending on relative speed.

Verses 77-82 - Astodaya time without dṛkkarma - For Orissa, udayāsta degrees for each planet has been stated according to sāyana sun in each rāśi. At the time of cakra, the degrees of udayāsta are divided by difference of speeds of sun and graha. For cakrārdha of budha and śukra, division is by sum of gatis. Result will be time in days etc. For that period after cakra or cakrārdha, graha will be set, then it will rise. It will be set for that period before cakra/cakrārdha also.

Sāyana sun is calculated for the time of udaya or asta found approximately. Again, we calculate the difference of sun-graha distance and kśetrāmsa. This is divided by gati antara or sum and udayāsta times are corrected.

For this corrected udayāsta kāla, we take the average position of graha and sun. Speed of sun and graha for that position is the sphuṭa gati for both for purpose of udaya or asta times.

At any time we calculate the difference of sāyana and dhruvāmśa corrected sun and the sphuṭa graha. That is converted to kalā and divided by difference or sum of speeds of sun and planet. That will give days since udaya or asta or remaining days according to rules explained earlier.

Notes : (1) At cakra and cakrārdha planet has same position from earth as sun. Then they will be set due to closeness of sun. Assuming the speed at end of cakra/cakrārdha to be average speed upto period of udaya, we calculate the time when the planets will be separated at distance of kśetrāmśa, when they will rise heliacally. By successive approximation by speeds at approximate time of udayāsta, we calculate more accurate time.

(2) The days since udayāsta or remaining days are calculated by calculating as to when sāyana sun - sāyana graha = kśetrāmśa.

(3) 1/2 (sun + graha) at udaya or asta time is the mid position of sun and graha. With sufficient accuracy, speeds of sun and graha at that position can be considered sphuṭa.

Verses 82-84 : Conclusion -

Other astronomers have stated the udayāsta degrees of graha 1° less than values stated here. According to them the degrees are - guru 10, budha 12, śani 14, maṅgala 16, vakrī śukra 7, mārgī śukra 9.

This is not the real setting or loss of a planet. In course of rotation of earth and their own motion, they keep coming east, west, up or down. As eyes are dazed due to brightness of sun, tārā graha become lightless like light flies and become invisible. Due to bigger angular diameters, candra, guru and śukra are seen in day light also.

At equator 22°30′ kālāmśa before rise or after setting of sun, its light starts reaching horizon. At other places this kālāmśa is multiplied by lambajyā of the place and divided by trijyā. Light of sun will go upto that distance. (kālāmśa).

At equator, light of moon is visible 8 kālāmśa after, setting or before rise. Light of śukra is visible 1 kālāmśa before rise or after setting. Kālāmśa at other places is obtained by multiplying it with trijyā and dividng with lambāmśa jyā.

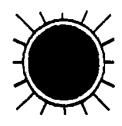
Notes : It has already been explained, how planets set hetically. Kālāmśa of sun here has been taken as 22°30′ at equator against 18° taken in modern astronomy. Kālāmśa of moon and śukra is not calculated, as it is ineffective compared to light of other stars.

At other places, kālāmśa depends both on krānti and akśāmśa as calculated earlier. Roughly we can assume that, sun rays will reach same

mānāṁśa at other places also which is equal to kālāmśa X trijyā / lambajyā

Verses 85-86 - Prayer and end

I pray thousand times to revered lord Jagannatha, whose brightness is like jewel of Indra (Indra nīla maṇi is blue), whose lotus feet are worshipped by Vāsuki, Gaṇesha, Śiva, moon and sun.

Thus ends the thirteenth chapter discribing rising and setting in Siddhānta Darpaṇa written for correct calculation and a text book by Śrī Candraśekhara of a famous royal family of Orissa.

Chapter - 14

LUNAR HORNS

Candra Śṛṅgonnati Varṇana

(Elevation of lunar horns)

Verse 1 - Scope - For knowledge of persons of sharp intellect, it is proposed to describe accurate daily rising and setting of moon, elevation of lunar horns (candra śṛṅga) and diagrams (parilekha).

Verses 2-11 : Time after sun set when moon sets.

Rising time and setting time of moon are calculated roughly according to method described earlier. At the time of sun set, accurate moon and sun are made śāyana (ayanāṁśa is added). Then dṛkkarma sanskāra is done.

6 rāśi is added to sāyana and dṛkkarma corrected sun and moon. By difference of their lagna (rising times of rāśis between them), kālāmśa is found out.

Among sun and moon, bhogya (remaining) asu of lesser rāśi, bhukta (lapsed) asu of bigger rāśi and rising time of other rāśis in between in asu - all are added and divided by 60. Result will be sphuṭa kālāmśa difference between sun and moon.

Kālāmśa difference divided by 6 gives the result in ghati etc. This is multiplied by gati of sun and moon and divided by 60. Result in kalā

etc. is added to sun and moon. Again, we find the difference of their rising times. After repeated procedures, rising time difference between sun and moon will become steady or fixed.

Here, āyana and ākśa dṛkkarma of moon is to be done every time, otherwise śara will be different due to change in distance between moon and its pāth.

When the rising time difference between moon and sun becomes constant thus, 3 rāśis are substracted from moon. For this vitribha lagna, nata and śara are found, and its āyana and ākśa dṛkkarma are done.

To make it more accurate, the dṛkkarma correction is made to moon and sun with 6 rāśis added to them. The rising time difference in asu is found. Sphuṭa gati of moon is divided by 14 and multiplied by trijyā and divided by lambajyā to get the lambana asu of moon. On substracting this from the rising time difference, we get correct difference in asu.

This period after sun set, moon will set. While finding instantaneous sun and moon, asu should be considered sāvana (21,659 part of sāvana day) and while finding moon at any time it will be nakśatra (i.e. 21600 part of a nākśatra day). At sun set time, asu will be candra sāvana (i.e. 22, 390 asu).

Notes : (1) Second half of fifth and Ist half of sixth verse are quoted from sūrya siddhānta, which was considered by many to be an interpolation. However, here, they are further specified by giving the values of asu to be taken in these

calculations. This is the method for calculating setting of moon in śukla pakśa (bright half). Though this is not specified anywhere, but next verse tells about procedure for kṛṣṇa pakśa. It has been clearly specified in sūrya siddhānta.

(2) Rough method for rising and setting time of moon - This has been stated in chapter 8 - candragrahaṇa verses 60-65. That is for pūrṇimā and can be used for 8th of śukla pakśa to 7th of kṛṣṇa pakśa. This ignores śara and doesn't do dṛkkarma sanskāra.

Rising time - At sun set time sāyana sun and moon are calculated.

Rising time of moon after sunrise

= Rising time of remaining part of rāśis of sāyana sun + for lapsed part of sāyana moon + for rāśis in between sun and moon + 56 asu as laṁbana correction for moon = A asu

$$= \frac{A}{360} \text{ ghaṭi}$$

Setting time of moon : Moon set time after sunset = rising time of remaining rāśis (of sun at sun rise time + 6) + for lapsed part of rāśi of (sāyana moon at sun rise + 6 rāśi) - 56 asu = A asu $= \frac{A}{360}$ ghaṭi.

Śara correction - Śara kalā × palabhā / 12 is added to rising time if śara is south and substracted if śara is north. Reverse correction is done for moon set time.

Around pūrṇimā, moon rise is around the time of sunset, hence the position of sun and moon at sunset time are taken for better approximation.

Rising time of moon – rising time of sun in east = rising periods of ecliptic between rāśi of sāyana sun to sāyana moon.

Due to lambana, moon will appear lower when seen at horizon and rise 56 asus later or set 56 asus earlier, the time needed by moon to cover earth's radius in its orbit.

When moon is setting (moon + 6 rāśi) is rising in east. Similarly at sunset time sun + 6 rāśi is rising. Hence moon set - sun set.

= rise of (moon + 6 rāśi) - rise of (sun + 6 rāśi)

= rising time between (moon + 6 rāśi) to (sun + 6 rāśi)

For equator, rising time of a rāśi and 6 rāśi away from it is same.

If we use the time of setting of rāśis instead of rising, addition of 6 rāśis is not needed.

Sign	Time of setting in asus		Sign
	'at the equator	at the local place	
1. Meṣa	1675	1675 + a	12 Mīna
2. Vṛṣa	1796	1796 + b	11 Kumbha
3. Mithuna	1929	1929 + c	10 Makara
4. Karika	1929	1929 - c	9 Dhanu
5. Simha	1796	1796 - b	8 Vṛścika
6.Kanyā	1675	1675 - a	7 Tulā

Here a, b, c are the rising time differences for meṣa, vṛśa and mithuna.

Due to north śara, effective krānti is increased, hence carajyā will increase. Component of śara parallel to krānti i.e. perp to equator is s cos ε where ε is inclination of moon's orbit with equator. Hence, corresponding carajyā increase is

$$s \tan \Phi = \frac{s \times \text{palabhā}}{12}$$

This is substracted from rising time as day length increases due to increase in krānti.

(3) Successive approximation and dṛkkarma -

Due to ākśa and āyana dṛkkarma, difference between sun and moon is corrected as visible from the place.

Difference in moon set - sun set

= rising time diff (sāyana moon + 6 rāśi) (sāyana sun + 6 rāśi)

as the rāśi at 6 rāśi difference is rising when sun or moon are setting.

That will be sphuṭa time difference in asu.

$$\frac{\text{Asu}}{60} = \frac{\text{Kalā}}{60} = \text{degree}$$

360° = 60 daṇḍa (nāksatra time)

Hence 1 danḍa time = 6° Kālāmśa

Speeds of ravi and sun are calculated for sāvana dina, hence sāvana asu is to be used (1 day = 21659 asu). For calculation at sunset time, we take candra savana dina because moon set to moon set time is equal to candra sāvana dina.

From speed of sun and moon at the asta time of moon, further corrections are done.

(4) Lambana correction at setting time is sphuṭa gati of moon divided by 14. This is lambana aṁśa at local akśāmsa. To convert it into kalā at equator or asu, it is divided by cos ϕ i.e. R cos ϕ / R or multiplied by trijyā and divided by lambajyā = R cos ϕ. This is substracted from setting time.

Verses 12-13 - In kṛṣṇa pakśa, sun at sunset time is calculated, 6 rāśi is added to sāyana sun. Difference in rising times between (sāyana sun + 6 rāśi) and sāyana moon at that time is the time after sun set when moon wil rise.

Here also dṛkkarma is to be done for sun and moon both at sun set time. At rising time, lambana asu is added. After repeated calculations with sun and moon positions at moon rise time we get steady value of differnce in sun set and moon rise in east.

Notes : In kṛṣṇa pakśa, difference between sun and moon is more than 6 rāśi. Hence at sun set time, moon is below east horizon, Hence, we calculate the difference between east horizon ecliptic point (i.e. sun + 6 rāśi) and moon. However, while the position of moon at sunset time comes on ecliptic, moon goes further east due to its motion, hence real rising will be later. This difference is corrected by successive approximation.

Verses 14-18 : Position of moon at desired time

Now, method is described for calculating position of moon at sun set time or any other time as observed from earth's surface.

By method explained in chapter 6, sphuṭa candra is found at desired time. Its position east or west half of sky is found (from lagna etc.)

Lagna for desired time, vitribha lagna and vitribha śaṅku is calculated. Dṛgjiyā for moon in east or west half of sky is multiplied by dṛggati (vitribha śaṅku) and divided by trijyā. Result is multiplied by first sphuṭa gati of moon and divided by 14 X trijyā 3438 (= 48132). Result in kalā etc will be added or substracted to moon, if moon is east or west from vitribha lagna. Then we get lambana corrected moon at desired time. After that, we find nati of moon again. Nati and śara are added or difference is taken according to same or different directions to get sphuṭa śara. From that, we do āyana and ākśa drkkarma. By making dṛkkarma correction, we get the samaprota vṛtta moon as seen from earth surface.

Notes : Methods of laṁbana and sphuṭa candra have already been explained in chapter 9 on śolar eclipse.

Verses 19-27 : Elevation of lunar horns

There are two types of elevation of lunar horns (śṛṅga). Generally horn means, pointed ends of the bright portion of the disc. But some authorities consider elevation of horns of black portion also. This horn is not seen but it can be known from calculations.

From one rise of moon to its next rising time is called sāvana day of moon.

From sphuṭa krānti of moon, its nata kāla is found by method explained earlier.

In first half of śukla pakśa (Ist day to 8th) and second half of kṛṣṇa pakśa (9th day to 14th), elevation of bright horns is found. For other days i.e. 2nd half of śukla and Ist half of kṛṣṇa pakśa,

elevation of dark horns is calculated. Out of bright or dark parts, whatever is less than half, its elevation is calculated.

Moon in its vimaṇḍala (inclined orbit) is lighted by rays from sun in apavṛtta (krānti vṛtta or ecliptic) and is seen in many shapes.

Even when both the horns of moon are equidistant from sun, they appear small or big and inclined.

At a place where midday sun at the end of uttara-ayana (i.e in sāyana mithuna) is above head (i.e. akśāmśa of the place is equal to maximum krānti - karka rekhā place), the sun at beginning of sāyana meṣa will be in sama maṇḍala at sunset time (i.e. in east west circle). At that place moon with zero śara will move exactly in east direction.

This shows that according to position of krānti vṛtta, position, speed and horns of moon are decided. That also changes due to change of śara.

As in candragrahaṇa, in finding elevation of horns also, āyana and ākśa valana are calculated.

Verses 28-29 - Śara valana

We consider the right angled triangle whose sides are

(1) Perpendicular side is the jyā of difference between moon and sun.

(2) Base is the bhuja of śarajyā (or śara with direction)

(3) Square root of sum of these squares is karṇa - i.e. linear distance between sun and moon.

Śara is multiplied by trijyā and divided by karṇa. Arc of the result in kalā is divided by 60 to know the valanāmśa of śara (i.e. angular deflection).

Notes : Here we form the right angled triangle for deflection from ecliptic only. For sun it is zero. In sūrya siddhānta, it is calculated for defleciton from equator. For that we take the difference of krānti of sun and moon = p

Then, base

$$= \frac{p \times \text{chāyā karṇa of moon} \pm 12 \text{ akśajyā}}{\text{lambajyā}}$$

Perpendicular = Śanku of moon i.e. koṭijyā of natāmśa

Then, Karṇa $= \sqrt{\text{base}^2 + \text{perp}^2}$

Here karṇa has been termed as madhyāhṇa candra prabhā karṇa i.e. straight distance (like a light ray) of mid day moon. This has been confused with mid day of sun. Ranganātha in his gūḍhārtha prakāśikā ṭīkā on Sūrya siddhānta, interprated it as mid point of civil day between sun rise to next sunrise i.e. sunset time. Accordingly, he derived the formula. This was followed by Burgess who wrote the commentary in 1860 at Chicago U.S.A. after he got Ranganātha Tīkā in Mahārāṣṭra in 1835. Svāmī Vijñānānanda followed it in his Baṅgalā commentary in 1909 and Sri Mahāvīra Pd Shrivastava in his Vijñāna Bhāṣya in 1940.

When moon is at meridian or its midday, sun is at horizon or above it i.e. within ± 90° of moon, as the bright portion is less than half for a horn

to follow. Hence it will be almost correct for other positions also.

Proof :

SZ is a quarter of the yāmyottara vṛtta. Let

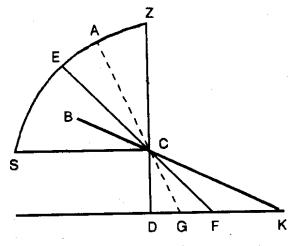



Figure 1

C be its centre and Z zenith. Let EC be nāḍī maṇḍala. ZC is produced to D, so that CD represents a śaṅku of 12 aṅgulas.

When sun is at E, DF is equinoctical shadow of CD or palabhā. When sun is at A, DG called bhuja is shadow and GF agrā. When sun is at B, DK called bhuja is shadow and FK agrā.

Thus Bhuja = Palabhā ± Agrā; or ± Agrā, when sun is on horizon.

Similarly bhuja of moon = Palabhā ± moon's agrā in the sphere whose radius is candra chāyā karṇa i.e. hypotenus of right angled triangle whose one side is śaṅku of 12 aṅgula and other is shadow caused by moon.

Thus in this sphere, sun's agrā

$$= \frac{\text{Sun agrā} \times \text{candra chāyā karṇa}}{\text{Trijyā}}$$

$$\text{Moon's agrā} = \frac{\text{Candra agrā} \times \text{Candra chāyā karṇa}}{\text{Trijyā}}$$

But agrā = $\frac{\text{Krānti jyā} \times \text{Trijyā}}{\text{Lambajyā}}$

Hence difference between sun's and moon's bhuja

= Palabhā ± (Candra krānti jyā ± Sun krānti jyā)

$\times \frac{\text{Candra chāyā karṇa}}{\text{Lambajyā}}$

$\frac{\text{Palabhā}}{\text{akśajyā}} = \frac{12}{\text{Lambajyā}}$, hence we get the formula for base.

From this base and śanku of moon's height, we get the karṇa which is direction from sun to moon in meridian circle, i.e. projection of sun moon line in this circle.

(2) Due to the confusion about this interpretation and apporximate formula, siddhānta darpaṇa has given more direct and accurate formula which can be used for any position of sun and moon.

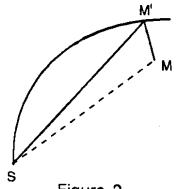



Figure 2

It is known that sun is always on ecliptic, but position of moon at M on ecliptic is perpendicular foot on ecliptic. Thus MM' is perpendicular on plane of ecliptic, i.e. on line SM' of this plane also.

Arc SM' is difference in moon and sun on ecliptic i.e. their rāśi difference. Line SM' is the jyā of that difference. MM' is śara of moon i.e. śara jyā (arc MM' is the śara).

Hence SM = $\sqrt{SM'^2 + MM'^2}$

gives prabhā karṇa of moon at any time.

(3) When we know the karṇa, at this distance śara will make an angle $= \dfrac{\text{śara}}{\text{karṇa}}$ radian

$$= \frac{\text{śara} \times \text{trijyā kalā}}{\text{karṇa}} \text{ in kalā}$$

We are following the scale of 1 aṅgula = 1′ on khagola circle in diagram. Hence, kalā is converted to degree or aṅgula by dividing it with 60.

Verses 30-43 - Diagram of lunar horns

We draw a khagola circle for same radius (57°18′) as in diagram of eclipse. Directions are marked.

Here also, moon is shown as a circle of radius 6 aṅgula (i.e. 12 aṅgula diameter). When moon is in east kapāla, sphuṭa valana is given in eastern point and if moon is in west half of sky, valana is given near west point in its direction (north or south).

Valana of śara is given in opposite direction from valana given earlier in both kapālas (east or west half of sky).

End point of śara valana is assumed to be sun and from that, a line is drawn upto centre of moon and extended. The point where it cuts the circumference of moon will be the border point between bright and dark portions of moon due to sun.

Difference of moon and sun in kalā is divided by 900 to get the aṅgula width of bright portion. From centre of moon, on the sun line (the end

point of śara is sun), we give two points on kha-vṛtta at 90° distance from sun on both sides. From these points also two lines are drawn to the centre of moon.

These two points cut moon on ends of a diameter. On sun line, from circumference, a point at distance of width of bright portion is given.

To draw a circle through these points, we draw arcs with 5 aṅgula radius from each of three points. They form two fish figures, whose head - tail lines cut at the centre of circle through these points. From the arc through the three points, the portion towards sun will be the bright portion of moon.

In sūkla pakśa, if moon is in western sky, or in kṛṣṇa pakśa moon in east sky, the bright side will be towards sun point. For śukla pakśa moon in east, kṛṣṇa pakśa moon in west, the bright side of moon will be on opposite side of sun point.

In śukla pakśa, less than half bright moon will be shown by putting the diagram on west side wall. Horns will be bent towards north or south.

In kṛṣṇa pakśa, this will be shown on eastern wall. For more than half portion of moon bright, it will show elevation of dark horns.

Notes : (1) Valana of moon depends on ākśa and āyana valana and due to its śara from ecliptic. Hence both are marked.

Since moon circle is of 12 aṅgula diameter, complete diameter 12 aṅgula will be bright when moon - Sun = 180° = 10,800 kalā

Hence 1 kalā difference = $\frac{12}{10,800}$ aṅgula bright part

= $\frac{1}{900}$ bright part.

This assumes that moon's speed is constant, which makes little error. But another assumption is that bright part is proportional to angular difference between sun and moon. Actually, it is proportional to utkrama jyā i.e. R (1-cos θ) as shown below.

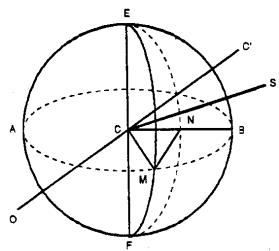



Figure 3 - Phase of moon

C is centre of moon, CO is direction of observer, BEAF is the face of moon, perpendicular to direction of observer. CS is direction of sun and EMF is the face of moon perpendicular to CS direction of sun. Thus the portion of moon between EMF and EBF is the bright portion seen to observer. But the circle EMF is seen obliquely by observer, hence it is seen as half ellipse as projected on BEAF plane whose major axis is EF and semi minor axis is CN. This projected ellipse ENF is the internal boundary of bright portion. CM is radius of moon hence equal to CB and CN is projection of CM, hence

CN = CM cos MCN = CB cos SCC′

Because ∠MCN is the angle between planes which are prependicular to the directions of observer and sun. Hence bright portion NB

= CB - CN = CB - CB cos SCC′

= CB (1 - cos SCC′) = CB. Utkramajya SCC′

∠SCC′ is roughly the angle between directions of sun and moon. If they are considered in ecliptic it is difference between longitudes. More accurately, it can be found from triangle OCS whose sides OC, OS and CS are known.

(3) In first half of śukla pakśa, when sun is setting, moon will be in west half of sky as it is less than 90° ahead of sun. Hence, the diagram will be shown on west wall with direction of sun downwards. In later half of śukla pakśa, moon will be in east half of sky, hence its dark horns will be shown in east sky because more than half part in bright.

Verses 44-61 - Modern method of showing lunar horns. Thus the method for finding lunar horns has been described according to old siddhānta texts. Now, I describe accurately, observed bright part of moon according to my experience and logic.

Jyā of difference of moon and sun rāśi etc is multiplied by yojana karṇa of moon and divided by yojana karṇa of sun. Result in kalā etc is added to moon of śukla pakśa and substracted from moon of kṛṣṇa pakśa. That will be sphuṭa moon.

From this sphuṭa moon, rāśi of sun is again substracted and utkrama jyā is found. That is

divided by 573. Result in aṅgula etc. will be measure of bright portion or dark portion which ever is less than half.

When less than half of moon is bright, this resultant aṅgula will be marked as bright portion. If more than half is bright, then bright aṅgula measure is (6 aṅgula - the result).

As before, from the end point of śara valana in the direction of sun, three points on bright dark boundary are found. Through fish lines, we find the centre and draw a circle through these points.

Bright portion of moon less than 1-1/2 aṅgula (i.e. 1/8th of moon's diameter) is not seen, because the end portions of horn are very thin. Increase in phase of moon, or its decrease should be shown to people through diagrams. On 4th day of bright half (śukla pakśa), at the sunset time, moon circle is drawn in north direction on earth's surface. 4 diameters are drawn through directions points and angle points. All the diameters bisect each other at the centre. West from moon at a distance, sun is shown. Due to this sun, west half of moon will be bright. To see the bright portion, earth point is given at a distance of 5 hands (5 X 24 aṅgulas) from moon's centre in agni koṇa (south east direction). North of this earth point will indicate zenith of sky. Though half the moon is always lighted to sun, the portion seen from earth is much less than half due to angle between white circle and visible circle. From southeast direction, we see the diameter through nairṛtya (south west) and īśāna points (north east). Of the bright portion touching the north south line, lower half portion will be seen from earth.

The line from earth centre to south point of moon touches west point and cuts the north east - south west line. From this point in direction of south west, bright portion will be seen. Rest part upto north east point will be dark.

When difference of moon and sun is 45°, ancient texts, have assumed 3 aṅgula bright portion. But in this calculation only (1/45) aṅgula is actually seen. Hence, scholars calculate the bright portion of moon from utkrama jyā only, because in a sphere, any object will be seen in line of sight (in perpendicular plane only).

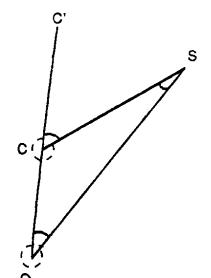
Notes : (1) Use of utkrama jyā - The formula as proved in previous section is through utkrama jyā as shown, Logically we can infer it because we see the sphere from curved side, not from side of centre. Hence the distance of plane surface will be proportional to utkrama jyā, from centre it is proportional to koṭijyā.

(2) Proof of the formula-

As in figure 3, we need to know the angle C′ C S as seen from moon between directions of sun and direction from observer.

$\angle$ SCC′ = $\angle$ CSO + $\angle$ COS - (1)

$\angle$ COS is the angle between directions (rāśi) of sun and moon).

Figure 4 - angle from moon between sun and earth

In śukla pakśa (moon - sun) is less than 180°, hence it is smaller angle COS itself. In kṛṣṇa pakśa it (moon - sun) is more than 180°, hence we

calculate the outer angle (360° - ∠COS) Then ∠SCC′ = ∠COS - ∠CSO - - - (1a)

Now in ΔCOS, by sine rule

$$\frac{OC}{\text{Sin } CSO} = \frac{OS}{\text{Sin } OCS} = \frac{OS}{\text{Sin } SCC'}$$

as Sin SCC′ = Sin (180°-OCS) = Sin OCS

or, Sin CSO = OC/OS Sin SCC′

∠ CSO is very small, because OC is very small compared to OS. Hence Sin CSO = ∠ CSO and sin ∠ SCC′ = ∠ SOC approximately.

Then ∠ CSO = OC/OS (∠ SOC) - - - (2)

Here, OC = candra karṇa, OS = sūrya karṇa

Putting the value of ∠ CSO in (1) or (1a), we get the SCC′ whose utkraṇa jyā is to be found.

(3) Aṅgula value of bright part.

For angle of 90°, utkrama jyā is 3438 kalā and bimba is 6 aṅgula bright

Hence for utkrama jyā 1 kalā,

bright portion is $\frac{6}{3438} = \frac{1}{573}$ aṅgula

(4) Diagram

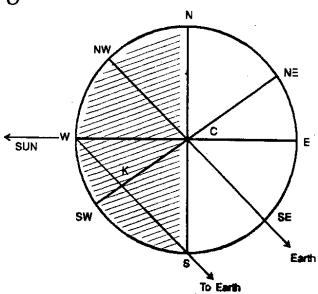



Figure 5 - Bright portion seen from earth

NWS is the face to wards sun and is bright. Face SW-S, E - NE is towards earth. Hence west of point S only, bright portion of moon is seen. WS line is hence the boundary of bright portion. it cuts SW-NE line on K. Hence from K to SW, is bright portion and remaining part from K to NE is dark portion.

(5) Modern method - The great circle from sun's centre to moon's centre is perpendicular to line joining lunar horns. The great circle from zenith to centre of moon is at angle from sun moon great circle, which is the angle of lunar horns with horizon. This angle can be known from spherical trigonometry, as discussed in tripraśnādhikāra.

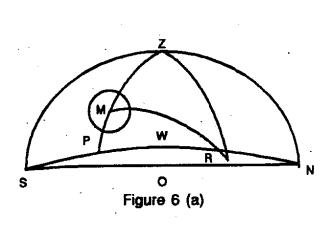
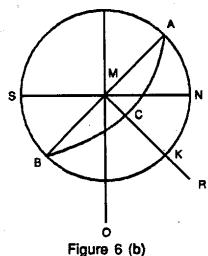



Figure 6 (a)

Figure 6 (b)

IN figure 6 a,

NZS = yāmyottara, Z = Zenith (Khasvastika)

O = observer, NOS = north south line

NWS = western horizon

M = Moon in western sky

R = position of setting sun

ZM = natāmśa of moon

MR = Distance between sun and moon

$\angle$ RZM = Difference between directions of moon and sun (digamśa) from zenith.

Natakāla can be known from viśuvāmśa (rising times) of sun and moon and their krānti.

cos (nata kāla)

$$= \frac{\cos(\text{natāmśa}) - \sin(\text{akśāmśa}) \times \sin(\text{krānti})}{\cos(\text{akśāmśa}) \times \cos(\text{krānti})} \quad \text{-- (1)}$$

This equation will give the natāmśa

Then digamśa will be known from the following equation

cos (digamśa)

$$\frac{\cos(\text{dhruvāntara}) - \cos(\text{natāmśa}) \times \sin(\text{akśāmśa})}{\sin(\text{natāmśa}) \times \cos(\text{akśāmśa})} \quad (2)$$

These equation have been derived for calculation of natamśa and calculation of karṇa vṛttāgrā in Tripraśnādhikāra verses 71 notes (3) and verse 44 (notes).

Thus in spherical triangle ZMR, we know ZM, ZR, MR, and $\angle$ MZR. ZR = 90°. Hence we can know $\angle$ ZMR and the elevation of lunar horns.

In figure 6(b), M = centre of moon

OM = vertical circle of moon centre (dṛk=maṇḍala)

RM = Direciton of sun from moon

ANKBC = Bright portion of moon

$\angle$ OMR = $\angle$ AMN = angle of elevation of lunar horns

In figure 6 (a), from spherical trigometry

cos MR = cos ZR cos ZM + Sin ZR Sin ZM Cos $\angle$ RZM

After finding MR from this equation,

$$\cos \angle ZMR = \frac{\text{Cos } ZR - \cos ZM \times \cos MR}{\text{Sin } ZM \times \text{Sin } MR}$$

180° - ∠ ZMR is the angle of elevation of horns, because it is equal to ∠ PMR. If sun is north from moon, then north horn will be upper and if south, then south horn will be upper. If digamśa of both sun and moon are same then horns will level. After knowing this, diagram of horns should be drawn as per figure 6(b).

Verses 62-63 : Horns of budha and śukra also are visible through telescope.

In India, north horn is mostly seen higher in both west and east kapāla. Very rarely, south horn is seen higher.

Notes : For akśāmśa more than 28-1/2° north, both sun and moon will be always in south. As we see from north, northern portion of bright horn will look bigger.

Verses 64-67 : Reasons for new methods —Earlier astronomers used to find difference of krānti's of sun and moon through a śaṅku of 12 aṅgula and from that, elevation of horns was found. Since this method doesn't give results as observed, I am rejecting it. When sun is prependicular to equator, half disc of moon in sāyana makara beginning is seen cut by meridian line at zenith. Hence, half disc is seen bright.

Hence utkrama jyā of (moon-sun) is multiplied by Ist sphuṭa gāti of moon and divided by (173452). Result in kalā is added or substracted from half disc of moon to find the bright portion width. This is added to half diameter when bright portion is more than half, otherwise substracted from it. This

will be correct measure of biṁba in both east and west sky.

When moon is at 11° from sun, its light is more than budha biṁba of diameter 17 vikalā and less than bright biṁba of guru. Hence, it is not proper to consider heliacal rising and setting of moon at 11° kālāmśa difference. The author considers it to be between 11° and 12°.

At the end of Ist day of bright half, 108th part of moon's disc is seen, even though it is very thin. On 4th day, its 1/6 part will be seen bright. At the end of 5th day, 1/4 parts will be bright. At the end of tenth day 3/4 part will be bright. On pūrṇimā, complete disc will be bright. On 8th day end half disc and on 11th day end 5/6 parts will appear bright.

Notes : (1) Brightness of moon has been calculated according to value of utkramajyā for the angle between sun and moon.

(2) Madhya biṁba kalā x madhya karṇa of candra = spaṣṭa bimba kalā x spaṣṭa karṇa

Hence bright portion in kalā

$$= \frac{\text{spaṣṭa bimba kalā} \times \text{utkrama jyā}}{2 \times \text{trijyā} \ (= \text{madhya karṇa})}$$

$$= \frac{\text{madhya bimba kalā} \times \text{utkrama jyā}}{2 \times \text{spaṣṭa karṇa}}$$

$$= \text{madhya bimba kalā} \times \text{utkramajyā}$$

$$\times \frac{\text{sphuṭa gati}}{2 \times \text{trijyā} \times \text{madhyagati}}$$

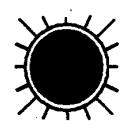
$$= \text{utkramajyā} \times \text{sphuṭa gati} \times \frac{\text{madhya bimba kalā}}{2 \times \text{trijyā} \times \text{madhya gati}}$$

$$= \text{utkramajyā} \times \text{sphuṭa gati} \times \frac{444 \times \text{trijyā}}{48705 \times \times 2 \times \text{trijyā} \times 790/35}$$

$$= \frac{\text{utkramajyā} \times \text{sphuṭa gāti}}{173452} \text{ as given}$$

Verses 68-69 - Prayer and end

On sea beach, Lord Jagannātha protects people from anger of yama with his sudarśana cakra, and destroys all diseases borne out of desires. May he end all our illnesses due to passions.

Thus ends the fourteenth chapter describing elevation of lunar horns in Siddhānta Darpaṇa, written for consonance in calculation and observation and education of students, by Śrī Candraśekhara born in famous royal family of Orissa.

Chapter - 15

MAHĀPĀTA VARṆANA

Verse 1 - Scope - I am describing mahāpāta as told in scriptures, which destroys the good deeds (karma) earned in pilgrimage, sacred thread wearing, marriage etc, in whose discussion mathematicians are also confused, and on whose occasion, results of charity, japa and bath become as auspicious as in an eclipse.

Notes : Mahāpāta is a fictious conjunction of sun and moon and is as good or bad as an eclipse. It destroys results of good deeds which accrue due to marriage etc as described in scriptures. But if good works like charity are done during mahāpāta, they are as fruitful as in eclipse. This is a difficult topic, as the conjunction is observed only mathematically not as a real phenomenon.

Verses 2-8 : Two mahāpātas -

Pātas are of two types - Vaidhṛti and vyatīpāta When their (sun moon) krāntis are equal, then these pātas occur. Out of gola and ayana, if ayana is same, then pāta is vaidhṛti and if gola is same then it is vyatīpāta.

When moon and sun are in same diurnal circle, they have gola sandhi. When both are in place of parama krānti, they have ayana sandhi.

When moon and sun are in one gola but different ayanas and their krāntis are equal then it is vyatīpāta yoga.

When moon and sun are in different gola but same ayana and their krānti are equal, it is vaidhṛti yoga. When krānti is same, their aspects are added (i.e. they are at same angle with equator plane).

(In Sūrya siddhānta) when moon and sun both have same krānti, due to combination of their rays at same angles there is flow of fire which is destructive for living beings.

Atipāta yoga is always bad and destructive. Other names of this yoga are vyatīpāta and vaidhṛti.

Each pāta has dark colour, very ferocious body and red eyes. Both are valiant and occur every month. Pāta from spaṣṭa position (of moon and sun) is more destructive than pāta from mean position. (Quotation ends)

Notes (1) Two yogas are named vaidhṛti and vyatipāta, but these have no relation at present with the two mahāpātas. However, these can be calculated from sum of longitudes of sun and moon and in that way they are related to yoga cycle.

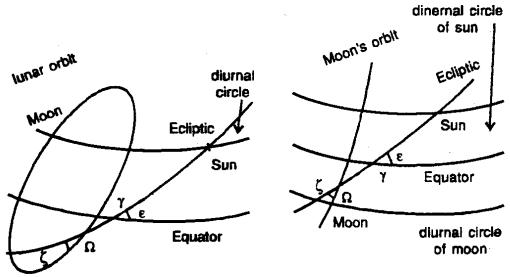
Figure 1 - (a) Vyatīpāta Figure 1 - (b) Vaidhṛti

Vyatipāta is 10th yoga and vaidhṛti is the last.

Figure 1 - Mahāpāta

Figure 1 (a) shows vyatipāta when, moon and sun have common diurnal circle i.e. same krānti but at the other end of orbit.

Figure 1 (b) shows, vaidhṛti yoga, in which the krānti of moon and sun are equal and opposite, i.e. diurnal circles of moon sun are at equal distances from equator, but in opposite direction. Sun and moon are in same side of sphere i.e. in same gola.

In these figures ♈ is 0° sāyana meṣa and ☊ is pāta of moon i.e. rāhu. ε is inclination between equator and ecliptic and ξ between ecliptic and moon's orbit.

If latitude of moon's orbit is neglected (it is less than 5° always), both moon and sun are on ecliptic. If their longitudes S_2 of sun and M_2 for moon from sāyana 0° are taken, then for equality of krānti.

$\text{Sin } S_L = \text{Sin } M_L$

$= \text{Sin } (180°-M_L)$

or $S_L = 180°-M_L$ or $S_L+M_L = 180°$

When they are numerically equal but in opposite direction, then $\text{Sin } S_L = - \text{Sin } M_L = \text{Sin } (360°-M_L)$

Hence $S_L + M_L = 360°$.

(2) In vaidhṛti yoga, $S_L + M_L$ - 2 ayanāṁśa = 360°

For Vyatīpāta yoga $S_L + M_L$ - 2 ayānaṁśa = 180°

Thus vaidhṛti yoga coincided with vaidhṛti mahāpāta when ayanāṁśa was 0°. But vyatipāta

yoga is only 10th yoga starting at 120° and it will not tally with the mahāpāta.

Verses 9-15 : Calculation of yoga -

When sum of sāyana sun and sāyana moon is 12 rāśi then vaidhṛti yoga is near.

Similarly, if sum of sāyana sun and sāyana moon is 6 rāśi or 18 rāśi, then vyatīpāta yoga is imminent.

When sun and moon are in different quadrants of ecliptic, then only pāta can happen. Both vyatīpāta and vaidhṛti yogas occur once each month. In some months vyatīpāta occurs twice, sometimes it doesn't occur in a month.

Pāta are possible when viṣkambhaka etc yogas occur. For that, we multiply ayanāmśa by 2 and added to minutes (kalā) of a circle (21,600) or half circle (180° = 10,800') if substracted earlier, and substracted if added earlier. When result is more than (21,600), cakra (21,600) is substracted. By dividing it with (800), result will be past no. of yogas from viṣkumbha etc. Adding 1 to quotient it will give the number of current yoga. Remainder multiplied by 60 and divided by 800 gives the part of current yoga lapsed.

If moon has no śara, then it is also the time of pāta. When moon has śara, pāta will be slightly before or after this time. Hence we should roughly calculate pāta, first according to madhyama krānti (i.e. Krānti of ecliptic point of moon without śara).

Notes : If has been explained earlier that pātas will occur when sum of sāyana moon and sun is 6 rāśi (for vyatīpāta) or 12 rāśi for vaidhṛti.

For, if S_L amd M_L are sāyana longitudes of sun and moon, when their krāntis are equal, for vyatīpāta

$$\text{Sin } S_L = \text{Sin } M_L = \text{Sin } (180^\circ - M_L)$$

$$\text{or } S_L = 180^\circ - M_L \text{ or } S_L + M_L = 180^\circ \quad --- \ (1)$$

When krānti is equal and opposite for vaidhṛti

$$\text{Sin } S_L = - \text{Sin } M_L = \text{Sin } (360^\circ - M_L)$$

$$\text{or } S_L + M_L = 360^\circ \quad --- \ (2)$$

While pāta is calculated with sāyana sun and moon, assuming madhyama krānti without śara, yoga is calculated for nirayana moon and sun.

Hence, if ayanāmśa is A and nirayana moon and sun are S and M, then

$$\text{yoga} = \frac{(S + M) \text{ Kalā}}{800 \text{ Kalā}} \quad ---- \ (3)$$

because each yoga extends for 800 kalā of sum of sun and moon position.

$$S_L = A+S, \ M_L = A+M$$

Hence for pātas

$$S_L + M_L = 6 \text{ rāśi or } 12 \text{ rāśi } (180^\circ \text{ or } 360^\circ)$$

$$\text{or } (S+A) + (M+A) = 6 \text{ or } 12 \text{ rāśi}$$

$$\text{or } S+M = (6, \text{ or } 12 \text{ rāśi}) - 2\,A$$

Putting this value of S+M in kalā in (3), we get the yoga number as stated in text.

Formulas (1) and (2) give krānti depending only on ecliptic. Since śara is very small it will be approximate time of pāta also. As ayanāmśa remains almost constant, the yogas for occurrance of pāta are fixed for some years. We can thus know the approximate time of pāta by the current yoga. After knowing sthūla pāta, we get it corrected

for śara of moon to know when sphuṭa krānti is equal.

(2) Since yoga is sum of sun and moon, it changes with sum of speeds i.e. (790/35+59/8) = 849/43 average speed. At this rate, rotation takes

$$\frac{21,600 \text{ Kalā}}{849/43 \text{ kalā/day}} = 25.4 \text{ days approximately}$$

In a lunar month of 29.5 days it will definitely complete one cycle, hence both the pātas will occur once at least. Due to extra length of lunar month, sometimes, one pāta may occur twice. If true krānti of moon is more than 23-1/2°, a pāta may not occur.

Verses 16-20 : Sthūla pāta for present ayanāmśa.

At present (1869 AD - writing of book), ayanāmśa is 22°. (It can be almost same in 1996 also with only 1° difference). Hence in śukla (24th yoga) and vṛddhi (11th yoga), vaidhṛti 3rd quarter and vyatīpāta Ist quarter often fall. That is their madhyama time.

Hence, we assume the śukla yoga and vṛddhi yoga as cakra (21,600) and cakrārddha (10,800 kalā) aproximately. On the day of that yoga, we calculate accurate value of sun and moon (at the end of these yoga times). Ayanāmśa is added to both sun and moon. Then we find the difference of (sāyana sun+moon) from (10,800) or (21,600) Kalā. That will be divided by sum of sun and moon gatis and multiplied by 60 to get time in daṇḍa etc. (It can be calculated from proportionate duration of current yoga also) This time is added to time of śukla or vṛddhi yoga if (sun+moon) was less than that, otherwise it will be substracted.

After successive approximations, sum of sāyana sun and sāyana moon will be equal to 6 or 12 rāśis at the calculated time. Then we calculate the śara of moon.

These yogas are not visible, hence dṛkkarma or lambana, nati are not needed for moon. Pāta is calculated from earth's centre only.

Notes : According to method described after verse 15, the yogas at the times of pāta have been calculated (based on madhya krānti of moon, assuming zero śara), for 22° ayanāmśa. At present also for 23-1/2° ayanāmśa, it is almost same.

Accurate time of madhyama pāta is found by method of successive approximation.

Verses 21-33 - Pāta from sphuṭa krānti

(From Sūrya siddhānta) - In odd quadrants, if sphuṭa krānti of moon (i.e. krānti of its ecliptic point corrected for śara) is more than krānti of sun, then pāta has already passed. If sphuṭa krānti is less, then pāta is yet to come. In even quadrants if sphuṭa krānti of moon is more than krānti of sun, then pāta is to come, if it is less then pāta has passed.

Persons conversant with gola (spherical trigonometry) can know the time of sphuṭa pāta through their methods. But detailed calculation method is explained for common men.

When sum of rāśis of sun and moon (both sāyana) is exactly squal to cakra or cakrārdha kalā (21,600 or 10,800 minutes), then if pāta has lapsed, then 60 daṇḍa is substracted from that (mean pāta) time. If pāta is yet to occur, then 60 daṇḍa is

added to that time. For that revised time, we calculate sun, moon's pāta and śara and difference between sphuṭa krānti of sun and moon. If sign of (candra krānti - sun krānti) has changed after this revised time then, pāta has occured during this 60 daṇḍa interval. If sign is same, then pāta is beyond that interval.

To find the correct time of pāta, we find the difference of krāntis of sun and moon, both at the mean pāta time and at interval fo 60 daṇḍa. If they are of different sign, they are added. If difference is of same sign their difference is taken. This will be the first krānti gati for finding pāta.

First krānti difference in kalā is multiplied by 60 and divided by first krānti gati. Quotient in daṅḍa etc is added to mean pāta time, if pāta was to come and substracted from it, if pāta had already passed.

At first corrected pāta time, we again find the krānti difference of sun and moon and find the second krānti gati kalā. Krānti difference of Ist corrected pāta time is multiplied by the time difference and divided by second krānti gati. By the result in ghaṭi etc, we again correct the Ist corected pāta time. Krānti gati is found by multiplying the change in krānti difference by 60 and dividing by the time difference.

By repeating this process, by successive approximation we get the time of mid-pāta. Last krānti gati wil be the gati of krānti antara at mid pāta time.

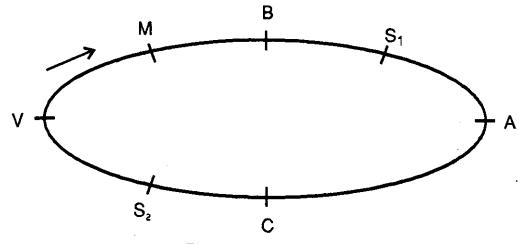



Figure 2 - Mahāpāta

Notes : (1) Whether pāta has gone or not -

In figure 2, VBAC is the ecliptic where V is vernal equinox. (or sāyana meṣa 0°) and A is autumnal equinox. B and C are solstice points in summer and winter at 90° from these. Thus the 1, 2, 3, 4th quadrants from V are VB, BA, AC and CV in the direction of motion shown by arrows.

When moon is in odd quadrant (VB or AC), eg. M in VB, then for vyatīpāta sun is at S_1 so that VM = AS_1 and VM+VS_1 = VM + VA - AS = VA = 180°. Similarly for vaidhṛti, sun will be at S_2 in VC where VS_2 = VM. Thus sun will be in 2nd or 4th quadrants i.e. in even quadrants.

At V and A, krānti (madhya krānti for moon) is zero, in VB portion it increases in north direction and in AC portion in south direction. Thus the krānti increases in VB and AC which are odd quadrants and decreases in the even quadrants BA and CV.

Thus when moon is in odd quadrant and its true krānti is more than sun (when madhya krānti is equal) then krānti of moon will further increase and sun will decrease for even quadrant. Hence they will be equal at an earlier time i.e. pāta had

already passed. If moon's true krānti is less than sun, it will increase and sun's krānti will decrease and they will be equal after some time. Hence pāta will come after some time.

When moon is in even quadrant, sun will be in odd, hence moon krānti will be decreasing and sun krānti will be increasing. If moon's krānti is more, it will be equal to sun after some time and spaṣṭa pāta will come. If moon's krānti is less, pāta has already passed.

This analysis has considered increase of only mean krānti of ecliptic point. Śara of moon also changes, which will change the true krānti. Hence, for correct calculation, moon's śara also has to be calculated.

Suppose moon in first quadrant has 5° north śara (maximum value). Then its true krānti at madhyama pāta will be 5° more than sun. Moon's pāta decreases at average rate of 5/6.8 degrees perday, because its quarter revolution is in 27.3/4 = 6.8 days. Krānti of sun will decrease and madhya krānti of moon will increase at the rate of 23.5/91 = 1/4° per day approximately. Hence total increase in moon's Krānti will be 5/6.8 + 1/4 + 1/4 per day = 1.24° per day compared to sun. Thus the true pāta will be about 4 days before madhyama pāta. Suppose it is vaidhṛti pāta. If previous vaidhṛti is 3 days later, then they will be in 25-7 = 18 days and in 12 days of the month another pāta can occur. Thus there will be two vyatipāta which comes about 12 days after vaidhṛti and one has already passed between two vaidhṛtis.

(2) Calculation of true pāta time

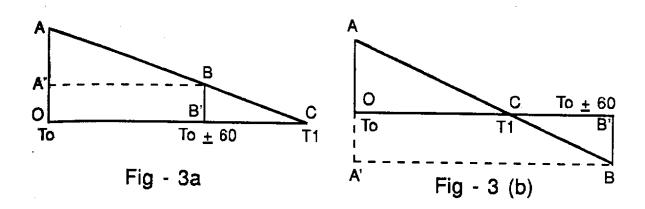



Figure 3 - Calculation of true pāta time

T_O is the madhyama pāta time when madhyama krānti of sun and moon are equal. But the true krānti of sun and moon are unequal due to śara of moon. Let AT_O be the difference of krānti of moon and sun at time T_O (it is negative if krānti of moon is less). To make a first aproximation of true krānti time, we calculate the position at 60 ghaṭi difference according to the pāta time is earlier or later. Then krānti difference is BB′ where B′ indicates time $T_O \pm 60$ daṇḍa. When krānti difference has same direction i.e. at O (T_O) and B′ (moon-sun krānti) is both positive or negative, the true krānti will be equal at time C outside B′ i.e. outside the interval of T_O to $T_O \pm 60$. We assume here that krānti difference has same gati (or rate of change), hence it will be zero where line AB cuts the line OB′ where krānti diff. is zero. This is shown in figure 3(a).

Figure 3(b) shows that the sign of krānti diff. changes. Then AB line cuts OB′ between the interval at C.

In both the figures we draw a line BA′ parallel to OB′ which cuts AO (or AO extended in fig b) at A′. Then AA′ is change in Krānti diff. in 60 daṇḍa time. Here AA′ = AO-BB′ = diff of krānti diff in fig (a) when krānti difference has same sign.

AA′ = AO + OA′ = AO + BB′ = sum of krānti diff. in figure (b) when they are of different signs.

Thus speed of krānti diff. is AA′/60 in each daṇḍa. Hence it will be zero in time T_0 T_1

$$= \frac{AO \times 60}{AA'} \text{ daṇḍa}$$

Here AA′ is the gati of krānti antara in 1 day or Ist krānti gati.

Thus we correct the madhya pāta time according to difference of krānti antara in 1 day.

By calculating the krānti difference again at point T_1 we get more accurate value of true pāta.

(3) Sūrya siddhānta has given another method, using difference of moon from rāhu. Here we have not described the method of calculating śara of moon, which is necessary for sphuṭa krānti. Śara depends upon bhuja jyā of difference between moon and rāhu, hence, we take this as difference of sphuṭa krānti in sūrya siddhānta.

Verses 34-42 - Sparśa and mokśa of pāta.

This was the time, centres of moon and sun were having same krānti i.e. mid point of pāta. When the first points of moon and sun have equal krānti, this is called sparśa time as in eclipse and When the last point has equal krānti it is mokśa time. Thus full pāta time is from sparśa to mokśa.

Now, method to find sparśa and mokśa time is being described.

Like method of lunar eclipse, we find the biṁba of moon and sun at mid time of true pāta and add their semi diameters (mānaikyārdha). Sum of semi-diameters is multiplied by 60 and divided by last krānti gati (i.e. krānti difference gati at mid pāta time).

Result will be madhyama sthiti ardha time in daṇḍa. By adding to pāta mid time, we get mokśa time and by substracting we get the sparśa time.

At these approximate times of sparśa (or mokśa), we again find the difference between sphuṭa krāntis. If this is less then mānaikyārdha (sum of semidiameters of sun and moon) then sparśa time has passed (or mokśa time is to come), as the krānti difference decreases from sparśa time (equal to mānaikyārdha) to mid time of pāta, where it is zero. When krānti antara is more than mānaikyārdha, then sparśa is to come (or mokśa time has passed).

Krānti antara at sparśa (or mokśa) time multiplied by 60 and divided by madhya sthiti ardha will give Ist krānti antara gati at sparśa (or mokśa)

Mānaikyārdha vikalā at sparśa (or mokśa) is divided by first krānti antara gati at sparśa. It will give sthiti ardha for sparśa (or mokśa) in daṇḍa. By substracting them (or adding) to mid pāta time, we get the time of sparśa (or mokśa) - Ist sphuṭa value.

Now at the first sphuṭa value of sparśa (or mokśa), krānti antara kalā is multiplied by second sthiti ardha time in ghaṭī for sparśa (or mokśa). We get second krānti antara gati at sparśa (or mokśa). Again we can get second sphuṭa value of sparśa (or mokśa) times and sthitiardha. By successive approximation, we get the steady value of sthiti ardha etc.

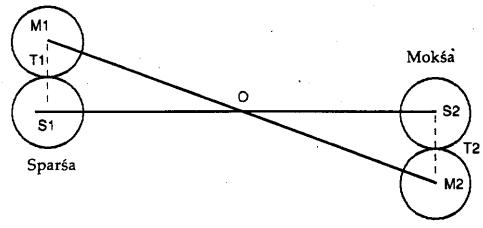



Figure 4 - Sparśa and mokśa of pāta

Notes : In figure 4, fictitious joining of sun and moon have been shown. Krānti's are equal in a pāta, but they may be in different direction. They have been shown in same direction. Sun and moon are always in adjacent quadrants for pāta as shown in figure 2, but they are shown at one place for explaining equality of krānti of different parts of sun and moon. At point O, krānti of centres of sun and moon are equal. Hence, it is mid time of pāta. When centres of moon and sun are at M_1 and S_1, their krānti antara is $M_1 S_1$ and discs touch each other at T_1. Krānti difference is moving along $M_1 OM_2$. At position M_2, S_2, the discs just touch at T_2.

Thus before M_1 or after M_2 position, the krāntis of no part of sun and moon will be same and there will be no pāta. Between these positions,

some point of moon will have same krānti as some point of sun, hence it will be pāta.

It is clear that at sparśa time

Krānti antara $M_1S_1 = M_1T_1 + T_1 S_1$

= semi diameters of (sun + moon).

Similarly at mokśa time also it will be equal to mānaikyārdha.

First we calculate the approximate position S_1, S_2 from the krānti antara gati at O. Then we correct it with the gāti at approximate positions of S_1 and S_2 to get more correct value. By repeating this process, we get the accurate value.

This is only a diagram to explain equality of krānti of different points, there is no closeness of sun or moon as in ecliptic.

Verses 43-45 : Effects of pāta - According to Sūrya siddhānta the time of pāta from beginning to end is fiery like burning fire and all auspicious works like marriages, sacred thread ceremony etc are prohibited during this.

Pāta arises due to equality of krānti's of sun and moon. That destroys all results of noble deeds.

By knowing the period of pāta, penances like bath, charity, mantra, śrāddha, worship, offers in fire all give good results, as on the occasion of eclipse.

Verses 46-49 : Duration of pāta

Average duration of pāta is two prahara or 15 daṇḍa. Minimum duration is 9 daṇḍa and maximum duration is 2/20 days.

When ucca and pāta of moon is in the last part of 12th rāśi (mīna), sāyana sun is near mīna and sāyana moon is near kanyā, then pāta duration is minimum.

When candra, its ucca, and pāta all are at beginning of karka rāśi, sāyana sun is at the end of dhanu rāśi, then pāta is for maximum duration.

If pāta of candra is between mithuna 28° to karka 1° instead of karka beginning, then there is minimum time between two equalities of krāntis (i.e. mahāpātas). If at the end of ayana, spaṣṭa krānti of moon is more than 28°, then krāntis cannot be equal.

Notes : (1) Here pāta has been used as short form of two mahāpātas - Vaidhṛti and vyatipāta - when krānti of sun and moon are equal. But pāta means the point of inter section of a planets orbit with ecliptic which is sun's orbit. For moon's orbit, pāta's are called rāhu or kātu. As in case of all orbits, the pāta point after which planet starts having north śara, the ascending node (rāhu) is pāta of moon.

When moon, its ucca and its pāta are in beginning of karka i.e. 90°, then sun in 270° will cause vaidhṛti. Then moon has almost zero śara and its true north krānti is equal to sun krānti in opposite south direction. Speed of krāntis will be slowest and speed of moon also will be slowest near its ucca, hence its pāta will be longest.

If moon pāta is between mithuna 28° to karka 1° (i.e. 88° to 91°) then within this movement of 3° pāta, just before pāta position moon krānti will

be less then 23-1/2° equal to krānti of sun before 270°. After pāta, śara of moon will rapidly increase and spaṣṭa krānti will be equal to sun krānti maximum at 270°. Hence next pāta will come earliest.

Opposite to the longest pāta, moon at 180° and sun at 360° (vyatī pāta), if ucca and pāta of moon are near 360,° then speed of moon is maximum, 0° krānti period will be for lowest period as krānti speed is maximum at 0° kranti and śara. hence pāta is of smallest duration.

For such situations, maximum and minimum periods of pātas have already been given.

(2) Maximum krānti of sun can be only upto 23-1/2° in either direction. However, due to śara, moon can have krānti upto 28-1/2° due to its parama śara of 5°, when madhya krānti and śara both are maximum and in same direction. Then moon's kranti will be between 23-1/2 to 28-1/2° and sun's krānti will be always less than 23-1/2°. Hence true krāntis can not be equal and there can be no true pāta, though madhya pāta will occur.

Verses 50-54 : Gola and ayana for pāta

For calculating true pāta, śara of moon changes due to its orbit (distance from its pāta rāhu). But madhya krānti is same as krānti of sun in that ayana. At gola sandhi (zero śara) sphuṭa krānti doesn't change due to śara. But in ayana sandhi (maximum krānti but least krānti speed), krānti gati changes due to śara gāti. Reason is that krānti gati is more in gola sandhi (at equator) and least in ayana sandhi (maximum krānti position).

In south and north gola, north south motion of moon due to śara doesn't change its total krānti gati. Being deflected north or south due to pāta, moon still continues its motion on krānti vṛtta. It is not affected, whatever may be the value of śara.

Varāhamihira has described gola and ayana system for mahāpātas very logically in his Bṛhatyātrā book.

Notes : This is an objective description and needs no further comment. Bṛhatyātrā is not a well known book of Vārāhamihira who has written three texts in three branches of jyotiṣa - Bṛhatsaṁhita (Saṁhitā), Bṛhatjātaka (astrology - phalita) and Pañca siddhāntika (astronomy).

Verses 55-58 : Inauspicious times

In grahasphuṭa - chapter 5, 27 yogas have been described according to sum of rāśi etc of sun and moon. Out of them 27th yoga is vaidhṛti and 17th is vyati pāta. These yogas are very fiery because sun and moon become very angry, their aspects being inclined at same angle to equator, in same manner as two bullocks become angry when they are forced to move together round a pole for crushing oil seeds or separating grain chaff.

From Sūrya siddhānta - Last quarters of aśleṣā, jyeṣṭhā and revatī - rāśi and nakśatra both have their borders. Hence last quarters (1/4th part) of these nakśatras is called gaṇḍa. Half of first quarter (first 1/8th part) of next nakśatras (maghā, mūla and aśvinī) are called gaṇḍānta.

All auspicious works are prohibited in sandhi (junction) of rāśis. Last navāmśa of karka, vṛścika

and mīṇa rāśi are in mīna rāsi. First navāmśa of next rāśis (siṁha, dhanu and meṣa) falls in meṣa rāśi. Hence all these navāmśa are also bad. Like gaṇḍānta, these navāmśa also fall in the junction of rāśi and nakśatra, hence good works are prohibited in them. Viṣṭi (bhadrā) etc bad karaṇas are also to be avoided.

Notes : This has nothing to do with gaṇita jyotiṣa. This can be considered use of these calculations of pāta, nakśatra karaṇa and yoga.

Sūrya siddhānta explains that 3 vyati pātas, 3 rāśi sandhi and 3 nakśatra sandhi all are very bad.

Here 3 types of vyatipāta are - mahāpāta called vyatipāta and vaidhṛti, yogas named vyati pāta and vaidhṛti. Mahāpāta are of two types - one from mean value of krānti and one from true krānti, hence three types of vyatipātas.

12 rāśis or 27 nakśatras both are equal to 360° or full circle. Hence 1 rāśi is equal to 2-1/4 = 9/4 nakśatras. Thus when 4 rāśis are complete, 9 nakśatras also are completed, and their junctions combine.

To tally rāśi with nakśatra, each nakśatra is divided into 4 quarters, so that each rāśi has 9 quarters. Each rāśi is also divided into 9 parts called navāmśa. Thus, 1 navāmśa = 1 quarter nakśatra = 3°20′. Navāmśa also is counted like rāśi starting with Ist ṇavāmśa of meṣa as meṣa, 2nd navāmṣa as vṛṣa etc.

Thus at the and of 4, 8, 12 rāśis, 9th, 18th and 27th nakśatras i.e. mīna navāmśa is completed. Next navāmśa i.e. Ist navāmśa of 5, 9 Ist rāśis are

meṣa navāmśa. According to rules stated, last quarter of 9th, 18th and 27th rāśis or first half quarters of next nakśatras are bad. If a child is born during this period (i.e. if moon is in gaṇḍa or gaṇḍānta nakśatra), that nakśatra is worshipped when it comes again (on 27th day of birth).

As the seventh day sunday was not meant for work in christianity, 7th karaṇa viṣṭi is not good for starting any important work or for proceeding on a journey. It is also called bhadrā (meaning good - probably for holiday purpose).

Verses 59-62 : Comments on the siddhānta methods - Brahmā took 47,400 divine years in creation of world, which is called sṛṣṭi kāla (creation period). From next day after creation, revolutions of graha, their ucca and pāta etc started. Hence it has aheady been stated that for calculation of graha etc, the years of creation will be deducted from the years counted from beginning of kalpa.

After completion of creation, caitra śukla pratipadā was the first tithi. Then sun was rising in Yamakoṭipattana and it was mid night in Lankā. This day was named as ravivāra (sunday). From that instant Brahmā left graha, ucca and pāta to move in their orbits from first point of aśvinī nakśatra (meṣa 0°) From that time only days, months, years, krānti and revolutions of graha etc started. They had not started from start of day of Brahmā (called kalpa). From that time, only ghaṭī (1/60 of a civil day), yuga and manu etc started.

Sages like Parāśara have described king, ministers and protectors of the years, clouds like droṇa and puśkara, rulers of grains etc, parts of

fire, rain and deceases, rāja yoga etc for predicting good or bad results of future. Sometimes, they give the said results, sometimes they don't. Due to that, these have not been described here, as in other siddhānta texts.

Sun and moon complete their revolutions at the end of every yuga and also in 1/4th part of yuga. During a quarter of yuga (10,80,000 years), sāvana ahargaṇa (civil days) are (39,44,79,457). At the end of dvāpara, sṛṣṭyabda (years since creation end) was (1,95,58,80,000). This divided by years of a quarter yuga (10,80,000) gives quotient (1811) and zero remainder. hence there is no need to state dhruva (positions) of sun and moon at the end of dvāpara (after complete revolutions they are again at start of meṣa 0°).

Verses 63-66 : Start of Karaṇa for this book

From beginning of creation to dvāpara end, past years (years completed at entry of mean sun in meṣa) were (1, 95, 58, 80, 000), and at (4970) completed years in kali (1869 AD - Karaṇābda) the ahargaṇas are (7, 14, 40, 22, 96, 627) and (18, 15, 334) from creation and kali. Both are correct as checked by vāra (weekdays).

At beginning of karaṇābda, when mean sun had entered meṣa, first day according to mean value (sun and moon) was caitra sukla pratipadā. The dhruva stated for that day (mean positions at beginning of year), when added to daily motion for lapsed days, becomes dhruva of madhyama graha acording to sūrya siddhānta. Ahargaṇa of karaṇābda starts with tuesday (maṅgala vāra).

The day before beginning of karaṇābda has been assumed monday. That day was caitra sukla pratipadā. (mean speed). According to śpaṣṭa position it was vaiśākha adhimāsa (extra month) pratipadā. Hence the day before start of karaṇābda is correct caitra pratipadā according to mean speed and monday, which is convenient day for stating dhruvas.

From starting point of karaṇābda, (18, 15, 334-15) days before, kaliyuga had started at mid night at Laṅkā. According to ancient authorities, that was caitra śukla pratipadā by mean positions. Again first day of karanābda is in vaiśākha by true position. To find this caitra śukla pratipadā, dhruva at the end of dvāpara is to be added. Thus the dhruva have to be stated after specifying whether it is for mean time or true time.

Vikrama years	= Kali years - 3044
Śaka years	= Vikrama years - 135
Bhāsvatī year	= Śaka years - 1021
Bhāskara II years	= Bhāsvati - 51
Kuchannā year	= Bhāskara II year - 148
Darpaṇa year	= Bhāskara II years - 719

Verses 67-68 : Importance of Siddhānta etc.

The text which calculates graha from number of days since creation is called siddhānta.

The text calculating graha from days since yuga beginning is called 'tantra'.

The text which starts its count of days from beginning of śaka year or any convenient year nearby is called 'karaṇa.'

Siddhānta Darpaṇa contains all the three methods.

On seeing a knower of siddhānta, pāpa (result of bad work) done in ten days is destroyed. Knower of tantra destroys 3 days pāpa and karaṇa knower destroys 1 days pāpa. The man who beats his drum about time without knowing jyotiṣa is a multiple loafer.

Verses 69-70 : Discussion of dhruva etc.

Many astronomers like Bhāskara saw that moon is 3 rāśi ahead of sum in half fortnight, hence they added 1/3rd of the nakśatra dhruva. They had not observed it directly. Many astronomers differ about starting point of meṣa 0°. I have not accepted them in absence of proof.

Current age of Brahmā has been assumed to be 50 years by some, or 58-1/2 years by others. This has no importance for any practical use, because graha is calculated from the current day (kalpa) of Brahmā. We should be satisfied with that only.

Verses 71-73 : Prayer and end.

Lord Jagannātha is ocean of mercy for uplift of down-trodden and is beloved husband of Lakśmī, daughter of ocean. He has thrown my mind out of his bhajana, so dear to me, into the market of mathematics. He may provide shelter to my intellect, so that it doesn't lose its way in hard work.

Thus ends the fifteenth chapter describing pāta in siddhānta darpaṇa written as a text book

and for accurate calculations by Śrī Candraśekhara born in famous royal family of Orissa.

Thus first half of siddhānta darpaṇa is complete which contains 3 adhikāras containing 15 chapters where correct siddhānta and accurate planetary calculation has been described for happiness of the learned.

Chapter - 16

QUESTIONS ON METHODS

4. GOLĀDHIKĀRA

Verses 1-4 : Prayer and scope -

By meditating on whose lotus feet, learned men like Brahmā, Vaśiṣṭha etc. often wrote new siddhānta texts after seeing errors in earlier texts, logically explained the questions by students and even knowing all, wrote small texts like the author, I pray to the same lord of worlds Śrī Jagannātha.

I pray to the sun also who moves around earth with mean velocity carrying all the planets as a first among planets, removes misery of people by providing light, as true position is epitom of three vedas, and is like eyes of grand cosmos.

I pray my guru Śrī Bhāskarācārya (II) who wrote his siddhānta siromaṇi, because earlier texts developed errors as shown by him, who revised the new numbers of revolutions after seeing the grahas in his time, and who did a great work for my benefit as well as of world at large.

For lord Jagannātha staying at Nīlācala, whole world is a play field. He destroys sins. By keeping his radiance in my heart, I have completed grahagaṇita and start gola gaṇita with happiness about which old astronomers have written in detail.

Notes (1) Scope - Now all the methods pertaining to calculations of planets are over. Now

all the doubts regarding the methods and assumpitons will be explained. All the explainations arise from curiousity or doubts of students, hence first of all questions are framed. These are the doubts expressed by common people and critics, supporters of other theories and explainations for his new methods. All the questioners are symbolised as student. In astronomy, all the calculations are done on a spherical surface, as the planets and stars move on the imaginary celestial sphere. hence spherical trigonometry is the main instrument to explain the methods. Hence this part is named golādhikāra, i.e. section on gola (sphere). After questions, this will explain the details of two golas - bhū (earth) and bha (sky).

(2) Purpose of writing new texts is to explain the changes and errors whch have developed in old texts. This is not to say that earlier sages were at fault, they were knowing everything, but explained only those matters which were necessary at that time.

(3) In addition to lord Jagannātha, iṣṭa deva of the author, all the ancient and recent propounders of the subject have been prayed. Out of 18 ācāryas of jyotiṣa, Brahmā and Vaśiṣṭha have been remembered first. Out of the traditional 18 siddhāntas, only 5 survive in pañca siddhāntikā in which Brāhmā siddhānta is the oldest. The same brahma siddhānta was updated by Brahmagupta. Vaśiṣṭha siddhānta is available in fragments in addition to 5 siddhāntas explained by Varāhamihira. In the current siddhāntas, sūrya siddhanta is the latest and most accurate and popular. Sun god has been prayed in three forms

- as ācārya of sūrya siddhānta, as god he is form of viṣṇu and epitome of three vedas, and thirdly he is first among planets which move around him. Here it is assumed that mean sun moves around earth and all other planets move around it.

Lastly, Bhāskarācārya (II) has been prayed whose siddhānta śiromaṇi has been largely followed in this book and who is considered guru by the author.

Verses 5-11 : Importance of gola

To explain gola gaṇita to layman, I have explained the statements of Bhāskarācārya, and sometimes quoted others also. Due to study of old texts, my ideas also have emerged.

(Rest from siddhānta siromaṇi) By not knowing true meaning of mean and true planets and secrets of mathematics, astronomer remains confused and he is not respected in society of the learned. I am writing this 'gola' text so that secrets of astronomy are as clearly seen as an 'āmalā' fruit kept in palms.

An astronomer is useless without knowledge of mathematics in same way as food of all tastes is without ghee (butter), state without king and a symposium without learned speaker.

A clever speaker may tell a lot to gathering of learned even without clear knowledge of grammer. But finally he is humiliated with sarcastic remarks and glances. Similarly, an astronomer without gola knowledge also becomes a matter of joke when he fails to understand many questions and secret statements.

Gola (sphere) is a model to understand the location and size of earth, planets and stars. Gola has been considered an independant subject which can be understood only through mathematics.

Predictions of astrology have been considered authentic according to old astronomers. Predictions are according to lagna, and lagna is according to position of planet (earth). Without knowing true positions of planets, predictions cannot be made only from lagna. All planets move on celestial spehre only. Hence knowledge of gola is necessary for calculating true positions of planets. Gola can be understood only by mathematics. Thus a person cannot understand jyotiṣa without knowing mathematics.

Mathematics is of two types-'vyakta' (i.e. concrete - arithmatic and applied mathematics) and 'avyakta' (abstract - pure mathematics like algebra, geometry). After knowing these two types of mathematics and grammer, one can read and understand jyotiṣa. Without them, he is a namesake astronomer only.

Notes : Modern branch of gola is spherical trigonometry. Spehre is a model in the sense, that it is a coordinate system. For stars and planets, we only measure the direction and distance is immaterial. Hence they can be observed as points on a spherical surface which are from same distance from observer.

Knowledge of gola needs calculation methods (vyakta or pāṭī gaṇita) and avyakta (abstract) gaṇita like trigonometry, algebra etc. Grammer is needed to understand the text of jyotiṣa and to describe it correctly in own words.

Verses 12-39 : Doubts about earth

Now sky, earth and nakśatras are being described as dialogue between student and teacher, so that all secrets are explained. (12)

(Student) : O Teacher ! you have explained about graha gaṇita and gola in your nectar like voice. But you have discussed views of many ācāryas with too much description of scriptures. These various branches of thoughts have only increased my doubts, instead of clearing it (13-14)

Earth according to purāṇas ; Long ago, at beginning of creation, the creator formed earth for our living. First, explain me its clear form and location (15) - (question 1).

After that only, I can really understand the celestial sphere and planetary orbits. (16)

Puraṇas have described earth in shape of a circle. This earth has been held by varāha, snake (śeṣanāga) and kacchapa etc. Area of earth is 50 crore yojanas (17).

In central portion there is Jambū dvīpa of 1 lakh yojana extent (area) which is surrounded on all sides by salt-water ocean (18)

At centre of Jambu dvipa, there is meru mountain, 84,000 yojana above surface and 16000 yojana below surface (19).

Beyond Jambūdvīpa, there are two other dvīpas surrounding it and surrounded by oceans which are successively of double sizes. (20)

Outer most ring of puṣkara dvipa, has mānasottara mountain in its centre in shape of a ring on which chariot of sun moves. (21)

Outside these dvīpas and oceans there is lokāloka mountain whose surface is golden. That is surrounded by spherical shell of brahmāṇḍa. (22)

At 1 lakh yojana from earth, sun rotates, when it is far and oblique, we see its rising or setting. (23)

Above sun, planets like moon, maṅgala etc. move. Dhruva is above all, exactly above meru. Successively above dhruva are lokas like mahah, jana, tapa etc. (24)

Bauddha view : Nakśatras rotate around earth, hence base of earth is not located in any direction (otherwise it would have collided with nakśatras). Due to lack of a heavier base for earth, it is continuously falling down. (25)

Jain view : According to jaina view, there are 2 suns, 2 moons and 54 nakśatras in the sky. They rotate round 4 cornered pillar of meru and alternately enter and come out of water. Hence from meru, one of the two suns rises alternately. (26-27)

Modern view : Sharp minded scholars of England tell that earth itself is elongated sphere and rotates round much bigger sun in elliptical orbits like other planets maṅgala etc. being attracted by sun (28-29)

Thus one revolution of earth in east direction is completed in (365/15) days. Day and night are caused by earth's rotation on its axis. (30)

Thus due to two types of earth's motion - daily and annual - day - night and year are caused respectively. Due to annual rotation of earth, it is at different place each day. (31)

Due to earth's attraction towards its centre, people do not fall in any direction. As the persons in a moving boat think their boat as fixed and mountains moving, similarly on moving earth people feel earth as fixed and graha, nakśatras as moving. (32) When earth is in meṣa beginning (from sun), sun is seen in tulā beginning. Due to inclination of north south axis of rotation, sun has north or south krānti. (33)

Planets in increasing order of distance from sun are budha, śukra, earth, maṅgala, guru and śani. Due to increasing distance, their speed becomes slower. They rotate around sun with their own speeds. (34)

Small satellites revolve round planets and due to being attached with planets, they revolve round sun also. For example, moon is a satellite of earth in orbit round it. Along with earth, moon also rotates round sun. (35)

As on earth, on other planets also seasons change due to difference in light and heat for different positions in orbit round sun. (36)

On this assumpiton, planetary motions, eclipse etc. can be calculated from earth's position. Small objects move around bigger object due to its attraction, this is logical explaination of planetary orbits. (37)

Other old theories - According to other old theories (both in India and outside), earth is fixed in space and is surounded by the orbits of moon, mercury, venus, sun, maṅgala, jupiter and saturn. (38)

Āryabhaṭa I, stated that earth moves at its own place; i.e. rotates around its axis. Hence, persons living on earth rotating in east direction, feel that nakśatras are moving in west direction. (39)

Notes : Purpose of stating these theories is to know as to which of the theories is correct and why ?

Verses 40-45 : Other questions about earth

(Student to his teacher) - Now you tell that sun rotates around earth alongwith planets in orbits round it. Moon also rotates round earth. (What is the correct position ?) (40)

First you had stated that speeds of planets in yojanas is same. Now you tell that it is not equal. Why this new idea has come ! (41)

There are many explainations about this creation by God, whch one is correct? What is the order of creation of planets? How earth was created? (42)

Tell the size and base of earth. How many are the oceans, continents and mountains on earth? How many persons live on it? (43)

Please tell the circumference, diameter, surface area and volume of earth. How the loka, graha and nakśatras are situated from earth one above the other? (44)

How many living and inanimate beings live on surface of earth or below the suface ? Do similar creatures live on the surface of other planets like moon and mars also? (45)

Notes : These questions are similar to questions in sūrya siddhānta put by Maya to sun god, who taught sūrya siddhanta. All siddhāntas assume that linear speeds of all planets in yojanas is same, difference in angular speed is only due to change in distance from earth. According to this theory, sun was at 13.4 times distance of moon, because moon's rotation round earth is 13.4 times faster. But Candraśekhara revised the diameter of sun about 11 times the accepted value, hence its distance also was increased 11 times because angular diameter is same. Hence, ratio of sun distance to moon distance has become 150 : 1 instead of 13 : 1 according to same linear speed. However, linear speed of other planets have been assumed to be same as sun. From part following of equal yojana speed theory, a doubt about its correctness has arisen.

Verses 46-48 : Revision of bhagaṇas

As the graha were not observed according to their kalpa bhagaṇas stated in old siddhāntas, Āryabhaṭa etc assumed other values of bhagaṇas which gave correct observations. Bhāskara II etc again rejected them and determined their own values of bhagaṇas. But your bhagaṇas are again different from those of Bhāskara. It appears that bhagaṇas are not constant. Then what will be fate of your values ? Wil it bę revised by others likewise?

Verses 49 : Guru years

If 60 saṁvatsaras arise due to stay of mean guru in one rāśi, then why its cycle doesn't start

with first year prabhava when mean guru enters meṣa beginning.

Verses 50-53 - True positions of planets

How have you formed charts according to ahargaṇa numbers? Why so much trouble is needed to find true palnets. Is this method for true planets applicable for earth only, or on other planets also? (50)

Earlier ācāryas have stated 4 steps for finding sphuṭa (true) positions of tārā grahas like maṅgala. But you have described extra steps for sphuṭa of budha, śani and maṅgala. Why ? (51)

Why śīghra and manda paridhi have been assumed ? Why they are unequal at the end of odd and even quadrants. (52)

How manda kendra and śīghra kendra move? Why the bhujaphalas of manda and śīghra become positive or negative? Why the gati phala of graha is positive for kendra in karka beginning or negative in makara beginning? (53)

Verses 54-55 - Questions on krānti

All siddhantas have assumed parama krānti to be 24° Why have you assumed it to be half a degree less (23-1/2°)? Why krānti is north or south?(53)

Why duration of day and night increase or decrease ? All rāśis are in ecliptic (krānti vṛtta) only, still why their rising times on horizon differ?(54)

Verse 56 - Seasons

Why sun rays are hard in summer and pleasant in winter ? Why clouds pour more rain at the end of summer and not after other seasons?

Verses 57-62 : Qestions on eclipses

Why have you assumed diameter of sun 11 times big compared to other ācāryas ? Why moon diameter has been assumed smaller ? How both the eclipses happen according to you ? (57)

Why solar eclipse also doesn't occur (at all places) in its parvaśandhi (new moon day) like lunar eclipse? Where the laṁbana of sun is more or less? (58)

For parama grāsa value of solar eclipse, lambana was done, then why śara correction also?(59)

Please explain further about need of madhyama lagna and vitribha lagna for calculating laṁbana and nati, why earlier ācāryas have not stated about śarākśa correction ? (60)

To know the observed tamomāna at the beginning,. middle and end of solar eclipse (i.e. sparśa, madhya and mokśa), why have you assumed a śaṅku, which was not assumed by earlier ācāryas ? (61)

As in solar eclipse, why the grāsa and śara are not in same direction in lunar eclise also ? Why directions of valana correction are different in east or west kapāla (of sky) ? (62)

Verses 63-65 - Conjunction of planets

What is the difference of orbits in yojana for the grahas which are seen together on earth ? (63)

In āyana dṛkkarma, when kadamba prota śara is made dhruva prota by old method, it generally reduces. But according to your method why śara increases after krānti correction ? (64-65)

Verses 66-67 - Lights of stars -

There are infinite number of stars in sky, still why the darkness doesn't go in night as in day time ? (66) (This is called Olber's paradox in modern astronomy). How far in yojana does the rays of sun reach ? How far the light of planets and star go ? (i.e. from how far can they be seen ?) (67)

Verses 68-69 : Size of brahmānṇḍa

According to śruti (veda), circumference of brahmāṇḍa is found by multiplying the sāvana dina numbers in a kalpa by daily speed of planets in yojana. But here, according to you, daily yojana speeds of planets is not same. Thus śruti saying has been contradicted. Then how the circumference of brahmāṇḍa can be found according to you and what is its value?

Notes : Basis of calculating the dimensions of brahmāṇḍa are two assumptions - (1) All planets move the total distance of circumference of brahmāṇḍa in a kalpa, and (2) Linear speeds of planets are same. Assumption (2) follows from first, as

Circumference of brahmāṇḍa

= Linear speed X sāvana dina in a kalpa

Since sāvana dina and circumference are fixed; linear speeds of all planets are same.

This assumption is similar to observation on quantum mechanics that very fast particles have

very small life period (rather half life), hence they travel almost the same distance during their life as we travel in our life, proportionate to our size. Approximately this holds correct, but time period and dimension are not in direct linear proportion.

This principle has been rejected in siddhānta darpaṇa as moon's linear speed has been assumed 11 times that of sun. Then what can be the basis of calculating the dimensions of brahmāṇḍa ?

Verse 70 : Rising at meru

You have asked to calculate rising and settings of graha and nakśatras by both dṛkkarmas - āyana and ākśa. Will it be done in same manner in meru region also ? Please tell.

Verses 71-73 : Observing sun and moon

Devas drink rays of moon. Whether this statement of purāṇas in true or false according to you? (71)

Why lunar horn always appears elevated towards north as seen in India - whatever may be direction of moon, in east or west? (72)

Moon and sun are far from each other. Still, how their aspects are combined in mahāpāta (vaidhṛti and vyatīpāta)? (73)

Verses 74-78 - Measures of time

Do the lokas like mahah, tapa also move with pravaha wind? What is the order of owners of day, month, year? (74)

Day and night happen due to rising and setting of sun, hence a day night of 60 daṇḍa is understood. But days of pitara equal to a lunar month, 1 day of gods in a solar year and day night of brahmā in 2000 mahāyugas is not understood. Please explain. (75)

How may types are of pralaya ? What are the types of years ? What is the type of time measuring instruments ? How the rāśi, degrees and kalā etc of graha and nakśatras in the sky is observed according to you? (76)

How six seasons occur on earth? Are the durations for solar, lunar, savana years etc. same? What is the benefit to world from graha bhagaṇa or nakśatras ? Critiques of vedas tell that in vedas earth has been called 'jagat' i.e. with gati (motion) - thus vedas assume that earth is moving. Can you reply to that opposition to vedas (assuming that vedas presume fixed earth) ? Why there is spot on moon ? Can the persons get emancipation (mokśa) without samādhi which is obtained by persons engrossed in brahma ? (77-78)

Verses 79 : Easy methods

Please tell the methods by which people can easily calculate the tithis, nakśatras etc. of pañcāṅga for past or present. After hearing answers to all these querries, my doubts will be removed.

Verses 80-81 - Prayer and end

May god viṣṇu (Jagannātha) remove our fear, who has lotus in his navel, who is decorated with garland of lotus, who has special affection for brahmā evolved from lotus of his navel, and by whose darśana (seeing) alone elephant was rescued from king of water (crocodile) (80)

Thus ends the sixteenth chapter describing questions in siddhānta darpaṇa written as a text book and calculations confirmed with observations by Śrī Candraśekhara, born in famous royal family of Orissa.

Chapter - 17

LOCATION OF EARTH

Bhūgola Sthiti Varṇana

Verses 1-2 - Scope -

As an excuse to answer the questions raised by a good student, I attempt here answer to all in sanskrit through a discussion of the essence of scriptures.

Now I establish the lack of movement of earth and movement of sun and other planets and form of celestial spehre and spherical objects by three methods of proof - pratyakśa (direct), śabda (quotation) and anumāna (indication).

Notes : Purpose of this chapter is to describe location of earth in solar system. According to author, earth is fixed and the planets and sun move around it. Then, size of earth sphere and other planets are to be told. According to nyāya (logic) darśaṇa, proofs (pramāṇa) are of three kinds-

(1) Pratyakśa - direct method or observation

(2) Śābda - By accepting the authorities of vedic assertions (3) Anumāna - Induction due to relation of cause and effect. To prove one's point through these methods is called vyavasthā (establishment).

Verses 3-12 : Support for earth

Centre of orbits of tārā grahas budha, śukra, maṅgala guru and śani is sun, but it is not sphuṭa

sun (actual position of sun) it is madhyama sun (its fictitious position with mean speed). Earth is at centre of orbit of this madhyama sūrya. Madhyama sūrya sometimes comes near to earth or goes far. At the centre of sky, earth is fixed with its own force. It is surrounded by orbits of sphuṭa sun, nakśatras and moon. (3)

(Siddhānta śiromaṇi) - Earth is filled with mountains, orchards, villages, brick structures, domes etc everywhere, in same manner as pollen grains fill the flower of kadamba. (4)

This is my siddhānta (view) that earth is without any other base in centre of space, surrounded by orbits of sun, planets and nakśatra, and earth and all planets are spherical. I will prove this view by replying to questions of persons with different views. (5)

(Sūrya siddhānta) Earth is situated in the middle of sky being fixed from all sides. Since it holds itself by holding power given by Brāhmā, it is called dhāriṇī (i.e. holder). (6)

(Siddhānta śiromaṇi) - If we assume somebody else as holder of earth, then a third will have to hold the holder, then third will be held by fourth and so on. At last we have to assume some abstract force only as final holder. To remove this defect in logic, there is no harm in considering the abstract force as holder of earth in beginning itself. Is the earth not a form of god of eight forms? (7)

The incarnations of god like Kūrma had held the earth. That means that they held only a portion of earth, not the whole earth. (8)

Mountains also hold only a part of earth, hence they are called 'bhūdhara' (i.e. holder of earth). Similar was the holding of earth by incarnations such as Kūrma, due to which they are called bhūsthāyī etc. (9)

When king of serpents (seṣanāga) is tired of holding earth on his head, he takes rest by bowing his head, which causes earth quake. This sentence also doesn't mean holding of the whole earth, but only a part of it. (10)

If śeṣanāga holds only a part of earth, then why earthquakes occur in every part of earth ? Its explaination is that the whole body shakes when any limb of the body has vibration. Similarly, if any part of earth shakes, the vibration spreads to other parts also. (11)

Any matter is small part of earth only, hence earth attracts every matter. When it falls on earth, it gets a support and does not fall further. But there is no attractor of earth in any particular direction, so there is no reason for earth to go in any direction. (12)

Notes : Within solar system, earth and all other planets are held by sun due to its force of attraction. The planets are in their fixed elliptical orbits because they are balanced by two forces - force of attraction towards sun (centripetal force) and force due to circular motion (centrifugal force) away from sun.

It is doubtful if vedas claims that earth was fixed. It was merely a convenient assumption, because geocentric calculations are done by assuming earth as centre of coordinate system of celestial

sphere. Assumption of earth as centre is merely for calculation of events as seen from earth. It doesn't mean that earth is fixed and is centre of sky.

Yajurveda Taittirīya saṁhitā (3-4-11) tells -

मित्रो दाधार पृथिवी मुतद्याम्

(The sun supports the heaven and the earth)

Same views are expressed in Ṛk veda (1-164-14)

Sun is supporter of all world.

Cause of earthquakes, in siddhānta tattwa viveka (madhyamādhikāra (206b-7a) -

Earth's crust is hard and rocky where, however, a fissure occurs due to lack of strength, gases emerge forcibly causing the earth to quake, when there would also be a terific noise.

It is a philosophical question - what is the ultimate support of universe? Attraction of sun holds the planets of solar system. Sun itself moves round the galactic centre (chapter on conjunction of stars). Galaxies float in universe randomly like particles of gas (theory of Sir J.H. Jeans). This is universally accepted, with minor effects between galaxies of a clusture. About the ultimate infinity we don't know. Mach's theory, incorporated in steady state theory of universe is that the gravitational attraction between matter bodies is result of presence of matter spreading to infinity. The mutual attraction among galaxies is balanced by expansion of universe - each galaxy is moving away from the other. Whether this attractions will be able to halt the expension and reverse it, needs a minimum density of matter. We are doubtful

whether present density exceeds this critical value or not - hence both outcomes are possible. Universe may expand for ever, with or without creation of matter to fill up the gap. It may start callapsing, after its expansion has ceased - oscillating universe. Most popular view is that expansion of universe started with a big bang about 10-15 billian years ago - from a nucleus, which we may name 'Brahmāṇḍa' or cosmic egg as the term used in vedas. For further discussion books on cosmology may be referred - (1) An intelligent man's guide to science - I saac Assimov (2) Cosmology or (3) structure of Universe by Nārlikara or (4) History of time by Stephen Hawkins.

Verses 13-32 : Earth as a large spehre

(Brahma sphuṭa siddhānta) - Size of earth is 50 crore yojanas. If the kings having pride over their small kingdoms understand this, they will develop detechment from world. Devotees assume all things in small objects while worshipping a pīṭha (a small board or stone). For example small stones are assumed as god, rivers of India in a handful of water or ocean in a water pot. Hence imagining such a big earth also is not difficult or useless. (13-14)

Gross and very large size of earth has been well admitted by all, as we have assumed brahmāṇḍa (space) in a body (same rules of physics apply at small or large scale), śakti (energy or energy form of god as a female) in our veins or assuming ātmā (self) as part of paramātmā. (115)

At place of pujā also assumption of maṇḍūka (frog) etc is also as per a prescribed order. By

sphere. Assumption of earth as centre is merely for calculation of events as seen from earth. It doesn't mean that earth is fixed and is centre of sky.

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be stated about spherical shape and circumference of earth? (22) - Quotation ends.

If earth is plane instead of round, then speed of sun at far end in east will appear very slow as in case of a cloud. But there is no difference in motion of sun in east direction or in mid day. In all directions, speed of sun is uniform like motion of a golden bowl in water (used for measuring time - which water takes to fill the bowl through a hole). Thus it is concluded that earth is not plane but spherical (23-24).

When a ship is approaching sea coast, first its top portion, then middle portion and at last the whole ship (above water) is seen. If earth is not spherical, plane surface of water wil not obstruct the vision like this. At the time of lunar eclipse also, shadow of earth covering moon at any time is circular. Circular shadow at all times, proves that earth is spherical (25-26)

If earth is plane, not spherical, then sun would have risen at same time at all places from east to west. There is time difference between sunrise at places east-west to each other, which proves spherical shape of earth. Increase or decrease of phases of moon also is in circular shape whch proves that moon also is spherical. On the basis of śāstra and observation, ancient ācāryas have stated sun, nakśatras and all planets as spherical. (When all bodies in universe are spherical, earth also should be spherical - inductive inference) (27).

Many call the earth as 'Ananta' which is not incorrect. Since earth is round, if we move in any direction, we will keep on going round and round without end till parārdha. (28)

Earlier ācāryas have stated the location of moon over sun and planets over nakśatra. This is when observed from heaven side, not according to revolving bodies. (29)

Lord Kṛṣṇa had shown world vision to Arjuna in Kurukśetra, to Markaṇḍeya in water of deluge, and to king Dhṛtarāṣṭra in his royal court. That spherical world is different from our earth. That vision and our earth are not same. (30-31)

Hence, we are not concerned with the earth, mountains, oceans, sun and nakśatra etc which are described in purāṇas. Earth described in gaṇita jyotiṣa is different from that. (32)

Notes : Spherical shape of earth is well proved and needs no further discussion.

Reason of spherical surface of all planets and stars is that it is the equilibrium surface under a central gravitational field, hence liquid water of ocean maintains a spherical surface. Any deviation from that surface will cause a change in gravitational potential and there will be a force against that change to take it back to spherical shape.

Due to rotation of all planets and sun (probably all other stars) also around their axis, they are elongated at the equator. Gaseous planets are flattened at axis more, they also rotate faster. Since gaseous molecules have more internal motion,

they get move angular momenturm due to gravitational collapse.

This causes correction in akśāmśa, which has already been formulated in appendix to tripraśnādhikāra while calculating palallax.

Verses 33-38 : Critism of Bauddha view

(Siddhānta śiromaṇi quotation continues) O followers of Buddha! Your intellect is really dull. You clearly see that every body thrown upwards, falls back on earth. Even then, how do you assume that earth is continuously falling down ? (33)

(Notes - There is no up or down for earth, hence falling down of earth has no meaning. If earth also falls, then the falling object moving in same direction will not reach earth, assuming that speeds of both are same).

You see the polar fish (star group containing north pole star) as centre of daily revolutions of nakśatra, sun and moon around it. Still you thoughtlessly tell that there are two suns, two moons and two set of 27 nakśatras. You are really worthless. (Quotation ends). (34)

There is a star group in north sky in shape of a fish, called polar fish. In mouth of this there is pole star and a smaller star is in tail. The dhruvaprota through this smaller star in tail meets the ecliptic at 19° from viśākhā. Between mouth and tail star there are many stars in two rows. This tail star joins west horizon at sunset time while remaining in viśākhā nakśatra. At mid night, this is below pole star i.e. in south north circle

(meridian). Again at sunrise time it joins the east horizon. It is clear from that, that there is only one sun. Even then, O Buddhist brothers, how do you imagine two suns ? (35-38)

Notes : (1) Polar fish - This group is called ursa minor in modern astronomy as it has same shape as ursa major (saptarṣi maṇḍala) but has smaller size.

In Dhruva bhramaṇākhyatikā, the polar fish is described as follows -

Around that one sees a constellation of stars consisting of twelve stars and looking like a fish. It is named as the polar fish. From a distance, one sees a pair of bright stars, one at its mouth and the other at its tail. Star at the mouth is 3 degrees from pole and the star at tail in 9° away.

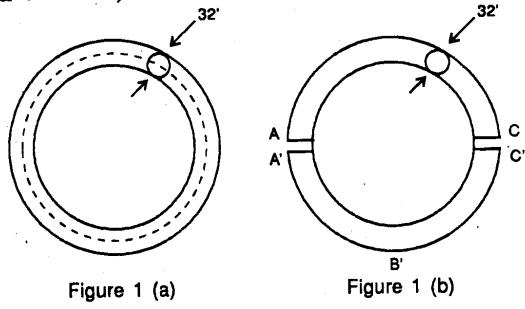
The Persian scholar Al - Birunī says :

The Hindus tell it śakvara and śiśumāra also Śiśumāra is a great lizard, as it is called susmāra in Persia. There is a similar aquatic species like a crocodile.

Diurnal rotation of polar fish has been used to refute the Jain theory of two suns and two moons rising alternately, by Brahmagupta, Lalla and Bhāskara II. It is not known as to why Bhāskara ascribed it to Buddhists.

(2) Concept of two suns, moons etc. - The gathā in Jaina text Sūrya prajñapti lists daily observations of sun at the time of rising and setting, for the purpose of calculating mean tropical year.

Position of two suns daily was noted (for sun set and sun rise):

Figure 1 (a) Figure 1 (b)

Hence two suns were thought wrongly. Average north south motion of sun is about 16′ per day (24 X 2° in 180° uttarayaṇa or dakśiṇāyana). Hence if the rotation of sun of 32′ diameter sun forms a maṇḍala (circular strip) then sun covers half a maṇḍala each day as shown in figure 1 (a).

Due to two observations of sun each day, two suns were thought and Malayagiri wrongly interprated half mandala as half circular strips of diameter width as in figure 1 (b). Thus one sun moves in upper maṇḍala ABC one day, next day it moves in lower maṇḍala A′B′C′. Then after two days, it will go to next maṇḍala on upper side.

Another reason of such interpretation is system of calculation of obliquity of earth's axis and declination of planets also on a flat surface. It is similar to projection of earth's map on a plane paper. Then Jambū dvipa was the first concentric circle from north pole to line of maximum north

krānti of sun. Next concentric circles will have much larger areas on plane projection. That will be explained while explaining dimensions of mount meru. However, a separate map will have to be prepared for south hemisphere, resulting in another sets of sun, moon and stars.

It may be noted that increasing sizes of concentric circles are due to projection on plane. Radius of Jambudvipa is 50,000 yojana and height of meru is 1 lakh yojana out of which 1000 is below surface. That gives attitude of pole star meru from 24°N, which is periphery of Jambū dvīpa. According to this system area of Jambūdvipa will be about 75 crore $(yojanas)^2$ out of which polar area is to be reduced to make it about 50 crore $(yojana)^2$. This is the value stated in verse 13 of this chapter.

Verses 39-78 Arguments against motion of earth-

If your intellect is sharp, and according to you, sun is fixed and earth is moving, then you will be contradicted by your own theory and will be defeated in the logic. Hence, stop your illogical talks and listen to me. (39)

Everybody in this world sees earth without any movement, hence fixed earth is directly proved by observation (pratyakśa pramāṇa). Similarly motion of sun also is seen clearly. Hence, what is the use of assuming sun as fixed and earth as moving ? (40)

(1) First logic for earth motion - A person sitting in a boat moving in current of the river, sees the trees on river bank moving opposite to motion of boat. He also sees that an object thrown

upwards, comes back to him, as if boat is fixed. Similarly, from moving earth, we see both events. Due to motion of earth, fixed stars and sun etc appear moving in opposite direction. From earth also any object thrown upwards comes back. If this is the logic for thinking motion of earth, then listen to refutation. (41)

Refutations - If wind is blowing from east and if a person keeps a silk flag on a pole, will the flag always be towards west ? Does the motion of wind happen only from east towards west, due to eastward rotation of earth on its axis) ? (42)

(2) Establishing fixed earth - According to jyotiṣa, pravaha wind moves the nakśatras one round daily from east to west. If you refute that the planets cannot move to east compared to nakśatras, then listen to the answer. On earth also clouds and birds can move east-wards even when wind is towards west. Similarly the stars are rotated towards west, but planets can move eastwards due to their own power. (43)

(3) Śabda pramāṇa for fixed earth - Ancient ācāryas and knowers of veda have told the earth as fixed. If you don't accept this, how you can be inclined towards pious work ? (44)

You may tell that world is called 'Jagat' meaning always moving. But it has other explainations also. World including heavens, living world, lower words, is destroyed in 4 types of pralaya (deluge or destruction). In that sense only, earth is called 'jagat', meaning that it will be gone or destroyed. (45)

If you interprate the other meaning only, then please tell - why earth was called 'acalā' and 'sthirā'? (Both mean fixed). In addition, it holds everything, hence it is called 'dharā' also. (46)

(4) Logical inference for fixed earth -

If you do not accept the vedic statement as truth or standard, only due to your argumentative nature, then I am stating logical proof also to prove the opinion of sūrya siddhānta or Bhāskāra II. (47)

(1) Your assertion is that under attraction of sun the planets like budha move in their orbits in addition to moving on their axis. Similarly, earth also moves on its axis and in its orbit round sun. (48)

You also say that a body moves only under action of force and there are two types of force (on heavenly bodies) - Forces of attraction towards centre (centripetal force) and force of repulsion away from centre (centrifugal force due to circular motion). (49)

In that case both the forces should be in earth also, because earth must have been created first, due to abundance of atoms in it. But we do not see the repulsion force of earth. Similarly, no other object on earth moves on its axis on its own. (50). We have not seen a stone rotating on its axis even for a moment, when it is thrown upwards. But on plane surface, a round object like wheel is seen rolling. (51)

(Forces towards centre and opposite) - A round body is rotated by tying it with rope, exerting force by hand. When it is separated from rope, then it moves away, and only force on it is

attraction towards centre of earth. Thus we see only two forces - repulsion by hand, and attraction by earth. We do not see its rotation on axis or forces of attraction and repulsion both in hand. (52)

(Axial motion) Similarly, if earth is moving due to forces of attraction and repulsion by sun on its centre, then motion of earth should always be in the direction of sun. Then there should always be light on one side of earth and darkness on the other side. (53) If earth rotates round sun in this manner, then the region on equator where there is day, there would have been no night and at night places, there would have been no day. (54)

(2) You may argue that axial rotation of earth is like rolling of a round object like wheel on plane surface. But this statement also is contradicted with your own theories. (55)

In rolling motion, earth will move equal to its circumference in one day, only then the same place will face sun again after one day. But circumference of earth is (12,000) kosa (= 40,000 kms approx.) (56) But according to your view, daily motion of earth in east direction is (8,00,000) kosa. How can it move so much compared to circumference.

Again this motion in east is in plane of equator, how motion of earth can be in the plane of ecliptic ? (57) How a person going in īśāna koṇa (north east) can move towards east? (58)

(3) You may assume the side motion (rotation on axis) of earth like a cancer, then these are

different types of logic - one is nisarga (natural) and other is upādhi (artificial). (59)

According to you, earth's motion around sun is not natural, it is only due to attraction of sun. But cancer moves on its own, not due to any force of attraction. Hence this comparison is not proper.

(4) Similarly it cannot be accepted that motion of earth is not in the plane of krānti. (60) When moon is in plane of equator, its spots are seen in south east direction for south ayana, and in north east direction in north ayana. (61) If motion of earth is in equator plane, like motion of planets, then spots of moon would have been in the direction of ayana (north or south). (62)

(5) Moon doesn't leave ecliptic, even when it has north śara in south krānti or south śara in north krānti. But earth has no śara, how can it leave ecliptic, by moving in equator plane? (63)

Further logic against axial rotation in different direction - (1) When rotating nut fruit in thrown towards sky, wheel is rolled on surface or in air, or rolling the ball during play - in all cases rotation around axis is always in the direction of motion. But rotation of earth is in direction of equator. How can it move in the direction of ecliptic ? (64)

Moon moves in its orbit (vimaṇḍala) oblique to the ecliptic and also rotates on its axis. If earth also similarly rotates along equator but moves along ecliptic, then we could have seen hills or trees in east, moving in angle direction - as we see rising of nakśatras in different directions. (65)

(2) You may interprate my saying that if with equator motion, the ecliptic motion of earth is in direction of sun, then due to elongation of ecliptic (ellipse shape), krānti gati (north south direction) will be straight like a chord, not curved like an arc. My answer is that sāyana sun should appear smaller in karka and makara beginning (due to distant part of eclipse) and should be bigger in meṣa or tulā beginning. But it is not so. (66)

Similarly, orbit of moon should be straight and its spot should be seen always in same direction. But it is seen in different directions from earth. (Hence earth does not move). (67)

(3) You may say. that motion of earth is in direction of ecliptic, inspite of attraction towards sun due to god's desire. Then I will say, that same god may desire, that earth should remain fixed, it should not move. (68)

Force of attraction : You may tell that sun attracts due to its big size. But force of attraction should be in earth also (69). A small fixed iron magnet also attracts bigger mass of iron. This proves that reason of attractive force is not the big size, but its natural power. (70)

Even after so much explaination, if you say that only a big object can pull a smaller object, and not vice versa, then I will say that it is possible only for an independent body. But objects, small or big do not have their independent force, their attractive force depends upon will of god. (71)

In a pond, big boats also revolve round a small pillar to which they are tied. Very big sun

along with planets revolves round the small earth, according to desire of god, for welfare of the world. (72)

Axial rotation of different planets : (1) You say that sun and all other planets rotate on their axis, so earth also must rotate on its axis. This doesn't contradict the fact that earth is fixed. Whether one moves or the other is fixed - depends on their nature. It is not necessary that all should act in same manner.

(2) You have seen the axial rotation of planets with talescopes. Why you couldn't see any axial rotation in moon also, which is nearest ? (That contradicts your statement, that all planets should move on their axis also). (73)

(3) You may say that satellites don't move on their axis like bigger planets. But I say that the satellites like moon have same relation with planets like earth, which the planets are having with sun (in both cases smaller objects are in orbit round the bigger object). Like planets, the satellites also receive heat and light from sun, why they are not similar in motion also ? (74)

Notes : Actually moon also rotates on its axis, but due to a strange coincidence its speed of axial rotation is exactly same as the speed of revolution round earth. Hence we see the same side of moon always. If moon was without axial rotation, different points on its surface would have faced earth due to its revolution.

(4) Earth is very close to sun and completes revolution of sun in 365 days and axial rotation in 60 daṇḍa (24 hours) Jupiter is farther from sun

and hence it takes 12 years for one rotation round sun. But it is very big compared to earth, still it completes axial rotation in only 25 daṇḍas (10 hours), i.e. less than half the time for earth. Form this, it seems there is no rule for these things, god's desire is the only cause. (75)

If it is stated that bigger planets have faster rotation on axis, then axial rotation of sun should have been in less than 25 daṇḍas time for jupiter. But it takes 25 days, though it is much larger than jupiter. (76)

Notes : (1) Mass and period of rotation on axis - All planets and their satellites rotate. The rotation of planets confirms to a certain regularity - the more the mass of the planet, the faster it rotates. There are some exceptions, due to special reasons, but in general the rule appears to be true.

According to modern concepts, the sun and planets had formed out of a rotating nebula composed of gas and solid dust particles. The particles collided and joined into larger bodies, thereby forming embryos of the sun and planets. The largest number of collision were at the centre of mass of the system, where almost all the gas of the nebula had been attracted. So the sun was formed. But almost all the initial moment of momentum of the nebula appeared to be concentrated not in the sun, but in planets. After the sun was ignited, its radiation dispersed the light gases from immediate surroundings to peripheral areas to form three giant planets. Planets of the terrestrial gorup turned out to be composed of solid particles.

The matter of the rotating nebula being compressed into dense spheres, the velocity of rotation increases due to principle of conservation of moment of momentum (i.e. angular momentum). Thus the giant planets have a higher velocity of rotation than that of smaller planets.

It can be assumed that in beginning, the rotation periods were faster according to bigger mass, but some planets deccelarated their rotation due to different reasons. About neptune, our knowledge is not sufficient. But pluto appears to have been its satellite in the beginning. Then neptune has partially lost its moment of rotation, after pluto has broken away.

(2) **Resonance in rotation of moon, venus and mercury** - The earth is braked by its satellite, the moon. The nearer and farther points of earth from moon, are attracted more and less by moon. Hence they are raised from the surface of earth. The hump of water in oceans causes tide. Since rotation velocity of earth is more than speed of moon's revolution round earth, the tidal humps drag the earth rotaiton. The humps appear not on straight line from earth to moon, they are turned in the direction of earth's motion by about 2°. This causes decrease in earth's angular velocity by 2×10^{-10} of its magnitude per year. Consequently the length of day increases each year by 2×10^{-5} seconds. Loss in angular momentum of earth is compensated by increase in distance of moon by 3 cms per year.

When the evolution of solar system had just started, the decceleration of rotation of earth - moon system was reciprocal. As moon applied brake on earth's rotation, earth also applied brake

on fast rotation of moon. As a result of this the moon is facing the earth always by one side - its sidereal perod of rotation P = moon's revolution period round earth T = 27.322 days. This is called resonance, when ratio of oscillation periods is integer, here it is 1. This resonance condition was achieved quickly because moon was closer to earth in past. The centre of mass of moon is 2-3 Kms. from its geometrical centre towards earth. This was when moon was 5-6 times closer to earth some billions of years ago. Since then powerful tidal forces have turned moon to face one side forever.

Abnormally slow rotation of venus and mercury is explained by the hypothesis that they revolved round sun together (like earth and moon) in one orbit. Then their rotations were in resonance with each other. But now mercury's rotation is in resonance with its revolution round sun, and venus rotation is in resonance with earth's revolution round sun.

Orbit of mercury has high eccentricity, hence its speed at perihelion is 1.52 times higher than at appelion. Its period of revolution T = 88 days on earth.

$$\text{Period of rotation } P = \frac{2T}{3} = 58.7 \text{ days}$$

Hence the solar day on mercury Po is given by

$$\frac{1}{P_0} = \frac{1}{p} - \frac{1}{T} = \frac{1}{2T}$$

Hence Po = 2 T = 176 days

Thus mercury's solar day is three times longer than its sidereal day and twice longer than its period of revolution. In perihelion, its one side or its opposite side only faces, in aphelion the points perpendicular to it only face.

For venus, the axis of rotation is almost perpendicular to the plane of its orbit, but the direction of its rotation is reverse. Hence its period of rotation P = - 243.16 days. Its period of rotation P is related to its revolution period T and earth's period of revolution To = 1 year accuretely by the following formula.

$$\frac{1}{P} = -\frac{4}{T} + \frac{5}{T_0}$$

Period of conjunction of earth and venus Tc is

$$\frac{1}{T_c} = \frac{1}{T} - \frac{1}{T_0} \text{ or } Tc = \frac{T - T_0}{T_0 - T} = 583.92 \text{ days}$$

In this period, an observer on venus will see 5 rises of sun and 4 rises of earth, as

$$\frac{Tc}{5} = \frac{1}{\left(\frac{1}{T} - \frac{1}{P}\right)} = 116.8 \text{ days}$$ (one solar day on venus)

$$\frac{Tc}{4} = \frac{1}{\left(\frac{1}{T_o} - \frac{1}{P}\right)} = 146.0 \text{ days}$$ (one 'earth' rise day on venus)

Thus, at time of conjunction, when venus is nearest to earth, it always faces us with one and same section of the surface.

Explanation by natural qualities : As the natural quality of sun and fire is heat, moon has coldness, water is fluid, rock is hard and wind is motion, similarly natural quality of earth is its remaining fixed. The different natures and abilities of objects is strange, it has no explaination. (77)

Even small qualitity of soil sinks in water, but big ship of wood floats. Similarly in wind (pravaha), small sized earth also sinks (i.e. is fixed) and sun etc of bigger size are floating (moving), because they are lighter (78).

Notes : Floating is due to specific grevity being less than water. Even a ship of iron, heavier than water will float if it is made hollow and flat shaped, so that it displaces more water than its volume which is equal to its weight.

Neither earth, nor planets float or sink in pravaha, which is very light (solar atmosphere). Even the concept of ether was as a lightest object. Thus the aguments are highly erroneous, due to incomplete understanding of 'pravaha' or modern theories.

Verses 79-93 : Planetry motions from fixed earth - What is the time when bhagaṇa (revolutions of the grahas had started according to you, (supporter of modern astronomy) or its time is not specifically known ? According to me (author) it had started just after creation was completed. Do you agree ? (79)

Whatever may be your opinion, the daily motion of a planet comes out to be same, whether calculated according to your period of revolutions or my values of number of revolutions in a kalpa. Hence this dispute is immaterial for calculation. (80)

If sun is the centre of all planetary orbits, then as viewed from earth, the sphuṭa sun will be (1) mean position for budha and śukra and (2) śīghrocca for maṅgala, guru and śani. If from the observed śīghra and mandaparidhis, we find the true planets from spaṣṭa sun (instead of mean sun) then we observe some errors. When true sun is in kanyā or mīna, then the positions of maṅgala and śukra near their ucca (aphelion - farthest point) differs from the true values by 52 kalā and 54 kalā respectively. Again when they are closest to earth, the error is 262 and 334 kalās for maṅgala and śūkra. Similarly other planets also will give errors if we assume true sun as their centre instead of mean sun as correct centre. (81-85)

Thus sphuṭa sun cannot be considered mean position for budha and śukra or śīghrocca for maṅgala, guru and śani. However, the persons who say that we are getting correct postions of planets by assuming true sun at centre, - are not giving correct logic. When a palm fruit (strongly bound to tree) drops just after a crow sits on tree, we cannot say that fruits was plucked by force of crow, which is much less than required force. Thus mere coincidence doesn't make a proof in support of heliocentric theory. (86-87)

By assuming mean sun as centre of planetary orbits, we have been calculating true planets

correctly since long. (88) Hence, the planets are not attracted by sphuṭa sun at the centre of orbit. Rather they rotate on their own being attracted by their gods at ucca like moon. This appears more reasonable (89). (This is again palm fruit - crow logic criticised above).

In cakrārdha, mandaphala of śukra is very little and for maṅgala, parocca and mandocca also are same. Hence they can be found according to your theory (heliocentric) also. (90)

Different planes for orbits - The planets starting with budha are rotating on their axis perpendicular to ecliptic plane and are revolving under attraction of sun. Hence there orbits should be in ecliptic plane only (which is not so). (91)

Earth moves under attraction of same sun, and under earth attraction, moon moves. In such a condition moon also is forced to move in the direction of axial rotation. It cannot move in a different plane (inclined at 5°). (92)

Again, rotation and revolution of earth being in same plane of ecliptic, dhruva will be on perpendicular side from sun's direction on equinox day. Then how, spaṣṭa krānti of moon can occur without attraction of ecliptic motion. From all these, fixed earth is well proved. (93)

Notes : (1) Due to concept of relative motion, it is possible to assume any body fixed, and to calculate relative motion of other bodies. In heliocentric theory also, we have to calculate geocentric positions to know the planets as seen from earth, because they are seen from earth only. Hence, method of calculation doesn't prove which

body is fixed or moving. This way both theories are correct.

However, considering sun as centre, calculations are simpler for heliocentric position. More important is that we can formulate theories of planetary motion (Introduction to chapter 5).

(2) Even in Indian methods, finding manda sphuṭa graha before śīghra phala calculation is for finding heliocentric position only. If mean sun only is the true centre of orbits, then śīghraphala could be calculated only in one step like mandaphalas of moon and sun. Hence, the four step calculation and approximation in the first two steps is due to assumpiton of heliocentric orbits. (Explained in chapter 5)

Verses 94-101 - Śara of planets

Planets are located above or below, south or north due to their mandocca, śīghrocca and pāta. The circles showing this motion are not real paths of the planets. (4)

They are unseen lines of circle. In this orbit only the planets have śara (at the end of which they are situated, not on the circle of orbits). As the western scientists assume the sun attraction to be reason of motion, we assume the gods of śīghrocca and mandocca having attraction power, and god at pāta having repulsion (vikśepa) power. (95)

If earth also is considered like other planets, then why guru, much bigger than earth goes far in north or south of its orbit due to pāta ? (96)

Like venus, earth also is near sun (compared to Jupiter), why earth is not having viksepa like venus ? If motion of śara depends on size of planet, then śara of earth cannot be less than jupiter (as shown from examples of nearer and farther planets). (97)

If śara of earth is assumed to be 3 kalā according to you, then correct sun cannot be found due to difference in equinox and solstice position due to śara. Hence eclipse shadow, time or lagna, nothing can be calculated. (98)

All these can be calculated if we don't assume śara for earth. Hence, assuming earth as fixed, is the correct principle. In both the golas (north or south) when bhuja of sun is same, krāntis also are exactly same. From that also, earth appears to be fixed. (99)

If we assume earth's motion, its śara has to be assumed which creates this confusion. Just as you think that sun has no śara, I think that earth has no śara. Disc of sun is separate from madhyama sun (mean fictitous position of sun). Then why can you not accept śara of sun ? (100)

You tell that earth is like all other planets of sun with only one difference, that earth doesn't have śara like other planets. In a similar way, I can say that all other planets revolve round sun, only earth is fixed. By nature itself, earth is fixed, why do you object ? (101) (Both assume exception for earth).

Notes : We regard earth orbit as the reference plane, hence earth has no śara or sun has no śara

as viewed from earth. More correctly the axis of angular momentum of the whole solar system should be taken as reference. That is inclined to the pole of ecliptic at an angle of 1.7° in direction of mithuna. Similarly polar axis of sun is inclined at angle of 7°15′ to the ecliptic pole. Rotation period of sun near its equator is 25 days and near the pole it is 35 days. Inclination of sun's axis to ecliptic is almost similar to inclination of nearest planet mercury.

However, taking sun's orbit as reference is useful for calculation of all events related to sun, year, lagna, seasons etc and also for calculating eclipse.

Verse 102-104 : Vapours from planets

If nearby planets like mangala are like earth and they also contain 6 seasons as on earth, then the vapours forming clouds in that planet would have acted as cover between earth and the planet, as moon covers sun in eclipse.

Planets budha and śukra are formed of water, earth is made of soil and bigger than them. This shows that all planets are not same. Hence there is nothing illogical in assuming earth as fixed and planets starting with budha as moving.

The water near earth becomes vapour due to heat of sun, which forms the cloud. On other planets, there is only water, hence there is no vapour and hence clouds are not seen.

Notes : All these are totally wrong and need no comment. All inner planets between guru and

sun are formed of rocks. Atmosphere and water vapour are contained due to higher force of gravitation and lower heat. Water vapourises anywhere, whether near earth or not. It depends only on temperature or pressure. There are much denser clouds on venus due to high temperatures.

Verses 105-111 : Centre of mass

At the ends of a high rod, one heavy and one light body is suspended. If the rod is supported at the balance point, it remains horizontal. That balance point is called centre of mass. (105)

If the rod is suspended on that point by a thread and rotated, small object will move in a bigger circle and heavy object in smaller circle. Earth and sun are similarly assumed small and heavy objects. (106)

Their mass centre is within sun, hence sun has been considered fixed at centre (of solar system). In this manner earth is moving in a large circle far away from sun. If this is the reason explained for fixed sun and moving earth, then this too is defective. (107)

Refutation of fixed sun - Mass (weight) of sun is in proportion to its volume. Volume of sun (cube of its diameter) is divided by volume of jupiter (cube of its diameter) Then we get

$$\frac{\text{mass of sun}}{\text{mass of Jupiter}} = \frac{45{,}92{,}64{,}62{,}91{,}33{,}53{,}882}{51{,}23{,}42{,}00{,}467{,}707}$$

= 896 + remainder = 897 almost.

By dividing the distance between sun and jupiter by this ratio,

we get $\frac{24,50,10,000}{897}$ = 2,72,018 kosa. (108)

Thus mass centre is 2,72,018 kosa away from centre of sun. Radius of sun is 2,22,261 / 2 kosa, which is less than half the distance of mass centre. Thus mass centre doesn't come within sun. Hence your reason for keeping sun as fixed is not correct. (109)

Here also period of revolution is found by finding the revolution time around sun. Then why, you unnecessarily assume mass centre away from sun (instead of sun itself) ? (110)

Weight of planets can not be measured, hence discussion about their mass centre is useless. For benifit of students and teachers, this has been discussed in conneciton of mass centre. (111)

Notes : (1) Here mass and weight has been considered as one. Mass is amount of matter in an object. Weight is the force of earth's attraction on it, hence it is defined only on earth. It is proportional to mass, hence in common language both are used for same meaning.

Mass is measured in terms of action of force on it, and in that sense, it is intertial mass. Force per unit mass, is the rate of change of its speed, called acceleration. In heavenly bodies, only important force is gravitation, which is also proporitonal to mass. Mass measured as per its gravitational pull, is called gravitational mass. These are the same - gravitational or inertial mass, according ðt physics understood so far.

When two or more bodies are under action of parallel forces (like force of attraction on a body

small compared to earth), the resultant force passes through the mass centre.

Figure 2

In figure (2), O is mass centre of bodies with masses M_1 at A and M_2 and B. Then M_1 X OA = M_2 X OB.

(2) Mass is not proportional to volume for all objects, it is only for objects of same density.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Hence, iron of same volume in 8 times as heavy as water. Here it has been assumed that densities of planets are same which is not correct. Rocky planets like earth have more density. Gaseous planets like Jupiter and beyond are less dense. Even sun is less dense then earth. Density of earth is 5.52 and of sun it is 1.41 if water density is considered 1.

Verses 112-115 - Efects of earth's rotation

Due to rotation of earth, high mountains or houses move in a bigger circle. Hence their linear speed is more, than the speed of rotation of surface. If a stone is dropped from top of a high building or mountain, it will fall at a point slightly towards east, from the vertically down point (as earth is rotating in east direction). (112)

This would not have happened, if earth had no rotation. According to me (author), it is because

an earthly wind named 'Āvaha' moves steadily from west towards east always. (113)

Due to that wind, all grass, trees are slightly inclined towards east. Hence, the stone falls slightly eastwards. (114)

Hence, we need not assume rotation of earth to explain it. On plane surface, and covered space, free of winds, all objects fall directly downwards. (115)

Notes : Effect of earth's rotation is both static and dynamic. Due to centrifugal force, the force of attraction reduces. It is also slightly bend away from equator giving bulge at equator. Resultant force is also slightly towards east from the true vertical, hence definition of vertical itself changes. Plumb line is towards east from true vertical line. Due to that also stone will fall towards east.

Another effect is dynamic called coriolis force. All motions tend to turn clockwise in north hemisphere. This is seen in the climatic winds, which start moving north, then turn towards north east slowly.

Verses 116-128 : Relation between distance and period of revolution.

Distance of planet centre from centre of sun is called distance. The time taken by a planet to make one revolution of sun is called its period of revolution (bhagaṇa kāla). (116)

The ratio between cube of distance and square of bhagaṇa kāla is same for all the planets (Keplar's third law). (117)

$$\frac{\text{Square of mars's bhagaṇa kāla}}{\text{Square of earth's bhagaṇa kāla}} = \frac{687^2}{365^2}$$

= 3/31 = 3.519 (118)

$$\frac{(\text{Mars distance})^3}{(\text{Earth distance})^3} = \frac{(7{,}22{,}50{,}000)^3}{(4{,}75{,}00{,}000)^3}$$

= 3.519 = 3/31 = (119, 120)

Thus both ratios are same. If, on that basis, you tell that earth moves, then I become perturbed and many things will have to be told. (120)

There is no effect without cause. Planets move around sun following this rule, only if sun itself rotates around its axis by the same rule. (121)

If this proportion is correct, then any planet very close to sun should move round the sun in 25 days, becuase rotation of sun around its axis takes 25 days, as seen by telescopes. (122)

Comparing this with distance and rotation period of budha,

$$\frac{(\text{Bhagaṇa of sun's surface})^2}{(\text{Bhagaṇa of budha})^2} = \frac{(\text{Sun radius})^3}{(\text{Budha distance})^3}$$

or $(\text{Bhagaṇa})^2$ of a planet at sun's surface

$$= 88^2 \times \frac{(2{,}22{,}161/30)^3}{(\textit{Budha distance})^3}$$

Bhagaṇa of that planet comes only 6/52 daṇḍas, i.e. sun should complete 1 quarter less than 9 round on its axis in one day. This is against the observed period of rotation of 25 days. (123-125)

Again if we assume the rotation period of sun to be 25 days, then

$$= \frac{(\text{sun radius})^3}{\dfrac{(\text{Mars distance})^3 \times (\text{sun bhagaṇa})^2}{(\text{Mars bhagaṇa})^2}}$$

Then sun's radius comes out to be 36 times its true value. (126-127)

Thus, if diameter is considered correct, then rotation period in wrong. If rotation of sun is considered correct, then its diameter is wrong. Thus sun's own period of rotation and diameter are against this rule of proportion. Thus, under its influence, planets like maṅgala will not revolve under this rule. (128)

Note : Sun's own rotation is not the result of an orbital motion, it is residual angular momentum since its formation. Other planets move in the field of sun's gravitation, hence they follow this rule. Sun doesn't attract itself, and this rule cannot be applied to self rotation.

Verses 129-133 : Manda and śīghra kendra -

In their own orbits, śīghra kendra of grahas starting with maṅgala lies in cakrārdha in nearer position and in cakra for farther position. (129)

Similarly manda kendra is at closest point and farthest point from madhyama sūrya. In maṅgala etc, śīghra phala and mandaphala is of same types (nearest or farthest points are reference for measuring). Since direction of earth (from sun) is opposite to direction of sun, it is more convenient to assume that sun rotates around earth along with planets. (130)

At the end of odd quadrant mandaparidhi of maṅgala is 3° more. This mandocca is attracted by

parocca in direction of earth from sun. Hence mandaphala of maṅgala is slightly more than śīghra phala, as it is attracted by mandocca, which is in turn attracted by parocca. This is due to manda kendra of mangala. (131)

For mercury, increase in śīghraphala or decrease in mandaphala is not due to sun alone. (This is due to manda-kendra of sun and mercury both). (132)

Mars and mercury, thus revolve around sun and earth both. Since manda and śīghra both are seen, it is clear that at least one of earth or sun moves. If sun is assumed fixed, then correct positions of planets will not come. Thus it is settled, that sun moves round earth, while carrying planetary orbits with it. (133)

Note : Whether sun is considered fixed or not, we have to calculate both - distance and direction of sun from earth, then distance and direction of the planet from sun. The smaller distance is called sīghra paridhi. In heliocentric position also, same calculation method is adopted. As explained in chapter 5, siddhānta formulas are symmetric for both inner and outer planets, as smaller orbit is always considered śīghra paridhi. Hence, a common formula is sufficient for both inner and outer planets. However, this doesn't contradict movement of earth.

Verses 134-138 : Distance and nature of stars

Scientists (western) say that stars are very far, and so, they appear fixed. The stars are all like suns and situated all over sky in different planes. (134)

Other suns also are with their family of planets. Nearest star is at distance 8,000 times the distance from earth to sun. (135)

Many stars are larger than sun also. How all can rotate round the much smaller earth? Hence, movement of stars in seen, only due to rotation of earth. (136)

If you say like this, then listen to my explaination. Nakśatras rotate round earth at distance of 360 times the distance of madhyama sun. (137)

(Distance of stars from earth = 360 X mean distance of sun = 360 X 76,08,294 = 68,47,46,460 yojāṇa).

Based on this distance, the difference in śīghra paridhi at the end of even and odd quadrants should be 1°, which is actually observed. hence the śīghra phala stated by me is correct. Earth's rotation in east like a worm (cancer) is illogical. It will be proved at last by discusson of krānti.

Notes : Differences in śīghra paridhi is not due to finite distance of stars as explaiend in chapter 5. Distance of star given here is as per old estimates. Nearest star is at least 3,60,000 times more distant than sun. If nearby star vega at 26.5 light years moves at a speed of 100 Kms/second perpendicular to its direction it will move only 1° in 1400 years. Other stars move much less.

Verses 139-142 : Motion of nakśatra sphere.

When earth is at the end of dhanu rāśi (from sun), then a star is seen at some point. When

earth is at 47° krānti difference from that place (i.e. in end of mithuna sāyana rāśi), then the star should be seen at 8 kalā south of its earlier rising point. (47°/360° = 8 kalā) based on star distances. But this does not happen. So I consider the earth as fixed. (139)

You cannot say that stars are fixed and they don't move. Reason is movement of ayana. Due to change in north and south krāntis, movement of nakśatra circle is clearly seen. If you explain that it is due to movement of krānti vṛtta, then listen to my explaination. (140)

Earlier ācāryas had stated observed values of dhruvāmśa and śarāmśa of naksatras. It continues the same today also. If we assume movement of krānti vṛtta and consider the nakśatras as fixed, this will not happen. Hence nakśatras as well as krānti vṛtta both move. (141)

Movement of ayanas, daily rotation of nakśatras from east to west, show that earth is fixed. If there were other stars in sky like sun, then there should not have been darkness during night. (142)

Notes : The difference in calculated positions of stars for maximum 47° krānti difference is on assumption that stars are only 360 times more distant than sun. They are at least thousand times more distant than assumed here.

Verses 143-145 : Darkness in night

With the telescope of magnifying power of 100, 'lubdhaka' star is visible even at 8 kalā difference from sun. If lubdhaka also, is same as sun, then the light from two suns in same direction

will create terrrific brightness. (Hence there is not an infinite number of stars like our sun). (143)

You explain that stars are very far, and their light is absorbed by vapour and ice particles. Hence their light is not as bright as sun. My objection is that stars cannot be as bright as planets, who have various types of motions (stars are thus dull). Assumpiton of ice or vapour particles is only due to confusion. (144)

Due to nearness, light of planets like mangala is constant. But light from far stars absorbed by snow etc. is flickering. Hence they vibrate like flames of fire. But like fire, their light also may be extinguished. Has it been seen ? (145)

Notes : If infinite sun like stars are assumed uniformly spaced in all directions, then there will be infinute number of stars in any cone of vision. Area of any spherical shell increases in proportion to square of its radius (R^2). Hence the number of stars will be proportional to R^2. Light from an individual star at distance R will have intensity proportional to $1/R^2$. Hence all spherical shells contribute the same amount of light. From infinite shells in space, light at earth will be infinite. This is called Olbers' paradox.

This doesn't happen because, light is absorbed by gases spread in sky. But even then the gases will became heated and will start emitting light. Then the temperature at each place will be 6000°K which is average surface temerature of a star.

Its real explaination was provided by expansion of universe observed by Hubble in 1928. Due to expansion, the stars at farther distance have

lesser influence. Hence, we are mainly affected by nearby star sun only. Thus, when we don't face sun side, there is darkness in night.

Verse 146 - Equatorial bulge : If an earthen sphere is rotated along its diameter, the sphere is pressed along axis. Similarly, on earth also polar regions (meru) are depressed and equator portion has a bulge. This confirms the rotation of earth along its polar diameter. Reply to this logic is that, in creation of god there are many similar strange things. To explain coincidence of both these results, it is not necessary to assume axial rotation of earth.

Verses 147-150 : Desire of god

If the west rotation of nakśatras is due to east rotation of earth, then why god had to create pravaha wind with hard labour for west rotation of stars ? Reason is that god can keep rye as fixed and make the mountains move. To show his abilities, he has made the earth fixed. (147)

Playful god has done impossible acts many a times to punish the cruel people for good or bad works. But in earthly creation there is no blemish. Human body is transitory, but the earth created with great labour from stones and gems will be kept fixed by god. As man doesn't leave his nature, God is always engaged in rotatig stars, while keeping the earth fixed. (148)

Worldly people are always hankering after house and other properties. This is not done by adepts. Similarly god acts only for justice (not to satisfy his habit). Against this logic, my reply is that, god has not achieved any goal by giving lesser life period to men, elephant and horse and

longer life to crow, jackal etc. God's desire is the only reason. (149)

Earth is created out of internal (detached) and external (visible) abilities of god. As a body of god, earth is living place of all beings and having all attributes. Hence grahas also which are parts of god, revolve round it. There is nothing strange in it. (150)

Verses 151-152 - Summary of evidence

Through many reasonings like fruits of siddhānta tree, theory of three types of earth's motion has been demolished-daily motion, annual motion and krānti. May it remove the tiredness of people out of motion of earth, through good taste of the logical fruits. (151)

Thus fixed position of earth has been proved by the following arguments - (1) rising of same star in celestial sphere always at the same place (2) Increase or decrease of manda and śīghra phala of budha and maṅgala due to their manda kendra (3) Spots of moon always look alike from earth (4) From the nature of light and heavy bodies (5) Movement of attracted body in the direction of attraction (6) sun has no śara (deviation from ecliptic). (152)

Verses 153-154 - Earth as gem

(Siddhānta Śekhara) - As the iron embedded in gem is round, so the earth, without any support and shelter for all, is definitely round. (153)

(Varāha purāṇa) - As iron is embedded within a gem, similarly earth of 5 mahābhūta (fundamental elements) is within the sphere of stars. (154)

Verses 155-159 : Background of discussions

Due to all these reasons, I hold that earth is fixed. Sun through its mean form carries all planets and revolves round the earth. The planets do not move in any fixed order (they have independant motions). The scholars who know the value of labour, should give a thought to my theories. I will be satisfied only with that. I am not concerned with praise or criticism of my views. (155)

O learned astronomers ! You are friends to all. Please pardon my out-spoken views to prove the fixed earth. To explain the fixed earth to students, I told whatever came to my mind. Who is without fault in this world ? (i.e. there is possible fault in my logic also). (156)

Man wants his rise only. While constructing idol of god, if it becomes a monkey idol, what can be done ? It can be thought as fate only, not fault of the man. (157)

Universe has originated from ocean in which world is continuing. It is not affected by strong winds etc. and remains as such in the space. As described in purāṇas, it is full of seven seas, continents and mountains all around. Here I have not described the universe. Only its base, earth and sun etc. have been told. (158)

Movement of earth has never been heard in past. In 144th part of kaliyuga (I.e. kali samvat 3000 or 102 B.C.) there was a cursory mention of earth's motion by Buddhists. The great scholars like Bhāskara also replied it lightly. hence much had to be told here to counter that view. (159)

Notes : The arguments are mainly directed against concepts of modern physics and Keplar's law of planetary motions, which the author heard during discussions with Prof. Jogesh Chandra Rai of Cuttack, who wrote introduction ot siddhānta darpaṇa in 1899 AD. His logic depends on the fact that both the calculation methods for planetary motion are equivalent. Even in heliocentric theory we have to calculate geocentric position as all observations are from earth only.

The author has partly understood the concept of centre of mass. But concepts of relative motion, attraction and repulsion at different points, motion in direction other than direction of force have not been grasped by him. Force causes acceleration of a body in its direction, but if initial motion is in different line, the resultant motion will not be in direction of force, i.e. velocity can be in a different direction. This can be understood from any graduate level text book of physics or mechanics.

Candraśekhara also, expresses doubts about correctness of his theory of fixed earth in verses (155-157)

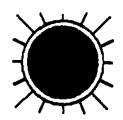
Though nowhere in the vedas, earth has been told as fixed, siddhānta jyotiṣa calculates on basis of earth as centre of cordinate system as it is done now also. Hence without specific mention of fixed earth, it creates this impression.

In supporting this view, author has felt that atternate theories will demolish the siddhānta jyotiṣa which is not so. Another reason is pride in Indian thoughts and the feeling that European theories were popular only due to political and

financial influence of those countries. That was definitely so, as Greenwich became reference for 0° longitude in place of Ujjain, or Gregorian calender was accepted worldwide after rise in British power. In next verse he has felt that, movement of earth has been felt only due to hope of getting gold medals - and this has been named golden theory.

Verses 160-161 - Prayer and end

Theory of moving earth and fixed sun has been proved true, undeservingly due to influence of gold (gold medals or economic and political power). May Lord Jagannātha living in Nīlācala bless me, by whose grace many thoughts were expressed by me and the fire of logic melted the golden theory of moving earth.

Thus ends the seventeenth chapter describing position of earth in the sky in siddhānta darpaṇa written as a text book for accurate calculations by Śrī Candraśekhara born in famous royal family of Orissa.

Chapter - 18

DESCRIPTION OF EARTH

Bhūgola Vivarana

Verses 1-2 : Prayer and scope

I pray to the essence called Kṛṣṇa, whom devotees call Bhagavāna (i.e. with influence and prosperity), sāṁkhya, and yoga call detached puruṣa and parama Śiva (supreme god), vedānta calls Brahma (Big or ever increasing), nyāya (logic) calls Paramātma (the grand soul of universe), mīmānsakas call karma (action) and sun worshippers call it Kula deva (i.e. family god). (1)

Sages has said a lot about creation. God willing, I will tell about it in detail in an independant work. Here I am telling some useful facts about creation as per Sūrya siddhānta. Then I will also tell about situaiton and extent of world as answer to questions posed earlier (in chapter 16). (2)

Note : This prayer is almost similar to classical 'secular' prayer of god

यं शैवाः समुपासते शिव इति ब्रह्मेति वेदान्तिनः

बौद्धाः बुद्ध इति प्रमाण पटवः कर्त्तेति नैयायिकाः

अर्हन्नित्यथ जैन शासन रताः कर्मेति मीमांसकाः

सो ऽयं वो विदधातु वाञ्छित फलं त्रैलोक्य नाथो हरिः

Different names of god are -

Śaiva - Śiva i.e. well doer or tranquil

Vedāntī - Brahmā - Grand universe, ever expanding

Bauddha - Buddhá - Enlightened

Nyāya followers (logicians) - Karttā (agent)

Jain - Arhat (Able)

Mīmānsaka (thinkers of customs etc.) - Karma (action)

In this book Bauddha and jain views are not included. 'Karttā' name by nyāya followers has been stated as paramātmā. The additional names are for

Sāṁkhya and yoga - Parama siva (supremely calm and composed)

or Nirlipta (detached)

Sun worshippers - Kula devatā - (Head of gods).

Verses 3-22 : Creation as stated in Sūrya siddhānta (Sun god to Maya asura - in sūrya siddhānta) Listen to attentively because this matter is related to philosophy (adhyātma) and hence very sacred. I have nothing which I would keep off from my devotees, with myself. (3)

Parama puruṣa Vāsudeva himself is the form of Parama Brahma (infinite universe). He is formless, calm, unchanging and beyond 25 elements. (4)

As part of Parama Brahma, Saṁkarṣaṇa adopted body form and entered into nature (energy). He is all pervading but knowable (due to body form). First of all 'āpah' (vapour or gas) was created. Seed was put into it by Sankarṣaṇa. (5)

That seed developed into golden egg (radiant from out side, spherical), inside which was only darkness. Within that, eternal Aniruddha took form (or became bright to be seen). (6)

This 'Aniruddha' has been called 'Hiraṇya-garbha' (cosmic egg of light or energy - Hiraṇya means gold as well as light). Being the first born, he was also called 'Āditya' or 'Sūrya'. (7)

He was called 'Param jyoti' because it removed darkness. Because he created world, he was called 'Savitā'. He produces living beings and also revolves round all bhuvanas (3 or 14). (8)

Being brightness or energy himself, Sūrya destroys darkness and is famous as 'mahāna' (The great). Ṛk veda is his maṇḍala (spherical spread), Sāma veda is his light rays and yajur veda is his body. (9)

Thus sūrya is embodiment of three vedas. He is soul of Kāla and creator who has produced Kāla. (Time measures are based on sun, creation or destruction are another meaning of Kāla, which are also due to sūrya). He is soul of all beings. In microscopic form, he is within every body. In his grand form, everything is included in him. (10)

The whole world with moving and non-moving is his vehicle (ratha or chariot). Chanda (rhyme or vibration) are his horses. They are connected with rays of light and revolve with sun. All are controlled by sun. (11)

Only one quarter of sun's energy (teja) comes out, three quarters are hidden as indestructible. He created 'ahaṁkāra' (individual differentiation) and then crated Brahmā to create the world. (12)

Sūrya gave 'vara' (method) of vedas (knowledge) to Brahmā and put that grandfather of all beings in a great egg (Brahmāṇḍa). He himself rotates with brightness. (13)

Becoming in form of Ahaṁkāra, Brahmā desired to create the beings. Moon was formed from his 'mana' (mind - hence the words mind and moon evolved from 'mana'), fire from ears and sun from eyes appeared. (14)

'Mana' gave birth to ākāśa (space), from ākāśa came vāyu (air), agni (fire) from vāyu, water from fire, and earth (soil) from water appeared. Thus five mahābhūtas (elements) appeared with one guṇa (attribute) more at each step. (15) Thus appeared agni (fire), jala (fluid), sun, moon and then mars etc. Five star like planets (tārā grahas starting with mangala) appeared from five elements - teja, earth, ākāśa water and wind. (16)

Again Brahmā divided himself into 12 parts and took form of 12 rāśi's (signs of zodiac). The same 12 rāśis were divided into 27 nakśatras. (17)

Then gods were created and through creation of prakṛti (female form of god or energy), he created the whole world with moving and non-moving bodies in order of upper, middle and lower planes. (118)

With his knowledge of vedas, he assumed different classifications according to guṇa (attributes) and karma (functions). (19)

From these classifications or regions, following were created in that order - graha, nakśatra, stars, earth, biṁba (disc of planets etc), brahmāṇḍa, deva, asura, man and siddha (adepts). (20)

In the hollow space of brahmāṇḍa exist lokas like bhū, bhuvah, svah. Brahmāṇḍa is shaped like two big bowls joined at their edges. (21)

Internal circumference of brahmāṇḍa is called 'vyoma-kakśā (i.e. orbit of the sky). Within this, move nakśatras etc. starting from periphery . (22)

Notes : This is a combination of ideas of sāṅkhya' of Kapila (one of the six systems of philosophy), purāṇas and in Gītā. The chart as per Gītā Rahasya by Bāla Gaṅgādhara Tilaka is given below (page 180 and 184)

Vāsuḍeva (abode of all)

Puruṣa (formless viewer) ← [Two forms, both without birth or beginning] → Prakṛti (unexpressed, with 3 quṇas, 'sakti, giver of birth)

Saṇkarṣaṇa (seed)

Golden egg (Pradyumna)

Hiraṇya garbha - Aniruddha

Mahān or Buddhi (Expressed and fine)

Ahaṅkāra (Pride)

Sāttvika, i.e. expressed and fine organs

Tāmasa i.e. without organs

Five Tanmātrā (Fine)

Five mahābūta (gross)

Five sense organs

Five organs of work

Mana

25 elements

Sāṅkhya Classification	Elements	Vedānta Classification	Gītā Classification
(Neither crea-	1. Purusa		Parā prakṛti
tionnor destru-	1. Prakṛti		Aparā prakṛti
ction)	1. Mahan	8types of gross	8 types of
Mūla	1. Ahankāra	forms of brahma	aparā prakṛti
prakṛti	5. Tanmātrā		
7 Prakṛti	1. Mana	Due to	These 15 are
(cration) Vikṛti	5. Sense organs	deformations	not comted as
(deformation	5. work organs	these are not	basic elements
or destruction)	5. Mahābhūta	considered	due to deform-
16 vikāra	(great elements)	basic elements	ations
(deformities)	25 Total		

5. Tanmātrās - Tanmātrās (micro forms for) śabda (sound), sparṣa (touch), rūpa (form - vision), rasa (fluid or taste), gandha (air or small).

5. Sense organs (one for each tanmātrā) - ears (for sound), skin (for touch - tvacā in sanskrit), eyes (for vision), Tongue (for taste) and nose (for smell)

5 work organs - speech (mouth), hands, feet, upastha (generative organ) and exretory organ (gudā)

5. Mahābhūta - Ākāśa (space) - Sound quality

Vāyu (wind or air) - (+) Touch quality

Agni (fire) (+) rūpa or vision

Jala (liquid) (+) Taste (rasa)

Bhūmi (soil) (+) Smell (gandha)

Tārā grahas corresponding to 5 mahābhūtas

Ākāśa - Jupiter (guru)

Vāyu - Śani (saturn)

Agni - Maṅgala (mars)

Jala - Śukra (venus)

Bhūmi (soil)	- Budha
Three Guṇas	
Sattva	- Light - Tretā
Rajas	- Action - Dvāpara
Tamas	- Static - Kali yuga
Three loka types	**- 14 types of beings (bhuvana)**
1. Daiva creations (Sattva guṇa)	1. Brāhma, 2. Prājāpatya, 3. Aindra, 4. daiva 5. Gāndharva, 6. Pitrya 7. Videha 8. Prakṛtilaya
2. Human forms (Rajas guṇa)	1. Man
3. Tiryak (lower forms)	1. Animals, 2. birds, 3. Repliles (Tamas guṇa) 4. Insects 5. Static (planets).

This is the order of creation from higher to lower. 12 rāśis and 12 nakśatras have already been explained.

(2) Theory of creation in vedas - Only one section Nāsadīya sūkta of Rk veda (10-129/1-6) is being explained here according to Dr. P.V. Vartaka of Puna and Dr. A.S. Rāmanāthana, nuclear Scientist at Madras.

नासदासीन्नो सदासीत् तदानीं, नासीद्रजो नो व्योमा परो यत् ।
किमावरीव: कुह कस्य शर्मन्, अम्भ:, किमासीद् गहनं गभीरम् ।१ ।

Meaning - At that time (of the creation of the universe) there was no 'asat', no 'sat', no solar system (rajas), no space, nothing else was present. Then what covered on ? where ? For whose shelter? What was that unfathomable and profound thing emitting sound (ambhas) ?

'Sat' means the existant things, like water, horse etc., 'Asat' means non existant things.

Śatapatha brāhmaṇa (10-5-3) tells

नैव वा इदमग्रेऽसदासीन्नैव सदासीत्। आसीदिव वा इदमग्रे नैवासी द्वैतन्मन एवास। .... तस्मादेतद् ऋषिणाभ्यनूक्तम्। ना सदासीन्नो सदासीत्तदानीमिति। नेव हि सन्मनो नेवासत्।

It is not true that cosmos did not exist (asat), nor was it sat or did exist. It was there, and it was also not there. Only mind (manas) was there. Later on it says that manas created vāk, then come prāṇa, which created organ of vision end so on. At other place the brāhmaṇa says (6-1-1)

असद्वा इदमग्र आसीत्। तदाहु: किं तदासीदिति। ऋषयो वाव ते ऽ ग्रे असदासीत्। तदाहु: ते के ऋषय: इति। प्राणा वा ऋषय:। ते यत्पुरास्मात्सर्वस्मादिदमिच्छन्त: श्रमेण तपसारिषन्स्तस्मादृषय:।

The cosmos was asat in the beginning. What is asat? Ṛṣis were asat in the beginning, who are those ṛṣis? Prāṇas were the ṛṣis. Because they strained themselves desiring creation of this cosmos, they are called ṛṣis.

Thus asat means prāṇic energy, which Sāyaṇa tells nameless and formless brahman. It is non existant in the sense that it is energy and not matter which has mass or which covers space etc. Universe was created out of this pranic energy - it was neither matter (sat) nor energy (asat) - may be a combination of both as dual nature of matter or energy particles. Asat gave rise to 'manas' and from manas, a desire to create arose, and creation commenced.

Negation of both means, that neither of them existed alone - a combination of both sat and asat was present - or the entity which was both.

'Rajas' means 'loka' when used in plural. It has also been used in the sense of 'antarikśa' loka i.e. solar system (creation in sky). Thus it means

the primordial substance which formed the lokas later on. As a quality it is the desire which is cause of creation. Satva is only existance or pure knowledge. Rajoguṇa is action oriented, it means controlled action which creates. Thus there was no matter or solar system which can create and support life.

There was no vyoma (ākāśa) at that time. 'Om' means original sound or vibration. Creation started with vibration which is form of energy and it caused interaction between matter and energy particles. Vibration felt by us is sound i.e. śabda tattva. Vyoma means space without vibration. Vibration was not present when creation had not started. After vibration, vyoma becomes ākāśa.

'Vāyu' means central core and 'vayonādha' is its covering envelope. Vayonādha has been also called 'chandas' or śarman'. Śatapatha Brāhmaṇa (8-2-2-8) tells

प्राणा वै देवा वयोनाधाः । प्राणैर्हीद सर्वं वयुनं नद्धम् । छन्दांसि वै देवा वयोनाधाः । छन्दोभिर्हीद सर्वं वयुनं नद्धम् ।

(Śarma) has become 'carma' (skin) i.e covering surface (3-2-2-8) - शर्म चर्म वा एतत् कृष्णस्य (मृगस्य) तन्मानुषं शर्म देवात्रा । Thus there was no covering surface as there was no material to be covered. In modern astronomy, space expansion means expansion of materials (galaxies) which are spreading from each other. (Hubbles theory 1928). In beginning it was a point universe without cover.

The third question - for whose shelter all this was prepared ? - is unanswered so far.

'Ambhas' means fluid, though whch vibrations can pass, i.e. sound producing material. According to Gamow, the initial distribution of matter was uniform and very thin with density of

10^{-22} compared to 1 for water. This extreme rare matter has been termed as 'Ylem' which has no meaning. But ambhas indicates true qualities of that matter in which vibration was the only prominent effect.

In first line, it is told there was nothing. Next line questions - what covered on ? That means that during very short period all the elements came into being. (There are books on first 3 minutes of universe, History of time etc.).

(b) नमृत्यु रासीदमृतं न तर्हि, न रात्र्या अह्न आसीत् प्रकेत: ।
आनीदवातं स्वधया तदेकं, तस्माद्धान्यं न पर: किंचनास ॥२ ॥

There was neither death, nor immortality. Since there was no creation, there was no question of permanence or death. Here martya means changing and amartya means everlasting principles. Both are necessary for creation.

There was no day or night, as there was no sun or planets from where day night can be observed.

Second line - There was only that one breathing without air, with his own energy. Really there was nothing else.

There was only one thing - means there was only one primordial fire ball - from which universe appeared after big bang. This is called 'Hiranya-garbha' - fire ball. This was only thing present as per śāntipāṭha-"हिरण्यगर्भ: समवर्तताग्रे भूतस्य जात: पतिरेक आसीत्"

It is clarified that this breathing was without air, as it was not available in space. This was pulsation of surface found in star due to interaction between nuclear forces of expansion in interior of star and gravitational attraction which compresses

it. Thus Gamow writes - spectral lines of Cepheid variables prove that the stars are so to speak breathing - i.e. their surface layers are periodically rising and falling.

Here 'bala' is force in a dormant stage, in action it is śakti and result is kriyā. Śvadhā' means śakti which is able to create. It is also called māyā bala i.e. force seen due to its effect (oscillation etc.). In pralaya it merges with rasa as waves in ocean calms down and merge with sea. Thus, svadhā in star is its nuclear energy causing creation, which also causes breathing.

तम आसीत्तमसा गूलह, मग्रेऽप्रकेतं सलिलं सर्व मा इदम् ।
तुच्छ्येनाभ्व पिहितं यदासीत्, तमसस्तन्महिना जायतैकम् ॥३ ॥

There was darkness to begin with. There was something mysterious in the darkness. It was impossible to understand. It was all undulating matter (salila) 'Ābhu' originated from the surrounding was wrapped by lighter material. It developed further due to heat energy.

Initially stars are cold spheres of rarefied gases, which do not emit light. They become compressed due to gravitation. Then heat and light are produced, and finally nuclear reactions starts, resulting in creation of new atoms.

Gamow says - At dawn of universe, the stars must have been so dilute that they occupied all space forming a continuous gas. Later, due to some internal instability, the continuous gas must have broken up into a number of separate clouds or gas drops.

Śalila' means a matter with ripples, i.e. undulant cosmic gas. Separate clouds or gas drops

are 'ābhu' - i.e. a thing formed from surrounding material (आसमन्तात् भवति इति आभु) Though called drops, the gas drops were 2-3 light years wide with mass of about 10^{30} kg. So 'ābhu' is a better word than drop. Ābhu was covered with still lighter matter as stated in the verse.

कामस्तदग्रे समवर्ततांधि, मनसो रेत: प्रथमं यदासीत् ।

सतो बन्धु मसति निरविन्दन्, हृदि प्रतीष्या कवयो मनीषा ॥४ ॥

The great worldly desire comes from the minute, invisible, unworldly seed of mind, in the same way 'sat' come from 'asat'. The yogis with far reaching intelligence have recognised this fact after complete thinking in the mind and thorough scanning in their hearts.

A smallest molecular disturbance or a speck of vibration (or smallest bīja) started the chain reaction in rare gas clouds, which started creation. Thus the reason is smallest force or matter called 'kāma' or desire. First action was from 'mana' i.e. minutest.

तिरश्चीनो विततो रश्मिरेषाम्, अघ: स्विदासी दु परिस्विदासीत् ।

रेतोधा आसन् महिमान आसन्, स्वधा अवस्तात् प्रयति:, परस्तात् ॥५ ॥

Strands or rays scattered out; were they oblique or downward or upward ? They became germ holders and became mighty. Those who tried to keep themselves aloof (स्वान्धारयतिइतिस्वधा) remained small or those who surrendered themselves (स्वान् दधाति इति स्वधा) became inferior; while those who tried hard became superior.

'Raśmi' means strands or rays. 'Ābhu' (gas drops) emited light rays in all directions. They also scattered matter in strands which are seen in spiral nebulae. Our galaxy also has two spirial 'arms'. In

space there is no real direction, hence up, down or oblique has no meaning. Thus the strands were in all possible direction, or in spiral shape with changing directions.

Out of the matter in the strands, some lump came together to become germ holder. Those which remained aloof remained small - named 'svadhā'. Some bodies surrendered themselves to others, these are also 'svadhā'. Other bodies absorbed more and more matter and smaller bodies by gavitational attraction, and became enormous. These are 'Prayati'.

This is similar to Nebular Hypothesis of Laplace corrected by Weizasackr.

को अद्धा वेद क इह प्रवोचत् कुत अजाता कुत इयं विसृष्टिः ।
अर्वाग् देवा अस्य विसर्जनेनाऽथा को वेद यत आबभूव ॥६ ॥

Who really knows ? Who in this world can confidently give a talk on origin of this great universe ? The gods are subsequent to the creation of the universe; then who knows from what this universe originated ?

इयं विसृष्टि यत आबभूव, यदि वा दधे यदि वा न ।
यो अस्याध्यक्षः परमे व्योमन्, सो अंग वेद यदि वा न वेद ॥७ ॥

Does He, from whom this universe arose, support it or not ? Does He, who is the highest authority of the universe and who is in super space, know it definitely or not ?

Universe is supported by its creator. Thus living beings are supported by earth, the creator. Earth is supported by its creator Sun, expressed in worship of earth—

'देवी त्वया धृता लोका, देवि त्वं विष्णुना धृता'

Solar system is itself supported by its creator, the galaxy. The chain is endless and we don't know the answer. It is doubtful whether the power holding galaxy is the highest power, or is there any power still superior to it.

(3) Puruṣa Sūkta of Ṛk veda (1-90/13-14) imagines universe as a grand human being whose limbs are different parts of the universe.

चन्द्रमा मनसो जातः चक्षोः सूर्यो अजायत् ।
मुखादिन्द्रश्चाग्निश्च प्राणाद् वायु रजायत् ॥१३ ॥

नाभ्या आसीदन्तरिक्ष शीर्ष्णो द्यौः समवर्तत ।
पद्भ्यां भूमिर्दिशः श्रोत्रात् तथा लोकानकल्पयन् ॥१४ ॥

Mana - produced moon (i.e. mana)

(Lunar - moon's, lunacy - mental illness)

Eyes - Sun, Feet-Earth

Mouth - Indra and Agni

(Mouth and agni both eat or consume. Mouth and Indra both give verbal direction).

Prāṇa - Vāyu

Nābhi - Antarikśa (nearby space, core of matter)

Head - Sky (open and vacant space)

Ears - Directions

(4) Verse 12 - Light of sun is from nulcear reaction in which 3 out of 4 protons are preserved (amṛta) and 1 proton is converted to energy and comes out as light.

Verse 21 - Seven Lokas - Bhū, bhuvah, Svah, mahah, jana, tapah, satya.

Verses 23-32 : Comment on creation

In the above 20 verses of sūrya siddhānta order of creation has been explained. Now the form of sun mentioned there, is being explained in detail. Everywhere work is done at four levels (Saṅkalpa or decision, planning, execution and result) - as it has been stated in Śrī Nāradīya Purāṇa. Here Pradyumna is padmabhū brahmā (23-24).

Origin of brahmā is sun in form of mahattattva. From Brahmā another sūrya has originated. (25)

Hence two sūrya (suns) are stated. First sūrya (first sun) is the origin of self created Brahmā, from whom creation started through this sun. (26)

Second sun is creator of deva, asura and man. Two forms of sun are in form of sphere and in form of deva. (27)

Sphere of sun is formed of five elements, still it is very bright, so it emits light like a lamp. Its brightness can be felt by touch. (28)

There are 8 parts of sun. Out of that, 4 are energy (fire) and other are having one parts each of remaining four elements. (29)

Since fire (teja) element is dominant in sun, it is called taijas. But 'teja' of divine sun is 100 times that of mars, fire etc. (30)

Similarly in 8 parts of moon, 4 parts are water, hence it has coldness like snow ball. (31) Other parts are one each of remaining 4 elements. This is lighted with sun light only. (32)

Notes : Only one notion borrowed from vedas is correct. First sun was the original fire ball (Hiraṇya garbha) from which Brahmā was created. Second sun is the present sun, like any of the other stars. They are rather called first and second generation of stars.

Hotness of sun or coldness of moon are completely erroneous ideas of middle period or dark ages of knowledge. Temperature of sun at surface is 6000°K and at centre it is almost 20 million K. This cannot be felt by touch. Absolute temperature of fire is 300°C or about 600°K for wood fire. Surface temperature of sun may be called 10 times. Other comparisons have no sense.

Verses 33-35 : Drinking of rays of moon

It is stated in Śānti pāṭha of veda that moon drinks fire with its first ray or kalā. This is not real but metaphorical. When brightness of moon is less, it looks red, hence, it is said that it drinks fire. (33)

Light (Teja) cannot be drunk. Hence its meaning should be use or enjoyment. For example, Devadatta ate village, means he enjoyed the village. (In sanskṛta 'bhuj' means to eat or to enjoy). (34)

Kalā word has been used to indicate the time of increase or decrease of moon's phase. Otherwise, on new moon day, all rays of moon are lost. Then we cannot talk about increase of kalā. (35)

Verses 36-38 : Nature of moon

Small object is neglected compared to large one. Hence, due to abundance of cold in moon,

its minor heat is not felt. But it is not a fact that moon is totally devoid of heat. In absence of heat, it will be still colder like snow. (36)

Moon is full of water. It contains trees, mountains etc. They have been described as spots on moon. There are many old stories about it. (37)

Maṅgala etc. also are like moon only (lighted by sun). But stars have their own light, according to me. Some persons call them water bodies also. Since long, there is controversy. (38)

Verses 39-41 : Light of stars

Diameter of moon at distance of stars is only 5 vikalā. If stars are pure water bodies, then they cannot be as bright due to sun rays at they are seen. On the other hand, if they are bright like sun, there will not be darkness during night. Hence, stars are neither bright like sun, nor watery like moon. (39-40)

Thus, their light is reflected (aupādhika) like earth's (or like mars's). Planet is seen according to the extent of sunlight falling on it.

Verse 42 - Due to base, colour of fire changes. Similarly sun rays are of one colour only, but it gets different colours after falling on different planets.

Notes : Distance of stars is much larger than assumed here, hence the conclusion. Heat and coldness are same thing - different states of heat energy, which desperses like light at a distance. It is meaningless to talk of their combination. No planet, including earth is composed of mainly

water. Moon is totally waterless. Its spots are mountains and valleys on its surface.

Sun light has all the colours, predominent portion reaching earth is the light visible to human eyes. Actually eyes of living beings on earth have evolved to see only that part of light. Colour of an object is seen because, rays of particular colour are reflected more, remaining are absorbed.

Verses 43-44 : Beings on planets

According to scriptures, pitṛ loka is on back side (invisible side from earth) of moon. Similarly, on other planets also, different life forms exist. (43)

In any loka, living beings formed of that mahābhūta exist, from which the loka has formed. They use only that part of earth in whose contacts they come. (44)

Notes - So far there is no evidence that any living beings exist on any other planet except earth. There may be life forms within dense clouds of venus or in interior almosphere of Jupiter, where heat and organic matter are available. But so far no evidence has been found from samples of those planets.

On earth also the animals of different habitats are not different. A fish, a bird or an animal all have same body composition, it doesn't has excess of water, air or solid. Actually water, air or earth are not element at all - they represent different states of matter - solid, liquid or gas.

Verses 45-47 - Composition of earth.

Situated at centre of universe, earth is formed of five mahābhūtas. At the beginning, it had 4 parts of land (soil) out of 8 parts. (45)

Remaining 4 parts were 1 part each of the 4 other elements. On earth's surface 3 parts are water and one part is land. Thus on one fourth part of the surface only, all men are living. (46-47)

Notes - This composition of 4 parts soil and other parts have no meaning, as understood by modern science. It is mostly rocky and about half interior is hot liquid.

Verses 48-53 : Jambū dvīpa and Meru

Surrounded by salt water ocean, two continents are situated - one is upper and the other is lower continent. (48)

Upper part of the lower continent and upper continent has been called Jaṁbū dvīpa. Southern half of lower continent and 1/12th of north half is called asura bhūmi. This contains rivers and 6 oceans and continents forming boundaries and countries. (44-50)

Jambū dvīpa also is divided into many countries by mountain ranges. Its area has not been stated by Brahmagupta, Sun god (in sūrya siddhānta) or Bhāskarācārya. (51)

In detail descriptions of earth, mountain details are not real (there are contradictions). Hence they are described only by name by learned men. (52)

All texts have described height of meru as 84 yojana. Its width is 256 yojana at surface and 320 yojana below earth.

Notes : (1) According to Jain jyotiṣa texts followed by Purāṇas, earth is regarded as made up of concentric rings of land masses alternatively surrounded by ocean rings. Mount Meru is placed at centre of central island Jambūdvīpa. Meru or Sumeru is north pole and Jambū dvīpa is land

mass upto 23-1/2°N latitude. Hence almost the entire north hemisphere is occupied by Jambū dvīpa. North pole is considered area of deva, and south of equator is asura area. Thus the other six islands are mostly in asura bhūmi. Their order from nor h pole according to purāṇas is given below.

Countries & Oceans	Bhāgawata, Garuḍa, Vāmana Mārkandeya Linga, Kūrma Brahmāṇḍa, Agni Vāyu, Devi and Viṣṇu	Matsya	Varāha	Skanda	Mahābhārata & Padma	Siddhānta śiromani
1.	Jambū	Jambū	Jambū	Jambū	Jambū	Jambū
1a	Salt water (lavana)	lavaṇa	Lavaṇa	Lavaṇa	—	—
2	Plakśa	Śaka	Śaka	Śaka	Śaka	Śaka
2a	Sugar cane Juice (Ikśu)	Milk	Milk	Milk	—	—
3	Śālmala	Kuśa	Kuśa	Puśkara	Kuśa	Śālmala
3a	Wine (Surā)	Ghee	Curd	wine	—	—
4	Kuśa	Krauñca	Krauñca	Kuśa	Krauñca	Kuśa
4a	Ghee (Sarpi)	Ghee	Ghee	curd	—	—
5	Krauñca	Śālmala	Śālmala	Krauñca	Puśkara	Krauñca
5a	Curd (dadhi)	Wine	wine	Ghee	—	—
6	Śaka	Gomeda (ka)	Gomeda	Śālmali	—	gomeda (ka)
6a	Milk (kśira)	Sugar-cane	Sugar cane	Sugar cane	—	—
7	Puśkara	Puśkara	Puśkara	Gomeda	—	—
7a	Fresh water (Swāduda)	Fresh Water	Fresh Water	Fresh Water	—	—

a = Surrounded by an ocean of

Romaka siddhānta names - Jambū, Kuśa, Candra, Śālmala, Plakśa, Gomeda and Puśkara, Yogavāśiṣṭha (3-73/52-8) gives Jambū, Śaka, Kuśa, Śveta, Krauñca, Gomeda and Puṣkara

(2) Dimensions of Meru - Āryabhaṭa gives diameter of meru as 1 yojana, with same height. According to Purāṇas, Meru is 84,000 yojanas high

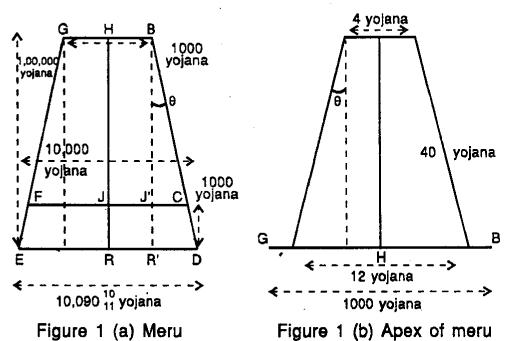
of which 16,000 yojanas lie inside the earth. (Vāyu purāṇa Ch.34, gāthā 1-45 ch.35, Ch. 35 gāthā 11-32, Visṇupurāṇa Aṁśa 2, Ch. 2, gāthā 5-19) Markaṇḍeya ch 54, gāthā 5-19, Matsya purāṇa ch 113, gāthā 4-40).

Abhidhamma koṣa of Vasubandhu, tells meru as 1,60,000 yojanas total height of which 80,000 yojanas are in water and remaining above earth.

Lokaprakāśa of Jain tells it 1,00,000 yojanas high of which 1000 yojanas lie inside the earth and 99000 yojanas outside earth. Jambūdvīpa Prajñapti adds that it has diameter of 10090-10/11 yojanas at its base (inside flat earth) 10,000 yojanas at base on flat earth and 1000 yojanas at top. Tiloya Pannati also states equivalent measures. Meru is made up of frustrum of cones. Its diameter at lowest base is 10090-10/11 yojanas and goes on decreasing uniformly upto 1000 yojanas at a height of 1,00,000 yojanas.

Besides, at centre of the top of meru, a cūlikā (apex or summit) having 12 yojana diameter at its base, 4 yojanas at top and 40 yojanas height.

Figure 1 (a) Meru

Figure 1 (b) Apex of meru

Figre 1(a) and 1(b) show the dimensions of meru and its cūlikā (apex) as per Jain texts.

Angle θ of inclination of meru sides with vertical is (Fig 1a)

$$\text{Tan}\ \theta = \frac{J'C}{BJ'} = \frac{4{,}500}{99{,}000} = \frac{1}{22}$$

$$\text{Also} \qquad \tan\theta = \frac{R'D}{BR'} = \frac{50{,}45\ \frac{15}{11}}{10{,}0000} = \frac{1}{22}$$

yoijanas

Angle θ' for apex from fig. 1 b is

$$\tan\theta' = \frac{4}{40} = \frac{1}{10}$$

Thus $\theta \neq \theta'$

Thus cūlikā is not having same slope as remaining part of meru.

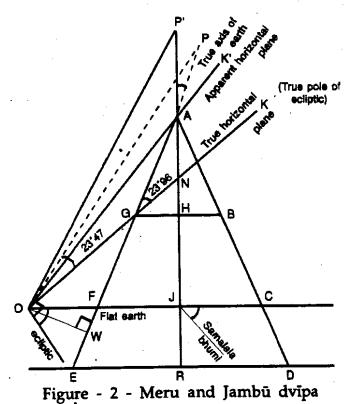
(3) Astronomical Model of Meru -

In figure 2, let OFJC be the plane of flat earth and FC the diameter of meru on it. ED and GB denote the diameter of meru at its lowest base, depressed inside flat earth and its top respectively. RJH is axis of meru. Join E, F, G, and DCB and extend them, till they meet at A on the extended axis of meru.

GB = Diameter of meru at its top = 1000 y

(y = Yojana)

FC = Diameter on meru of flat earth=10,000 y

Figure - 2 - Meru and Jambū dvīpa

ED = Diameter of meru at its lowest base depressed inside the flat earth = 10090-10/11 y

HJ=Height of meru above flat earth=99,000 y

JR = Depth of meru inside the flat earth = 1,000 y.

Now in ΔAFC, GB || FC

Hence $\frac{AJ}{AH} = \frac{FC}{GB}$

or $\frac{AH + 99{,}000}{AH} = \frac{10{,}000}{1000}$ (as AJ = AH+HJ)

or AH = 11000y - - - (1)

Similarly in ΔAED, GB || ED

So $\frac{AR}{AH} = \frac{ED}{GB}$ but AR = AH + HJ + JR

= 11000 + 99000 + 1000 = 111000y

So $\frac{111000}{11000} = \frac{ED}{1000}$

or ED = 10090-10/11 y as given

Thus AED is the approximate cone of meru. Traditional mount meru GEDB is frustrum of that cone. Cūlikā cannot be explained in this scheme.

Now let us assume -

(i) That the observer is situated at O, lying at circumference of Jambūdvīpa whose radius of 50,000 y.

(ii) that OGK represents the true horizontal plane of observer and it meets the direction of earth's axis at G such that P lies at the true celestial north pole and OW represents a plane parallel to the equatorial plane.

(iii) that OAK′ is the apparent horizontal plane of the observer

(iv) that P′ is chosen such that its apparent altitude ∠P′OK′ is equal to ∠PGK (angle between axis of earth and true horizontal plane OGK).

Now join P′ with A, the point of intersection of apparent horizontal plane with axis of earth. Extend P′ A till it meets perpendicularly the plane OFJC at J. The plane OFJC is inclined to equatorial plane at ∠ FOW which is equal to ∠ FAJ = angle between their perpendiculars.

Imaginary locus of revolution of P around P′ is projected on flat earth as locus of F revolving round J. This produces the cone AFC. This is cut at G by plane GHB, parallel to flat earth. True horizontal plane OGK meets the axis of meru at N.

Earth is concentric ring of landmasses, alternately surrounded by ocean rings with meru at centre. Hence, radius of Jambūdvīpa = OJ = 50,000y

Now in ΔNOJ, $GH \parallel OJ$

So $\dfrac{NH}{NJ} = \dfrac{GH}{OJ}$

But GH = 1/2 GB = 500y and NJ = NH + HJ, and HJ = 99,000y.

Hence $\dfrac{NH}{NH + 99000} = \dfrac{500}{50,000}$

or NH = 1000y

Also we have, JR = 1000y given

Thus height NH is preserved in terms of JR (depression of meru inside flat earth, and diameter ED was theoretically generated. Various angles are

$\angle OAJ = \tan^{-1} \dfrac{OJ}{AJ} = \tan^{-1} \dfrac{50000}{110000} = 24^\circ.45$

$\angle FAJ = \tan^{-1} \dfrac{FJ}{AJ} = \tan^{-1} \dfrac{5000}{110000} = 2^\circ.61$

$\angle AOJ = \tan^{-1} \dfrac{AJ}{OJ} = \tan^{-1} \dfrac{110000}{50000} = 65^\circ.55$

$\angle NOJ = \tan^{-1} \dfrac{NJ}{OJ} = \tan^{-1} \dfrac{100000}{50000} = 63^\circ.43$

And

$\angle ACG = \angle AOJ - \angle NOJ = 2^\circ.12$

$\angle OAF = \angle OAJ - \angle FAJ = 21^\circ.84$

$\angle PGK = \angle OAF + \angle ACG = 23^\circ.96$

By assumption, $\angle P'OK' = 23^\circ.96$, $\angle P'AK' = \angle OAJ = 24^\circ.45$

$\angle P' = \angle P'AK' - P'OK' = 0^\circ.49$.

This is almost equal to $LP \approx 0^\circ.49$

Hence ∠ POK = ∠ PGK - ∠P = 23°.47 = 23°.5 approx. - - (2)

Thus true altitude of celestial north pole is 23°.5 which is almost same as latitude of Ujjain (23°.90) It is exactly equal to the inclination of earth's equator with ecliptic 23°28′ instead of 23°27′ at present.

In this figure, obliquity of ecliptic ∠ PGK = 23°.96. But at maximum north krānti sun is overhead at latitude 23°.47 due to error in actual shape of earth.

Motion of sun - Terrestrial colatitude of O is OF as

F is north extremity of earth's axis

OF = OJ - FJ = 45000 y = 720 Y

where $Y = \frac{500}{8}$ y (Tiloya pannati units)

Let δm be maximum declination of sun and therefore

$\phi = \delta m$ is latitude of observer at O.

$90° - \delta m = 720$ Y - - - (3)

Sun moves from inner most maṇḍala (summer solstice day) upto outer most maṇḍala (winter solstice day) over a distance of 510Y and vice versa.

Hence $2 \delta m = 510$ Y - - - (4)

Solving (3) and (4), we have

$\delta m = 23°.54 = 23°.5$ approx - - - (5)

= Latitude of observer at O - - - from (2)

Conclusion : (1) Flat earth OFJC is inclined to equatorial plane at angle FOW = ∠FAJ = 2°.61

(ii) Circumference of Jambū dvīpa coincides with parallel of maximum north declination of sun. Axis of meru is instantaneously taken such that OJ = 50000 y wherever O lies on periphery (23°5 N). Earth's true axis passes through hypotenus of cone by meru, hence true radius of Jambū dvıpa = 45000 y.

(iii) Meru is an astronomical model for explaining attitude of celestial north pole as shown above. This view is further supported by the fact that the famous Kutuba mīnāra in Delhi is situated at 28°31′28″ north latitude and inclined at an angle of 5°1′28″ to the vertical to south, i.e. perpendicular to 23°30′ north latitude. Srī Kedar Nath Prabhakar explains in Varāhamihira Memorial volume that Kutubmīnāra is a model of meru at scale of 1 yard = 1000 yojanas. Inclination of Kutubamināra is equal to inclination of lunar orbit to the ecliptic. Thus its location is maixmum north latitude of moon. Hence kings of Delhi were called Candravamśī.

(4) Source of height of meru mentioned here is not known. Candraśekhara has not mentioned as to which śāstra he has quoted. However, 84 yojana a figure appears to be 84,000 yojana of purāṇas. he has ommitted thousands because it will far exceed radius of earth. Even 840 yojana is equal to radius of earth.

Since Jambūdvīpa is from 23°.5 N to 90°N only, the deva bhūmi or north hemisphere is extra land from equator to 23°.5 N latitude. Jambūdvīpa is thus most of Asia (above 23°5 N) and most of north America. Amoung southern land masses, parts of south America and Africa are north of

equator, whch are estimated here 1/12th of north hemisphere.

Oceans of water or sand (desert) serve as natural boundaries between continents. They are further subdivided by natural boundaries of mountains. Continents are called dvīpa, its divisions by mountains are varṣa. Varṣa are further divided into janapada according to political and racial divisions.

Verses 54-81 : Height and distance limit of vision.

Varṣa parvatas (mountains dividing continents) like Himālaya are each 50 yojanas high. Their width also is only 50 yojanas. Thus a related mathematics is being explained. (54)

From this method, we can find the distance of high objects like śaṅku or hills, their visible portions or their limit of vision on earth's surface can be found out. (55)

Take a circle of any size and imagine it to be earth of (21,600) kalā circumference (i.e. 360°). A śaṅku of desired height is placed at its top. Its height is multiplied by 21,600 - - - (56)

. . and divided by circumference of earth. Result will be śaṅku named sanskṛta. This sanskṛta śaṅku is kept at two places. At one place it is multiplied by trijyā and at other place it is added to trijyā. Sum is divided by product at first place. (57)

Result will be utkrama jyā. Its chord will be distance of visible limit is kalā. If it is multiplied by circumference of earth and divided by kalā of circle (21,600) Then (58)

. . result will be distance from which śaṅku can be seen. If utkramajyā is less than 7, another method is followed. It (utkrama jyā) is substracted from trijyā. Square of remainder is again sub-stracted from trijyā. Square root of the remainder will be the chord of utkramajyā. (59)

If utkrama jyā chord is more than its arc, then its value is taken in vikalā. (60)

Example : Height of himālaya is assumed to be a śaṅku of 50 yojanas. (61)

It is considered to be placed on circumference of earth of 5026 yojanas. By the method explained, its sanskṛta śaṅku will be 225 kalā (62)

By calculation at two places, its utkrama jyā is 202. Its arc 1182 is the limit of visibility in kalā. In yojana, it is 275. (Then 50 yojana high Himālaya can be seen from a distance of upto 275 yojanas). (63)

Distance between observer and observed (śaṅku) is multiplied by kalā of a circle (21,600) and divided by circumference of earth (5026). It will be bhuja kalā. (64)

If this bhuja kalā is less than 3 rāśi, then observer can see the śaṅku. Square of bhuja kalā jyā is substracted from square of trijyā. Square root of the remainder will be koṭi jyā. (65)

This koṭi jyā substracted from (3438) trijyā, will be utkramajyā. Then sanskṛta śaṅku is multi-plied by koṭijyā and divided by trijyā. (66)

It gives sphuṭa śaṅku. It is not seen, if it is less than utkramajyā. Its excess height over utkramajyā is seen over the obstructed part. (67)

Utkramajyā substracted from sphuṭa śaṅku is dṛśya jyā (jyā of visible portion). Dṛśya jyā is multiplied by trijyā and divided by jyā of difference between observer and observed. We get unnata jyā. (68)

Arc of this unnatajyā is visible portion of śaṅku over horizon of observer. This is multiplied by circumference (of earth) and divided by liptā of a circle (21,600). (69)

Result is visible portion of śaṅku i.e. dṛśyonnati. This dṛśyonnati is multiplied by trijyā and divided by koṭijyā to get visible portion height in yojanas. (70)

Again utkramajyā is multiplied by circumference of earth and divided by (21600). Result is multiplied by trijyā and divided by koṭijyā. (Result is unseen portion of śaṅku. (71)

Example of Purī : Assume Nīlācala (Puri) located 161 yojana south of Himālaya mountain base as place of observer. (72)

For 161 yojana distance, bhujakalā is 692, utkrama - jyā is 70 and koṭijyā is 3368. (73)

Sanskṛta śaṅku is 215, sphuṭa śaṅku 211, drśyajyā 141 and unnata jyā. (705) - - - (74)

Arc of unnatajyā (710) is unnatakalā, dṛśyonnati 33 and sphuṭa dṛśyonnati is 34 . . (75)

Visible portion is 16 yojana. Hence from Nīlācala, Himalaya will be visible 12° above horizon (arc of 16 yojana). (76)

These mountains are not visible due to obstruction by smoke, dust, frost, or cloud from

such a far distance. But many stars of polar fish are obstructed by Himālaya. (77)

That gives indication that himalaya is situated. But this doesn't really prove the height, length and width of himālaya, Nakśatras can be obstructed by cloud also (even if himalaya doesn't reach that height). Real height of himālaya is only 1/40 part of this value (50 yojanas). (78)

Thus, if himālaya is assumed to be a śaṅku of height 20,000 hands (30,000 ft.), then its sanskṛta śaṅku is 322 vikalā, whose utkrama jyā also is 322 vikalā (i.e. 5 kalā 22 vikalā). (79)

Its arc will be 192 kalā, hence visible limit is 89 kosa (44-1/2 yojana). Hence himālaya can be seen only from a distance upto 89 koṣa from its base. (80)

Kulācala (mountains dividing varṣa into janapadas) Mahendra parvata is only 1/8 as high as Himālaya. This can be seen from a distance upto 30 kosa from its base. (81).

Notes (1)

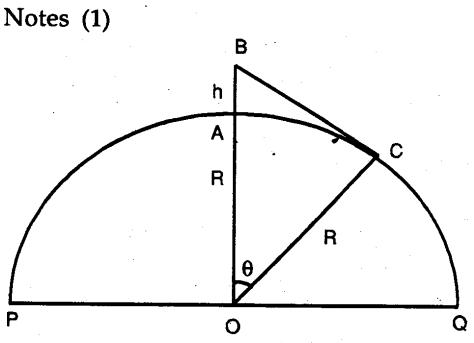



Figure 3a - Visible distance limit from height

Figure 3a shows half portion of earth circle PAQ with centre at O and radius OA = OC = R = 3438 kalā.

AB is śanku of height h which is visible upto point C on earth's surface. Then BC is tangent to circle and OC is perpendicular to it. Angular distance C from base A of śanku is $\angle AOC = \theta$

In $\Delta OBC, \quad \angle C = 90°, \quad \angle O = \theta$

$OB = OA + AB = R + h, \; OC = R$

$$\cos\theta = \frac{OC}{OB} = \frac{R}{R + h}$$

or, $h \cos\theta = R\,(1 - \cos\theta) =$ Utkramajyā of θ

or, Utkramjyā of θ

$$= h \cos\theta = \frac{hR}{R + h} \quad \text{- - - (1)}$$

Since R is expressed in kalā as well as utkramajyā, h is converted to kalā. If height in yojana is H then

$$\frac{\text{H yojana}}{\text{h kalā}} = \frac{\text{circumference of earth in yojana}}{\text{Circumference kalā (21,600)}}$$

$$\text{or } h = \frac{H \times 21{,}600}{\text{Circumference yojana}} \quad \text{- - - (2)}$$

Thus the steps in calculating AC are -

(1) Conversion of H yojana into h kalā by (2)

(2) Multiplying h by R and dividing by h+R to find utkramajyā by (1).

(3) From utkramjyā of θ we find θ from its chart

(4) Now $\frac{AC}{\theta} = \frac{\text{Circumference yojana}}{21{,}600}$

gives AC in yojanas.

When utkramajyā is less than 7, there is no smaller angle divsion for which its value is known. Then we substracted it from trijyā R to get

R - utkramajyā = R - R (1 - cos θ) = R cos θ

Then R^2 - (R cos $\theta)^2$ = $R^2 \sin^2 \theta$

Taking its square root we get R sin θ which is almost equal to kalā θ for every small angle.

(2) Bhujakalā is obviously less than 90° (3 rāśi) as $\angle OCA = 90°$ for point B of śaṅku of any height to be visible.

We find the minimum height of śaṅku visible from angular distance by the reverse process.

$$R \cos \theta = \sqrt{R^2 - (R \sin \theta)^2}$$

R - R cos θ = utkrama jyā of θ = X (suppose)

This should be equal to h cos θ = sphuṭa śaṅku

$$= \frac{h \times \text{koṭijyā}}{\text{Trijyā}}$$

i.e. we can see the minimum height h = X / cosθ

If height L is less than that we cannot see. If L is more than that then h portion is obstructed and we see the remaining portion.
L-h = L - X/cosθ - - - (3)

Sphuṭa sanku for L is L cosθ . Then dṛśyajyā = L cosθ - x .... (4)

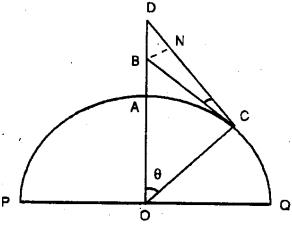



Figure 3 b - Visible from height from distance

In fig 3(b) AD = L,

AB = h is obstructed from C and we see BD = L-h.

However angular elevation of BD from horizon CB is equal to BN which is perpendicular to CD. BN is almost equal to arc at B.

$\angle$BDN = $\angle$OBC approximately as BD is small = $90° - \theta$

Hence BN = BD sin BDN = BD Sin $(90° - \theta)$ = (L-h) $\cos\theta$ = L $\cos\theta$ - X = dṛśya jyā as found in (4).

Visible height L-h is given by dṛśyajyā/$\cos\theta$ as given in formula. Obstructed portion AB is found as reverse process of the limiting distance.

(3) Since radius of Jambūdvīpa is assumed to be 50,000 yojanas, the varśa parvatas, dividing it into sectors should be equal to 50,000 yojanas. Then they will extend from ocean to ocean if they completely divide the continents. Thus himālaya, theoretically is assumed to touch ocean in both ends. See Kumāra sambhava of Kālidāsa, Ist verse-

अस्त्युत्तरस्यां दिशि देवतात्मा हिमालयो नाम नगाधिराजः ।
पूर्वापरौ तोयनिधीव ग्राह्यः स्थितः पृथिव्या इव मानदण्डः ॥

i.e. Himālaya touches ocean both in east and west and as such it is a scale to measure earth, in the sense that it is a standard for division of continents. We may assume that the extended direction of himālaya or other varṣa parvatas in both direction upto ocean are the dividing lines.

Thus height of mountains is not their height from earth's surface, it is their distance from north

pole along bifurcating lines of continents. Hence all are same, if continents are circles.

Even the reduced figure of 50 yojana height of himālaya is too much as felt by calculations here. Its highest peak is 29000 feet which has been taken as 30,000 ft. or 20,000 hands approximately here. This is almost directly north from Purī.

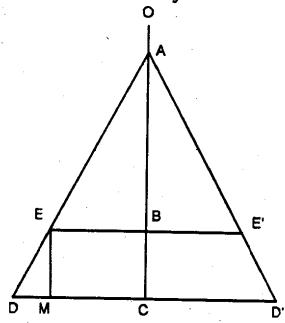



Figure 4 - Meru in purana

(4) Meru of 84 or 84000 yojana height of purāṇas is shown in figure 4.

OC is complete height of meru = 84 yojana. Its base at surface is EBE′ = 256 yojanas. Its base below earth is DCD′ = 320 yojanas

If E M is perpendicular on DC, then EM = BC is height below surface which is 16 yojana according to purāṇas.

DM = DC - EB = 1/2 X 320 - 1/2 X 256 = 160 - 128 = 32 yojana

In similar Triangles EDM and AEB,

$$\frac{AB}{EM} = \frac{EB}{DM} \quad \text{or} \quad \frac{AB}{16} = \frac{128}{32}$$

or AB = 64 yojanas

Thus AB + BC = 64 + 16 = 80 yojanas only and 4 yojana extra height OA remains over tip of cone. Reason for this model is not understood.

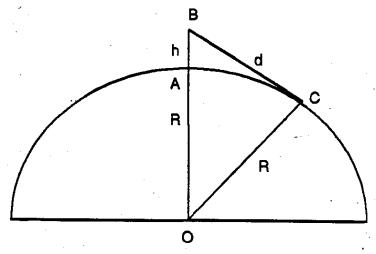



Figure 5 - Visibility limit

(5) Simpler formula for visible distance -

Much simpler formula can be derived without use of trigonometry. AC = BC approx = d. in Figure 5

Then $OB^2 = OC^2 + BC^2$

or $(h+R)^2 = R^2 + d^2$

or $d^2 = h^2+2\ hR = h\ (h+2R)$

If h is very small compared to R, then $h + 2R \approx 2R$

and $d^2 = 2\ Rh$ or $d = \sqrt{2Rh}$

Verses 82-87 : Meru and Kumeru

Meru has 84 yojana height and 256 yojana extent width. Then the gods situated at end of Amarāvatī (last portion of Meru) will not have night, even when sun is in south krānti. (82)

Becuase its lambāmśa will be 11°30′ (aksāmśa 78°30′) and dṛṣṭāmśa (vision limit) will be 25° (for 84 yojana height). On this arc, sun at southernmost

position also will be seen 13° above horizon. Thus it cannot be said that gods do not see sun for 6 months. (83)

On equinox day (when sun has 0° krānti), then in part eclipse of moon, shadow of meru within earth's shadow will be 4 kalā more. Hence, height of meru should be less than Himālaya. (84-85)

Sailors in ocean, see dhruva (pole star) above their head, but don't see meru. This doesn't mean that meru doesn't exist. (86)

Meru is abode of gods and Kumeru of asuras (demon) Due to their māyā (illusion), they are not visible like gods and demons, meru and kumeru also can be seen only by their worship and grace. (87)

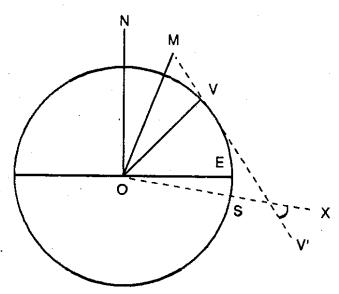



Figure 6 - Sun from meru

Notes : N is north pole, M is top of Meru. In figure (6)

∠NOM = angle of half extent of meru

= 128 yojana = 11°30′

Height of M is 84 yojana (Meru is taken flat top here, not conical)

Hence $\angle MOV = 25°$

S is southern most position of sun, E is equator.

Then OSX is direction of sun. SOE = 23-1/2°

$\angle EOV = \angle NOE - \angle NOM - \angle MOV$

$= 90° - 11°30' - 25° = 90° - 36°30'$

Then $\angle SOV = \angle SOE + \angle EOV = 23\text{-}1/2° + 90° - 36°30' = 90° - 13°$

Hence $\angle VSO = \angle XOV' = 90° - (90° - 13°) = 13°$. Thus sun will be 13° above horizon from M or V.

(2) $$\frac{\text{Radius of shadow}}{\text{Radius of earth}} = \frac{\text{Meru shadow}}{\text{Meru height}}$$

or Meru shaodw $= \dfrac{84 \times 33 \text{ Kalā}}{800} = 4$ Kalā approx.

This can be seen on moon only in part eclipse against bright portion of moon. Neither shadow of meru is seen, nor it is visible to sailors, because it is not a mountain at all. It is a projection of earth's axis.

Verses 88-91 : Location of India.

(Southern) Coast of Jambūdvīpa is at average distance of 270 yojanas from equator, but this distance is not uniform every where. (88)

Kumārī antarīpa (Kanyā Kumārī) is in nairṛtya (south west) direction from Nīlācala (Purī). Dvārakā is in vāyavya angle (north west) from Kumārī. Again Purī is east from Dwārakā. All the three places are on coast. (89)

Thus the triangle formed by these three places is in southern part of India (Bhārata varṣa). Again by seeing the difference (distance) between east, west and central portions, it is clear that (90) - - -

Bhārata is shaped like a conch shell (Śaṅkha) Therefore, Jambūdvīpa is not circular. Similarly oceans also are not circular (as stated in purāṇas) (91)

Verses 92-98 : Variations in oceans

Oceans also are of diffrent shapes and areas. They have been named as eastern, western, southern and northern. They have been named differently at different places. (92)

In lavaṇa (salt water) ocean there are more than hundred islands. It is learnt that more than 100 crore people live in earth. (93)

In 1 kalpa, earth sphere increases by 1 yojana in all directions, this principle has been well established. This is average increase, and actual increase differs from place to place. (94)

In many directions soil is eroded to different extent due to wind, ocean, rains and river flow. (95)

Due to unequal erosion, sometimes small and large islands submerge in water, sometimes they arise from ocean. Their shapes also change. (96)

Still this change of land mass or ocean doesn't change the spherical shape of earth. There is only change of place between land and sea. (97)

Salt water ocean is partly mixed with milk and curd at some places. Oceans of pure milk or curd stated in purāṇas don't exist anywhere. (98)

Notes : Population of earth at present is above 500 crores. There is no evidence of increase of 1 yojana radius of earth in a kalpa, though miniute quantity of meteors are added. Due to change in salt content, depth, currents or waves and im-

purities there is slight change in colour of rivers or oceans. To highlight those changes, they have been named as red sea, blue nīle, ocean of milk etc.

Verses 99-108 : Discussion on meru

There have been many changes from the creation. Earth as described in old siddhāntas and the situation at present time are different (99)

(Ādi yāmala) The meru mountain at centre of earth on one of whose hills, there are 33 crore gods as described in purāṇas, is only heard now, but not seen anywhere. (100)

(Sūrya siddhānta) : From centre of earth two meru mountains extend in opposite directions. They are golden and contain many jewels (101) On north meru, Indra and other gods and sages reside. On lower (south) meru or kumeru, demons (asuras), anti to gods reside (102) Around both merus, ocean has surrounded earth along its whole circumference. They divide the regions of gods and demons very distinctly. (103)

Devas live on sumeru and asuras on kumeru. The greatest circumference at equal distance from both is called equator (or circle with zero latitude). (104)

The circles parallel to equator, become successively smaller in direction of either meru. They are called akśāmśa (latitude) lines. The line between two merus cutting these circles is called deśāntara (longitude) line. (105)

Thus the lines which have been assumed on celestial sphere, have been assumed on earth's surface also. The place on earth, where line from

earth centre to celestial line meets, is the location of that line. (106)

As there are 'cakras' (nerves centre in energy body) in human body (6 cakra in spine and one at top of head), in the universe as grand body of god also, seven lokas are situated in its spine like 'pravaha' wind. (107)

Meru in sky has been supposed to be support of all lokas (worlds) by scriputres. It is said to be very windy. Its base is within water and it has 16 tops. (108)

Verses 109-112 - Hints on geography -

Earth is called 'Hiraṇmayī (golden, or full of energy or gems). It is viewed as a spherical body like bowl. Its first cover is land or soil. Beyond that, successive layers exist, each being 10 times the previous. (109)

Next to land is the layer of water, and at last there is prakṛti (energy field). Countries (seas, towns, etc are generally described in bhūgola (geography) only. (110)

Hence only a brief hint of that subject has been given here in a text of astronomy. Later on, in gola chapter, their real measures will be stated. (111)

For satisfying curiosity, and identifying locations of various places on earth, many verses are quoted from the standard text 'siddhānta śiromaṇi' (by Bhāskara II). (112)

Verses 113-152 : Geography quoted from Bhāskara - At the centre of earth, on equator, is located Laṅkā. East from Lankā (90°E) is Yamakoṭi

pattana, at same distance in west in Romaka pattana. Just below Lankā (i.e. at 180°) is Siddhapura. In north of Lankā (at 90°) is sumeru (north pole) and south is south pole. (113)

All these six places are at 1 quadrant from each other. Sura (gods) and siddha (sage or adepts) live in meru. At Aurva or Baḍavānala' (south pole - kumeru), asuras live. (114)

Wherever a man is located on earth, he thinks earth as below and himself as above. He considers places at 90° distance as oblique. (115)

As we see reflection in water vertically downwards, men on earth at 180° from each other see themselves as upwards and the other as head down wards. But as we are steady at our position, the persons at oblique and down places also sit like wise. (116)

North of salt water ocean is Jambūdvīpa (the great jambū continent), which is half of the earth (or half the land mass). Opposite to it in south hemesphere there are salt water and other oceans and 6 dvīpas (continents) (117)

First lies lavaṇa (salt water) and then kśīra (milk) ocean. From milk ocean moon and Lakśmī arose. Vāsudeva himself resides there, whose body is the whole universe. All the gods inḍluding Brahmā (creator) worship his feet. (118)

After milk ocean, lie curd (dadhi), ghee (butter), sugarcane juice, wine and lastly fresh water oceans at increasing distances. Within this fresh waters ocean lies Baḍvānala (fire within seas) or southpole. Deep places on earth are called Pātāla. (119)

In pātāla loka asura and serpents (sarpa) live. That is lighted by maṇi (jewel) in head of sarpas. Siddhas roam about with their jewel of virtues. Then the gold ornaments of their bodies shine. Beautiful bodies of siddha and divine ladies are always shining with brightness of ornaments. (120)

All seven oceans are circular (ring shaped). There are six continents interspersed between them. Continents are named as Śaka, Śālmalī, Kuśa, Krauñca, Gomedaka and Puṣkara. Another name for gomedaka is plakśa. (121)

North from Lankā is Himālaya, then Hemakūṭa and Niṣadha mountains. Each extends upto ocean. Again from Siddhapura, northwards lie Śṛngavāna, Śukla and Nīla mountains. The countries between two mountains are called 'droṇī' (trough) countries. (122)

North from Laṅkā, first comes Bhāratavarṣa, then Kinnar deśa, then Harivarṣa. North from Siddhapura are kuru, Hiraṇmaya and Ramyaka verṣas in order. (123)

North from Yamakoṭi pattana is Mālyavāna and North from Romaka is Gandhamādana mountains. Both mountains extend upto Nīla and Niṣadha mountains respectively. Region between Nīla and Niṣadha is called Ilāvṛtta varṣa. (124)

Region between Mālyavāna and ocean is called Bhadraturaga. Geographers call the land between Gandha-mādana and ocean as Ketumālaka varṣa. (125)

Ilāvṛtta varṣa is surrounded on four sides by Niṣadha, Nīla, Gandhamādana and Mālyavāna. In this large country devas live happily in their

houses. Soil of this land is beautiful and strange and contains gold mines. (126)

At its centre lies Meru mountain. Its soil is golden and according to Purāṇa knowers, it is abode of devas. This mountain itself is the stalk of lotus like earth from which Brahmā had taken birth. (127)

There are four viṣkumbha (branch) mountains of Meru - Mandara (mandarācala), Sugandha (Gandha mādana) Vipula and Supārśva. They abound in, respectively Kadamba (Ficus Kadamba), Jambū (Eugenia jambolana), Vaṭa (Ficus Bengalensis) and Pipala (Ficus religiosa). (128)

From the fall of juice of ripe jambū fruits, one river of jambū juice has formed. In contact of this water, soil turned to gold. Deva and siddha like its water so much that they leave even drinking of nectar also. (129)

Below the four branch mountains of Meru, there are four forest regions. Caitraratha is very picturesque, In Nandana forest apsarā (deva girls, literally, ladies moving in water) play. Dhṛti forest gives 'dhairya' (patience) to gods. Vaibhrāja is very interesting. (139)

There are four lakes in these four forests - Aruṇa, Mānasa, Mahāhṛada and Śveta - dala. When deva women are tired, they enjoy in these lakes. (131)

There are three top of Meru - each is formed of jewels and gold. On the three tops are the pura (towns) of Viṣṇu, Brahmā and Śiva. Below these tops lie the towns of Indra, (kings of gods), Agni (fire god), yama (Death god - Jamaśida of Persians),

rākśasa (a race of man in direction of Africa - south west from India), Varuṇa (Tāj or Arab race), Pavaṇa (north west direction), Kubera (wealth god) and Śiva. (These are in eight directions from India starting from east clockwise - and called lord of those directions). (132)

From feet of Viṣṇu, Gaṅgā river first landed on Meru. From there, it was divided into four parts, and proceeded below four branch mountains through their four lakes. (133)

Śīta gaṅgā in Bhadrāśva varṣa, Alakanandā (Gaṅgā) in Bhāratavarṣa, Vakśu gaṅgā in Ketumāla and Bhadra gaṅgā in north Kuruvarṣa. (134)

Many sinners are purified by Gaṅgā - by listening, desire to see, seeing, touching, bath, drinking water, devotion, memory or praising. (135)

Just by proceeding towards Gaṅgā, the bondage of pitṛ ṛṇa (debt of forefathers) is broken. Man jumps with joy as soon as he reaches banks of Gaṅgā and wins over agents of Yama (death). By entering water, he is free from hell and enters heaven. (136)

There are many parts of India - Indra, Kaśeru, Tāmraparṇa, Gabhastimān, Kumārikā, Nāga, Saumya, Vāruṇa and Gāndharva. (137)

Varṇa order (4 codes of conduct) exists only in Kumārikā khaṇḍa. There are 7 kulācala (mountains dividing the Kula or races of man) - Mahendra, Śukti, Malaya, Ṛkśa, Pāriyātra, Sahya and Vindhya. (138)

The place of zero latitude is called equator. Region near equator is called bhū loka, north region is called 'bhuvar loka'. Meru is 'Svah' loka. Above

that lie the lokas of Mahah, Jana, Tapa and satya. (139)

When it is sunrise at Laṅkā, it is midday at Yamakoṭi, sunset at Siddhapura and mid night at Romaka. (140)

Rising direction of sun is called east and setting direction is west. The directions between east and west are north and south on either sides. Four angle directions are bisectors of the angles between four directions. Meru is north from all. (141)

Yamakoṭi pattana is 1/4th circumference (90°) east from Ujjayinī. But Ujjayinī is not exact west from Yamakoṭi. Laṅkā is exact west from Yamakoṭi. (142)

It appears thus, that a place may be east from first place, but first place may not be west from that. Only on equator, this strange thing doesn't happen. (143)

On equator, a man sees both poles, sumeru and kumeru on his horizon towards north and south. Krānti vṛtta (ecliptic) is above his head like a water instrument (for measuring time). (144)

As a man proceeds north from equator, krānti vṛtta progressively dips towards south, and north pole rises above horizon. Altitude of north pole above horizon, or dip of ecliptic below zenith is exactly equal to the akśāmśa (latitude) of the place) north of equator. (145)

Distance from equator in yojana is multiplied by 360 and divided by circumference of earth. It gives akśāmśa of the place. By doing reverse, we

can find the yojana distance of equator from akśāmśa. (146)

As the people at sumeru, see north pole vertically above, similarly at kumeru, south pole is seen at zenith. In norțh pole ecliptic is seen moving in left (clockwise) direction. At south pole it looks moving in right direction. (147)

Due to matter expelled from earth (in volcanos etc.) radius of earth increases by 1 yojana in all directions in a kalpa of Brahmā. In night of Brahmā, this increase is nullified. (148)

Daily death of living beings is called daily pralaya. When whole creation merges in body of Brahmā after end of his day (kalpa), it is called Brāhma pralaya. (149)

After Brahmā completes his life period, whole creation merges in nature (Prakṛti - Energy field). All distinction between matter ceases. Mīmānsakas (thinker - one of 6 branches of philosophy) call it Prákṛtika pralaya. Again, at the time of creation, prakṛti creates distinction among particles. (150)

Yogīs burn their good work and sins both in their fire of realisation and engage their mind wholly in god. They are totally merged in god and don't desire to come into world again. That is called ātyantika pralaya. Thus there are four types of pralaya - daily, brāhma, prākṛtika and ātyantika. (151)

Thus these verses have stated locations of earth, mountains, deva, dānava, mānava, nakśatra, graha and their orbits, lokas mahas, jana tapa etc one above other. They are all located in the interior of universe (stomach like cosmic egg - brahmāṇḍa). (152)

Notes : (1) There is great confusion about lokas, dvīpas etc. and purāṇas are not consistant in their description. They do not describe the present cofiguration of continents. However Jain astronomy indicates that Jambūdvipa is the earth north of 23-1/2°N latitude and Meru is axis of earth's rotation. In that context meru and kumeru are very clear as north and south poles of sky. However, on earth, the Meru mountain is considered to be 'Pāmīra' plateau from where 4 great mountain ranges like Himālayas spread.

Thus Jambūdvīpa means most of Asia and complete Europe and North America except Mexico south portions (Central America). Other dvīpas are other peripheral regions of Jambūdvīpa or southern continents like Australia, South America and Africa. It doesn't include Antarcticā (south polar continent) which is said to contain 'badavānala' i.e. ocean fire - which may refer to land mass below cold ice or water.

Seven lokas in tantra are seven levels of man's existence or seven levels of physical, shadow or energy body etc. However, Brahmavaivarta purāṇa tells 7 lokas in svarga (i.e. north hemisphere) or Jambūdvipa -

Bhu, bhuvar, Svar (Kasmīra or Tibbet), Mahar, Tapa (Steppees) and satyaloka (snowbound polar regions) These are northwards from equator.

At base of meru (down ward portion kumeru), there is town of Ananta (Śeṣa nāga) who holds the earth. Ananta is 'Antarctica' and the town in that direction is Tiru - Anantapura in kerala which

is almost southern most town of India. South of it there is no land mass except Antarctica.

In lower regions (south hemisphere) or lower from Pāmīra, there are seven pātāla -

Atala, vitala, sutala, Talātala, Mahātala, Pātāla and Rasātala. (lowest region).

From Pāmīra, rasātala corresponds to Amajon river in Brazil. S. Muzaffar Ali has opined that tala means incline, which is also indicated in purāṇas. This has become 'Iklima' in Hebrew and Arabic, clime in Greek and climaṭe in English. If the regions are successively south of equator they are definitely climatic regions. However, viewed from Pāmīra, they start after end of Eurasian continent. Thus according to Bhagavaddatta, Talātala is north African region where a town Til-at-tala (amarnā) still exists. Atala is Italy and corresponds to old Atlantis contient and present Atlantic ocean.

8 Towns of Indra etc. are on eight sides of Meru (or Pāmīra). Varāhamihira opines that these directions are taken from central poriton of India, Ujjain. Thus Indra is east (Burma and Thailand - Irāvatī river). Agni region is Australia or Indonesia. Yama region cannot be in exact south hence it may mean Australia and New Zealand. South west (Nairṛtya) is Africa, West varuṇa is Arab land. Bhagavaddatta also opines that old name of Arab race is Tāj, derived from Varuṇa (yādas). Northwest is marut (windy areas of central asia). North is Kubera i.e. Tibbet and its north. North east is China, Japan (īśa) - of Maheśa. Muzaffar Ali opines Hwāng-Ho as Mahā gaṅgā.

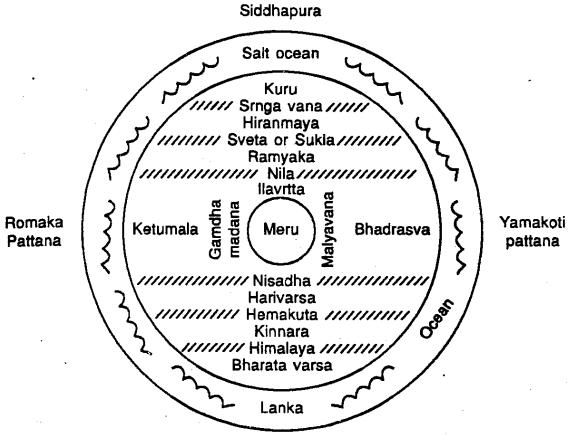



Figure 7 - Jambūdvīpa of Purāṇas

Location of cardinal towns : Lankā is south of Ujjain at equator by definition. The country named Lankā is very close to it, but slightly towards east and north. Hence Lankā has been assumed point on equator on longitude of Ujjain.

Yamakoṭi and Romaka have been described as pattanas (i.e. ports). Hence they must be coastal towns 90° east and same longitude west of Ujjain 75°54″ E. Thus Romaka pattana could be Conakry of Guinea in west Africa which is a port at that longitude and close to equator. 'Conakry' like 'Koṇārk' means a port. Yama means south direction and Koṭi is end point of a land mass protruding in sea. That is south western tip of south island of New Zealand or south eastern tip of New Celedonia (Nouma) which is also a port very close to south tropic near equator. Nauru island is almost near equator at that longitude, but it is not a Koṭi, but a coral island. However, it is very close to

Kiribātī islands where huge stone structures are found. At the location of Siddhapura, the town nearest to equator is west of Mexico - Aṭ present near lake Chapale or Guadalajara town. Nearby huge pyramids exist which were constructed to mark the end of east direction as remarked in Vālmīki Rāmāyaṇa (Kiṣkindhā Kāṇḍa).

At angle direction in south west is abode of Rākśasa - 45° west from Ujjain are the great pyramids of Egypt. Neighbouring Libyā was named after Prahlāda son of the first Daitya king Hiraṇya kaśipu, who ruled before gods. North of Libyā are 'Daitya' lands i.e. Deutsch land (Germany) and Dutch (Netherlands). That was approximately 'Atala' loka ruled by Prahlāda (around Italy).

(3) Seven continents : All seven continents are interprated within Eurasia and Africa by various scholars. However, the description of Jambu dvīpa means that it includes most of Eurasia and north America. The diagram in Fig 7 indicates that Jambūdvīpa has following parts -

Asia - From north-Ilāvṛtta - Scandinavia, Siberia, north Russia, Niṣadha is Varkhoyānsk range (varṣa parvata) in Siberia, Urala mountains (north Russia) Kjolen mountains of Sweden may be its west boundary. Then comes Harivarsa (Russian and Mangolian planes). Hemkūṭa is Altai, Nan shan ranges in China. Then China, Tibet area is Kimpurmṣa varṣa, whose mongoloid features have given the name Kinnar or Kimpuruṣa (Are they man?). Then is Bhārata Varṣa North Mexico, i.e. present Mexico, Texas etc are Kuru. Mexico plateau is Śṛngvāna, middle Rockies and Appalachian mountains are Śveta or Śukla (white

mountains). Plains of U.S.A are Hiraṇyaka varṣa ('Red' Indians). North Rockies and Labrador are Nīla mountains. Candian plains are Ramyaka varṣa.

Above Yamakoṭi, there is no land mass. Hence only one mountain has been named, which may be 'Sikhote Alin' range east of Siberia. Thus Bhadrāśva varṣa may be China, Japan, Korea and east Siberia.

Above Romaka pattana also is only one mountain range Gandhamādana. First range is Atlas which continues through Spanish plateau to Alps. Ketumāla may be west Europe and north Africa.

Other dvīpas should start from south perlphery in east or west of Jambūdvipa or south of Mexico.

(2) Kuśa Dvīpa is almost universally accepted as Africa. Egypt was called Kuśa in earlier days. Ethiopians call themselves Kuśa. Muzaffar Ali opines Persia to Isreal as Kuśa including north Africa. Himalayas in Afganistan are called Hindu Kuśa i.e. Indian part of Kuśa. It is by nature grass land or desert.

(3) Śaka Dvīpa is south east Asia which abounds in Teak trees accordig to M. Ali. (Śaka means Teak). It might also include land upto Australia and New Zealand, including Indonesia.

(4) Śālmalī dvipa is identified as south and east Africa. It included Madagaskara - Hariṇa dvīpa of purāṇas or Śaṅkha dvīpa (Zenj of Arab) - Zanjibar i.e. Tanzania coast.

(5) Krauñca dvīpa - It is named on mount Krauñca. Mahābhārata. tells it west from Meru,

(12, 14, 21-5) and in north (6-12). Bṛhat samhitā and Rāmāyaṇa locate it in the north. Kuśa and Krauñca are always mentioned together. It may be north west Europe according to Sri M. Ali and on similar considerạtions, eastern Canada and Greenland. It surrounds ghṛtoda (butter like) sea i.e. icy seas between north west Europe and Canada.

(6) Plakśa dvīpa - It is mentioned as Gomeda in varāha, matsya purāṇas and siddhānta śiromani. It is named after Plakśa or Pākara' tree which is characteristic of warm temperate or mediterranean islands. It is also in central America and Careabian islands. It is identified with fig tree. According to Wilford, the name still persists in Placia, a town in Mysia. There was a Peleasgi race in Cristone or Crotoen near Tyrrhanians in Itlay and Pelagsi who lived on shores of Hellespont. According to Herodotus they all spoke the same language. Sri V.V. Ayer identifies it with Greece and adjoining lands. The old names of America have been lost but it appears to be continued till central America through West Indies and old Atlantis.

(7) Puśkara dvīpa : It has two parts - one has no rain fall, no springs or vegetation. Other part is full of water, lakes. It has a huge circular mountain chain named Citrānśu in eastern half of dvīpa. Western half is surrounded by another circular range named Mānasa with Mahāvita (as its spur covering outer rim) son of Mānasa. Other purānas tell that there is a mountain range running through the whole of dvīpa, dividing it into two parts. According to Mastya, Mānasa is like a full moon rising near sea coast.

Sri M. Ali identifies it with Korea and Japan. Other efforts are also unsatisfactory if we limit

ourselves within Jambū dvīpa only. The mountain range on east coast is in Australia, which is actually called the great dividing range and is almost semi circular. Mountains of west and central desert are not exactly circular but the region between them is full of lake and rivers, while outer region is desert.

It tallies better with South America which is totally divided from north to south by Andes mountain in west portion. This may be mānasa range which is exactly semicircular from north coast to Bolivia. Guyana highlands are almost its continuation which may be called son of Mānasa or Mahāvita. Circular mountains in east coast are Brazilian high lands which are called Citrānśu due to extensive forest cover (hence colourful and picturesque).

West of Andes is desert, but east and specially north east portion is full of water.

Actually north America also is continuation of that mountain range. Rockies and Andes combined may be called Lokāloka parvata as it extends from north pole to south polar region. Beyond that is Pacific, the biggest ocean, called sweet water ocean.

(4) : Parts of India - The later descriptions, describe whole Jambudvīpa within Asia only. M. Ali identifies the mountains of Jambudvipa as

Nīla - Zerafshan, Trans Alai, Koksal Tan, Tienshan ranges

Śveta - Nura Tau, Turkistan, Altai, Atbashi, Akshai, Irak ranges

Śṛngavāna - Kara Tau, Kirghiz, Zailai, Ala Tau, Ketman

Ramyaka is between Nīla and Śveta

Hiraṇmaya is between Śveta and Śṛngavāna. Uttara Kuru or Śṛngāsaka is between Śṛngvān and northern ocean - the Arctic.

Bhadrāśva is Hwangho basin of north China, Ketumāla is Oxus basin.

Nīne divisons of India are -

(i) Indradvīpa - Burma (Myammar now) - 'amara' means deva whose king was Indra. (East of Brahmaputra)

(ii) Kaśerumān - Malaya peninsula - Between Mahendra and Śukti hills (Between Godāvarī and Mahānadī - M. Ali)

(iii) Tāmraparṇa - Srī Lanka called Tāṃbapanni in inscriptions of Aśoka, Taprobane in Greek. (Region south of Kaveri)

(iv) Gabhastimān - Between Narmadā and Godāvarī (M. Ali) Between Ṛkśa and Malaya mountains.Indonesia according to Cunningham.

(v) Nāgadvīpa - Jaffna peninsula of Srīlankā. Andaman & Nicobar, according to me.

(vi) Saumya - Coastal Belt west of Sindha or Tibet

(vii) Gandharva - Cis - Indus region - Gāndhāra or Kāndhāra of Afganistan

(viii) Varuṇa - West coast of India (M.Ali). Western islands of Arab sea according to me.

Cunningham suggests that these nine Khaṇḍas as the part of greater Bhāratvarsa which included the islands and parinsula of East Indies. Thus he identifies them with Burma, Malaya, Javā, Sumātrā, Ceylon etc.

It is supported by the fact that ninth and main Khaṇḍa has not been generally named. it is called Kumār or Kumarīkā Khaṇḍa where 4 classes and their rituals exist.

(5) List of Janapada or Communities : Greater India is assumed in shape of Kūrma (tortoise) facing east and floating on water.

Himālaya is varṣa parvata defining the country - south of it and north of ocean.

Sahya is a Kula parvata - western ghats

Malaya - Kerala hills, also Malaya peninsula

Mahendra - Eastern ghats in Orissa and Andhra Pradesh or mountain rage north east of Burma.

Pariyātra - Ring of ranges north of Narmadā river (Arāvali and west vindhyas)

Ṛkśa parvat - Modern vindhyas from source of Sonar to the eastern limit of catchment of Son river

Mahadeo hills, Hazaribagh range and Raj mahal hills.

Śukimat - north west of Mahendra, covering north west Orissa and Bastar region in a semi circle. Source of Ṛṣikulyā and Vamśadhārā (M. Ali) or India-Burma boundary.

(i) Janapadas of madhyadeśa

(a) Gangetic doāb (corruption of dvīpa)

Kuru - West of Yamunā from Delhi north wards

Jāngala - Wooded north eastern part of Kuru and was called Kurujāngala also

Pāñcāla - Rohilkhaṇḍ and Yamunā Gangā doab. North Pāñcāla had capital at Ahicchatra and

south had at Kampila. They are Ram nagar in Bareli and Kāmpilya in Farukhābāda distt. Gangā was boundary between two regions.

Kośala - Sarayu Rāptī doab, Ayodhyā was old capital.

Two later capitals were Śrāvasti (Sahet-Mahet near Balarampur) and Ṣāketa

Kāśi - One of the 16 mahā janapadas. Capital at Vārānaśī. South part of Ganga Gomati doab upto Son river in south part.

(b) South of Gangā Yamunā

Magadha - South of Ganga, east of Son and north of Vindhya hills upto Munger.

Kuntala = Mirzapur region in south east U.P.

(c) West of Yamunā - (western)

Matsya - Alwar and Gurgaon districts

Śūrasena - Bharatpur, Dholpur - Karauli region

Śalva - Shekhāvati - Loharu, Bhiwari Region. Its part Bodha was Hānsī, Hisar-sirasā tract. Bhuling was Luni river basin. Bhadrakāra was region west of Arāvalis

(ii) North Western Janapadas

(a) Makarān region - Angalok or Hingalaz - Shrine of Śiva

Pallava - Parikan river valley

Bāhu bhadra - Valley of Bāhu river on whose mouth Gwador is situated

Deśamanka - Valley of Dashta

Hārabhūsika - Armabel town towards Indus delta to Gāṇḍava near Kalāt

Carmakhaṇḍa - Mouth of Hab river and Churma islands, inhabited by pirates

(b) Baluchistan

Kālatoyaka - Kalata, valley of Malla river

Bāhlika - Baluchistan - valleys of Bolon, Nari and Gokh rivers named Balistan also. May be Balkha of Persia.

Vātadhāna - North of Bahlika - Valleys of Zhob, Kundar and Gomal. Waziristan

Toshara - Further north in valleys of Kurram and Tochi.

Aprita - Further north, west of Pashavar - land of Afridis.

(c) North mountain zone of Indus

Gāndhāra - Lower Kabul valley (Kandhār)

Śatadruja - Valley of Swāt river

Dārva - Valley of Pañjakora. C pital at Dir.

Kamboja - Valley of Kunar river

Lempāka - Lamghan now - upper Kabul valley

(d) North and north eastern mountains (Himālayan)

Auras - Urusa or Hazara distt in N.W. Frontier of Pakistan

Darada - Tribe of Darada in Kiśangangā valley of Kashmir.

Kāśmīra - Present Kashmir valley drained by Jhelum

(e) West Bank of Indus

Pārada - Dera Gazi Khan distt of Punjab

Sindha - Upto sea along Sindha river

(f) East Bank of Sindha -

Śūdra - Dry bed of Hakra (old saraśwati). Bahawalpur distt. Hakra and Sakkhar Town are corruptions of Śakra (Indra). These may be west boundaries of his empire.

Sauvīra - Rohri - Kharpur region of Sindha

Ābhīra-West part of Hydarabad distt of Sindha

(g) Punjab plains -

Sainika or Pidika - Rawalpindi and Pindi Ghali region

Jāngala - South half of Jhelum - Chenab Doāb

Kaikeya - North of Jangala Capital Rajāgarh or Girivraja is modern Jalalpur.

Madra - Rāvi - Chenab Doab - Capital Sukel is now Sanglawala Tibba

(iii) South Western Janapadas -

Bharukaccha - Broach region - north of Narmada delta and south of Māhī

Samahīya - Adjacent to Māhī river upto Sābarmati.

Saraswata - Patan - Mehsana plain between Aravallis and Cutch. Drained by Saraswatī river in past.

Arbuda - North west of Saraswata - Sirohi - Kotra Palanpur, Kachika - Cutch

Ānarta - North Saurāṣṭra - Dwārkā, Jāmnagar etc

Saurāṣṭra - South part - Jūnāgarh, Somnāth region

Surāla - Lower Tāpti basin round Surat and Navasāri

Tāpas (Tāmasa, Svāpada) - Khāndeś - Bhusaval, Pachora, Jalgaon, Tapal etc.

Turīyamīna - Tāpti valley between Badnur and Burhanpur (South Numar)

Rūpasa - Middle and lower Purṇa valley

Kāraskara - Upper Purṇa Valley - Karasgaon and Elichpur towns.

Nāsikya - Around Nāsik, Darna basin

Śūrparaka (Sūryārka) Surya Valley, Thane distt. Towns Safale, Mala, Sopārā.

Kāla vana (Kolavana) - Kalvan town on Girnā river, Girnā valley upto Chālisgaon

Kuliya - Kim river valley

Durga - Damangangā (old Durgā) valley

(iv) Eastern Janapadas

(a) Middle Gangetic valley

Malla - Doab of Gaṇḍaka and Rāpti - Gangā (Gorakhpur)

Videha - Gaṇḍaka to Kośī river. Capital Mithilā 35 miles north west of Vaiśālī

Magadha - East of Son, south of Gangā, north of Hazaribagh upto Munger. Capital Rajgir near Gaya

Aṅga - East of Mokāmā and west of Mandargiri, between Ganga in north and Rājmahal in south capital was Campā (near Munger)

(b) Kośī-Gaṅgā and Brahmaputra-Yamunā Doab

Puṇḍra - Capital Mahāsthān is 7 miles north of modern Bogra

Vanavāsaka - North and south Karnara distts. ruled by Kadamba family.

(b) Deccan Plateau

Māhiṣaka - Modern Karṇāṭaka, Mysore region, upto Tungabhadrā river (south Karṇāṭaka)

Kumāra - South portion, near Kumārī cape

Kuntala - Dhārwār, Bellāri, Anantapur, Raichur regions

Mahārāṣtra - Bhīmā basin

Kupatu - Modern Coimbatore and part of Salem

Aśmaka - Valley of Goldāvari below confluence of Manjirā, capital at Bodhan

Maulika - Upper Godāvarī valley. Capital at Paithan on north bank. Parts of Aurangabad, Ahmednagar, Bhir and Prabhāni distts.

Paurika (Paunika) - Valley of river Pūrana which joins Godāvarī at Nānder.

Vidarbha - Basins of river Wardhā and Penagaṅgā which forms south boundary.

Bhogavarddhana - Upper Purnā river valley below Sahyādri parvat. Bhokardon is 20 miles south of Ajantā.

(c) East coastal plain

Pāṇḍya - South of southern Vallaru river (Pudukoṭṭai) to Kanyākumāri. East upto Cholamaṇḍala coast, west upto Acchamakovil pass near south Kerala. Madurai, Tirunelveli, parts of old Travancore.

Cola - Cāromaṇḍala coastal plain from Tirupati to Pudukottai. Karur and Tiruchirāpalli.

Tāpas (Tāmasa, Svāpada) - Khāndeś - Bhusaval, Pachora, Jalgaon, Tapal etc.

Turīyamīna - Tāpti valley between Badnur and Burhanpur (South Numar)

Rūpasa - Middle and lower Purṇa valley

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(b) Kośī-Gaṅgā and Brahmaputra-Yamunā Doab

Puṇḍra - Capital Mahāsthān is 7 miles north of modern Bogra

Āndhra - Delta plains of Kṛṣṇā and Godāvarī rivers.

Kaliṅga - Coastal plain from Godāvarī delta to Mahānadī. Capital was Dantapura. Other cities were Rājapura, Siṁhapura (or Singapuram in Śrīkakulem) Kañcanapura and Kalinganagara (Mukhalinglam on Vaṁśadhārā banks)

Śavara - Valley of river Śabari - a left bank tributary of Godāvarī.

Pulinda - Region between Prānhitā and Bandia rivers - joining Godāvarī from north.

Mūṣika - Upper valley of river Mūsi, a tributary of Kṛṣṇā

Nala Kālika (Kalupa) - Basin of lower Mūsi, present Nalgoṇḍā distt of Andhra Pradesh.

Daṇḍaka - From hills of Orissa to source of Godāvarī. Mainly valley of Indrāvatī, left bank tributary of Godāvarī.

(vi) Vindhya Region -

North slope of Vindhya -

Avantī - Capital of Ujjayinī. Later on named Mālavā. It covered source of Chambala and area drained by Śiprā and Kālī Sindha.

Bhoja - Area around Bhilwara, areas drained by Chambal and Banas rivers (Parṇāśā of Purāṇa). Byolia, Maṇḍalagarh and Nīmach.

(b) North east slope of Vindhya -

Vidiśā - Basin of upper Betwa (Vetravatī)

Daśārṇa - Sagar plateau drained by Dhasān river

Karūṣa - North slope of Kaimur range, basin of upper Tons river. West limit was Ken river, north boundary was scarps of Vindhya facing Yamunā.

Mālava - Basin of middle and upper Ken (Karmanāsā)

(c) Intemediate Vindhya

Niṣadha - Narawar region near Gwalior, associated with king Nala

Tumbura - North of Narawar from foot hills of Vindhya to Chambal (Land of Tomar rajputs).

(d) Eastern and south eastern slopes of Vindhya -

Utkala - Present Balesore distt., north Orissa coast

Tośala - Whole Mahānadī delta. Tosalī (Modern Dhauli) near Bhubaneswar was centre.

Kośala - North margin of modern Mahākośal region

Mekala - Southern slopes of Maikal range, south of Amar Kaṇṭaka. present Bilaspur distt.

(e) Narmadā Basin

Tripurā - Around Tewar (10 miles west of Jabalpur). Upper Narmada valley covering Jabalpur and parts of Maṇḍalā and Narasimhapur distts.

Tuṇḍikera - Two towns of this name exist - north east of Narasimhapura beyond Bhander forest. Other is in Narmadā basin. It occupied south stertch of Narmadā basin. Town Sainkhedā on south bank is old name Śauṇḍikerā.

Tumurā - West of Tuṇḍikerā, southern besin of Narmadā. West half of Hoshangābād distt, centred around Tumurni - a town on Iṭārsi Khaṇḍawā line.

Kiṣkindhā - Further down in Khaṇḍawā - Khārgon region

Palavi - Foot hills of Satpura, facing Narmada. Pati town is south of Barwani.

Vītihotra - North of Narmadā and west of Tuṇḍikerā. Drained by Kolār, Jamner, Kanār rivers

Anūpa - Marshy or ill drained land. Alluvial tract of Narmadā basin just after Vindhya Satpurā trench.

Verses 153-166 : Dimensions of earth

Diameter of earth is 1600 yojana. Its circumference is calculated by multiplying with 3927 and dividing with 1250. For simpler method, multiply the diameter by (600) and divide by (191) to get circumference. (153)

By this method circumference of earth is 1600 X 600 / 191

= 5026/10/41 yojana, which is author's view also. Circumference multiplied by diameter gives, surface area which is

5026/10/41X.1600 = 80,41,885 (yojana)^2

Volume of earth is, Surface area X diameter/ 6

$$= \frac{80,41,885 \times 1600}{6} = 2,\ 14,\ 45,\ 02,\ 666/40$$

(yojana)^3

Land mass in north hemisphere = 15,55,175 yojana

Water " = 24,56,320 yojana

Land area in south hemisphere = 55,23,90 yojana

Water " = 34,78,000 (155)

(From Siddhānta Śiromaṇi)

$$\text{Area of circle} = \frac{\text{Circumference} \times \text{diameter}}{4}$$

This multiplied by 4, gives surface area of sphere of same diameter. It is like area of sphere covered by square net.

$$\text{Volume of sphere} = \frac{\text{Surface area} \times \text{Diameter}}{6}$$

In a solid object, length, breadth and height all exist. Volume means, number of unit cubes, contained in the object. (156)

Some opine that, the distance between ends of semi-circumference is diameter. Diameter multiplied by 22 and divided by 7 gives circumference. Product of circumference and diameter divided by 4 gives,..... (157)

area of circle. Area of circle multiplied by 2 gives area of curved face of hemi-sphere constructed on that. This is 1/4th more than the earlier result. (158)

Hence method for correct volume is stated. Circumference of a sphere is divided into (21,600) kalā. Its radius is (3438) kalā. Above and below the circumference we mark the two surface centres. (159)

Central circumference (21,600 kalā) is in exact middle of upper and lower centres. It is called equator. From top point upto equator, we draw circles at difference of 225 kalā, each parallel to equator. (160)

There are 23 rings between equator and these circles Width of each ring is 225 kalā. As we

proceed from the equator to top, circumference gets smaller. The width of each ring is perpendicular on circumference. (161)

Find the akśajyā at each place. Its square is substracted from square of trijyā and we take square root of remainder. Result will be lambajyā. (162)

Sum of all these lambajyā will be (52,532/38/24). Area of largest ring is found by multiplying its circumference (21,600) by its width. (225)

This area (48,60,000) is multiplied by sum of all lambajyā and divided by trijyā. (164)

We get the area of surface of hemisphere (7,42,60,800). Its double is surface area of sphere (14, 85, 21,600) in $(\text{kalā})^2$ - - - (165)

(Diameter X circumference) also gives the same value of surface area of sphere. Hence it is said that the surface area of sphere is equal to area of rectangle of length as circumference and width as diameter. (166)

Notes (1) The ratio $\frac{\text{circumference}}{\text{Diameter}} = \pi$ is fixed, but it is a transendental number, whose value cannot be expressed in fractions. It can only be approximated to desired accuracy.

Its value 3.1415926 - - - is approximated by 22/7 = 3.14 and 355/133 = 3.141592 upto 2 and 6 places of decimal. Mādhava had given its value upto 30 places of decimal in a Kāṭapayādi verse (See introduction to the book). These values or

even upto 1 lakh places of decimal can be found only through infinite series.

The approximations used here are middle of these formulas as

as 22/7 = 3.142 . . . and 355/113 = 3.1415929 both are slightly higher.

$$\frac{3927}{1250} = \frac{355 \times 11 + 22}{113 \times 11 + 7} \text{ and}$$

$$\frac{600}{191} = \frac{355 \times 2 - 22 \times 5}{113 \times 2 - 7 \times 5}$$

Thus if r is radius of circle, then by definition

Diameter = 2r and circumference = $2\pi r$

Mādhava (14th century) had derived the following infinite series in his Yukti Bhāśa.

$$\text{Sin } x = x - \frac{x^3}{\angle 2} + \frac{x^5}{\angle 5} \ldots\ldots\ldots\ldots$$

$$\cos x = 1 - \frac{x^2}{\angle 2} + \frac{x^4}{\angle 4} \ldots\ldots\ldots\ldots$$

$$\tan^{-1} t = t - \frac{t^3}{3} + \frac{t^5}{5} + \ldots\ldots$$

If we put t= $\pi/4$ in the third series we get Mādhva series

for π

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

These are known as Gregory's Series (1671) and Leibnitz series.

Nīlakaṇtha Somayājin quotes a result of Mādhava

बिबुधने त्रजगाहि हुताशन त्रिगुण वेद भवारण बाहवः

नव निखर्व मिते वृति विस्तरे परिधि मानमिदं जगदुर्वुधाः ।

i.e. $\frac{\text{circumference}}{\text{Diameter}} \times 9 \times 10^{11}$ is 28,27,43,33,88,233

i.e. II = 3.1415, 9265, 359 - - -

Karaṇa Paddhati and Ṣaḍratnamālā give two verses which yield the following approximations to π

(i) Circumference of circle in minutes be multiplied by 10^{10} and product divided by 31415926536, quotient will be the diameter of circle in minutes.

(ii) If you proceed thus (construct circle of unit diameter) and multiply the circumference by 10^{17}, it will be equal to 3 1415 9265 3589 79324.

Value upto 30 places is given by Mādhava in this verse read in Kaṭapayādi notation -

गोपी भाग्य मधुव्रात श्रृंगिशो दधि सन्धिगाः ।

खल जीवित खातावा॰गलहात रसन्धराः ॥

In Tantra saṁgraham, two series are given with a correction of the error term

$$(i)\ C \approx 4d - \frac{4d}{3} + \frac{4d}{5} - - \pm \frac{4d\ (\frac{1}{2}P)}{p^2 + 1}$$

(p = Last odd divisor - 1)

$$(ii)\ c = 4\ [1 - \frac{1}{3} + \frac{1}{5} - \pm \frac{\frac{p^2}{4} + 1}{(p^2 + 4p + 1)^{p/2}}$$

p = last odd number + 1 , d = 1

Karaṇa Paddhāti has given another series

$$\text{(i) } C = \left(3 + \frac{4}{3^2 - 2} - \frac{4}{5^2 - 5} + \frac{4}{7^2 - 7} + \ldots\ldots\right)$$

which gives

$$\pi = 3 + 4\left(\frac{1}{2\,.\,3\,.\,4} - \frac{1}{4\,.\,5\,.\,6} + \frac{1}{6\,.\,7\,.\,8}\ldots\ldots\right)$$

(ii) π

$$= 2 + 4\left(\frac{1}{1\,.\,3} - \frac{1}{3.\,.5} + \frac{1}{5\,.\,7} + \quad \pm \frac{1}{[(p-1)^2 + 2]^2}\right)$$

Rāmānujan discovered several rapidly converged series for π

$$\frac{4}{\pi} = 1 + \frac{7}{4}\left(\frac{1}{2}\right)^3 + \frac{13}{4^2}\left(\frac{1.3}{2.4}\right)^3 + \frac{19}{4^3}\left(\frac{1.3.5}{2.4.6}\right)^3 + \ldots$$

$$\frac{16}{\pi} = 5 + \frac{47}{64}\left(\frac{1}{2}\right)^3 + \frac{89}{64^2}\left(\frac{1.3}{2.4}\right)^3 + \frac{131}{64^3}\left(\frac{1.3.5}{2.4.6}\right)^3 + \ldots$$

$$\frac{3^2}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5} + 29}{64}\left(\frac{1}{2}\right)^3\left(\frac{\sqrt{5} - 1}{2^8}\right)$$

$$+ \frac{89\sqrt{5} + 59}{64^2}\left(\frac{1\,.\,3}{2\,.\,4}\right)^3\left(\frac{\sqrt{5} - 1}{2}\right)^{16} + \ldots\ldots$$

G. and D. Chudnovsky have given a very rapidly convergent series for II

$$\frac{1}{\pi} = 12 \sum_{n=0}^{\infty} (-1)^n \frac{(6n)!}{(n!)^3\,(3n)!}$$

$$\frac{1359\,1409 + 545140134n}{(640320)^{3n + 3/2}}$$

This series converges to 15 decimal places per term.

(2) Area of circle and sphere

Fig 8 a Fig 8 b

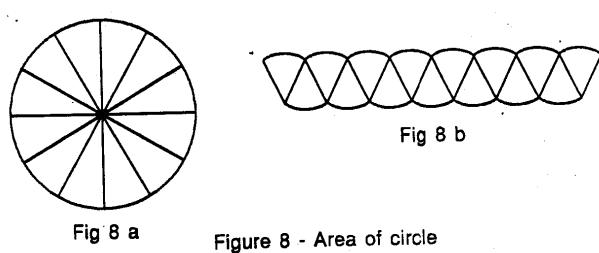



Figure 8 - Area of circle

In figure 8a, circle is divided into many sectors by radius in all directions. In 8b, they are arranged side by side, so that circumference parts are atternately up and down. Lengths are all equal to radius and they cover each other. Figure 8b becomes a rectangle if number of sectors becomes infinitely large. Then its width is radius and length is circumference X 1/2.

Hence area of circle = radius X circumference X 1/2

$$= \frac{\text{Diameter} \times \text{circumference}}{4}$$

Area of sphere is calculated by method of integral calculus, i.e. dividing its surface in infinitely small circular strips (or any other type of division) Here the smallest width has been chosen as 225 kalā because, we get the values of Jyā (R sine) only at this interval acording to methods in chapter 5.

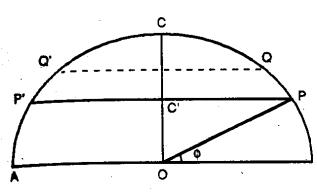



Figure 9 - Area of sphere

Figure 9 shows a section of hemisphere with AB as diameter of sphere and of greatest circle (equator). C is top surface centre. PP′ is a diameter of circle drawn paralel to equator i.e. perpendicular to line CO drawn from C to equator.

$\angle POB = \Phi =$ akśāmśa

Diameter of circle at P is PP′. If its mid point is C′ then OC′ P is right angled triangle.

$PC' = OP \cos \phi = R \cos \phi$

$= R \sin (90° - \phi) =$ Lambajyā

Here $R \cos \phi$ is calculated from

$R \cos \phi = \sqrt{R^2 - R^2 \sin \phi}$, as we know the

Jyā $= R \sin \phi$ from the chart.

If a circle at small distance QQ′ is also made, then the region between PQ is approximately a cylinder whose surface area can be found by wrapping a paper and spreading on plane area. Then it becomes a rectangle with length as circumference and width of arc PQ.

Then area of strip PQ is

= Circumference of PP′ X PQ arc

= Lambajyā at P X 225 kalā X 2π

Hence area of all strips

2π X (sum of lambajyā) X 225 - - - (1)

Here for first circle at equator

$$2\pi = \frac{\text{Circumference}}{R}$$

and area of first stirp = circumference X 225

Hence (1) becomes

$$(\text{Sum of Lambajyā}) \times \frac{\text{Circumference} \times 225}{R}$$

$$= \frac{\text{Area of first strip} \times \text{Sum of lambajyā}}{\text{Trijyā (R)}} \quad \text{- (2)}$$

which is the formula given in text.

(3) Volume of sphere is calculated by dividing it into very small cones, with apex at centre. Then the spherical surface can be considered almost plane, and volume of cone

= 1/3 X height (=radius) X area of base (part of surface)

By adding all cones, volume of spehre is

$$\frac{1}{3} \times \text{radius} \times \text{area of sphere}$$

$$= \frac{\text{Diameter} \times \text{Surface area}}{6}$$

as the formula given in text

Volume of a circular cone is again to be calculated by integral calculus, giving formula given above.

Verses 168 : Sphere and cube

Construct spheres and cubes of width equal to diameter out of mud. Ratio between their weights is the ratio between their volumes which can be thus measured. Then we find the ratio of cloth area needed to cover the surface areas. We

will find that the ratio of surface areas of cube and spehre is same as ratio of their volumes.

Note : Area of sphere of radius r = $4\pi r^2$

Area of a cube surface = $(2r)^2 = 4\ r^2$

Area of 6 surfaces of cube = $6 \times 4\ r^2 = 24\ r^2$

$$\text{Thus } \frac{\text{Area of cube}}{\text{Area of sphere}} = \frac{24r^2}{4\,\pi\, r^2} = \frac{6}{\pi} \quad \text{- - - (1)}$$

Volume of cube = $(2r)^3 = 8r^3$

Volume of sphere = $\frac{4}{3}\pi\ r^3$

$$\text{Thus } \frac{\text{cube volume}}{\text{sphere volume}} = \frac{8r^3}{\frac{4}{3}\pi\ r^3} = \frac{6}{\pi} \quad \text{- - - (ii)}$$

From (i) and (ii) we see that ratios of volume and area are same.

Verses 169-175 - Difference in units of length.

8 yavas put side by side become 1 aṅgula

4 aṅgula	= 1 fist
6 fist	= 1 hand
6 hands	= 1 daṇḍa
2000 daṇḍa	= 1 kosa
2 Kosa	= 1 gyvyūti
2 gavyūti	= 1 yojana

Some authorities consider 1 yojana = 28,160 hands.

(= 8 miles, where 1 yard = 2 hands in British measure), 28,160 hands = 14,080 yards = 8 mles X 1760 yards).

A man of average height goes 1 kosa in about 5,600 steps

(1 step = $\frac{3520}{5600}$ yards = $\frac{22}{35}$ yards = about 23 inches) (169-170)

Brahma and Sūrya siddhāntas have called the half chord as jyā. (Jyārdha). At some places gavyuti has been stated equal to yojana of 4 kosa. (Then 8 kosa = 1 yojāna). By this scale, diameters of earth, moon and sun are reduced by half to 800, 222, and 36,000 yojānas. (171)

According to Viṣṇudharmottara Purāṇa,

1 śaṅku	= 12 aṅgula
12 śaṅku	= 1 hand
4 hands	= 1 dhanu (bow)
1000 dhanu	= 1 kosa

As per this measure, value of diameters is as stated in this book earlier (Sun 72,000, moon 444, earth 1600) (172)

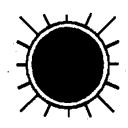
Quotation from the purāṇa gives the above measures (173)

If we raise our hands above head and stand on toes, then 1/5th of that height is called 1 hand. By this scale 1 yojana has been considered equal to 28160 hands (= 8 miles). In this scale circumference of earth is 3093/40 yojana and diameter is 984/45 yojana. (174)

Area of earth in this scale is 30,46,488 (yojana)^2 and volume is 50,00,04,843 i.e. approximately 50 crore yojanas. (Explaination of Brahma siddhānta value quoted in verse 13 of chapter 17). After concluding description of earth, now I will state about heavenly bodies (in next chapter). (175)

Verses 176-177 - Prayer and conclusion

I seek the shelter of younger brother of Balarāma (i.e. Kṛṣṇa) wearing blue clothes and who is shining blue like lotus, humming bee, indranīla maṇi, water of Yamunā river and black soot and is like crown of blue mountain (Nīlācala). (176)

Thus ends the eighteenth chapter describing geography in Siddhānta Darpaṇa written as text book for correct calculations by Śrī Candraśekhara born in famous royal family of Orissa. (178)

Chapter - 19

EARTH AND SKY

Bhūgola Khagola Varṇana

Verses 1-2- - Origin of spheres

Before creation of living beings; for creating sense organs of body and their subjects of experience, Brahmā (creator) created two types of seeds - internal and external types. External seed became cover of cosmic egg (Brahmāṇḍa). From internal seed, five spheres were created within universe - sky, air, sun rays, moon and earth. (1)

Meru of sky itself has been called marut (air). In sky spehre, Meru (line joining north and south poles) is like axle (yaṣṭi). Around it, nakśatras revolve. Universe is tied with this Meru-daṇḍa (rod of meru - also means spine of human body). From earth as root of the tree of universe, upto Brahmāṇḍa, seven airs, starting with āvaha, are located in spherical shells, one containing the previous. Within these 6 shells of air, lie moving and fixed creation, stars, planets, clouds etc.

Verses 3-6 : Air spheres as per Siddhānta Śiromaṇi -

Air on earth is called āvaha. After that lie the spheres of pravaha, saṁvaha, suvaha, parivaha, and parāvaha, one above the other. (3)

From earth's surface, earth's air or āvaha extends till 12 yojana. This contains clouds and lightning etc. Above that exists Pravaha whose motion is always from east to west. (4)

At its centre, orbits of planets and stars revolve in westerly direction being pushed by Pravaha. (5)

Within the west moving nakśatra sphere, the planets move with their small east motion, as an insect moves on a rotating wheel of potter with small speed against the rotation of wheel. (6)

Verses 7-11 - Nature of planetary orbits.

Madhyama sun moves in east direction. Around it, planets attracted by mandocca, revolve. They go in west direction also. Hence, the orbits of planets are of many types like madhyama and sphuṭa. (7)

Moon and sun revolve round earth. Due to small or big attraction of mandocca, their motion is changed slightly. Hence at first madhyama kakśā (mean orbit) is written. Then smaller orbits for finding true motion will be written. (8)

As the planets maṅgala, guru, śani are very far from earth and move with slow speed, their madhya kakśā has centre at madhyama sun. (9)

Orbit of madhyama sun will be their śīghra orbit. Between sun and earth, budha and śukra are faster, hence their madhya kakśā is sun orbit only. (10)

The śīghra orbit of budha and śukra around sun can be treated as mean orbit also to find the distance (manda karṇa) from sun, as the madhya

orbit of sun and moon is used to find their distance from earth. Distance of outer planets from sun is manda karṇa and from earth is sīghra karṇa.

Verses 12-18 : Orbit lengths

Madhya orbits of sun, budha and śukra are 4, 78,00,800 yojāna.

This is also the śīghra orbit of maṅgala, guru and śani.

Śīghra orbit of Budha is (1,84,56,420) yojana

Śukra Śīghra orbit is (3,46,55,580) yojana

Maṅgala manda or mean orbit (7,23,03,600)

Guru manda orbit (24,58,32,000) yojana

Śani manda orbit (44,12,37,600) yojana

Śīghra paridhi at end of odd quadrant

$$= \frac{\text{Śīghra orbit} \times 360^\circ}{\text{Madhya orbit}}$$

Moon orbit is (3,06,000) yojana

Nakśatra orbit is (17,20,82,88,000) yojana

Kakśa (orbit) length multiplied by 191 and divided by 1200 give distance of tārā graha from sun and of nakśatra and moon from earth.

Notes : Distance of planet mainly depends on bigger orbit called manda or madhya orbit. Minor differences are due to smaller orbit, which is faster also.

Orbit length is circumference of the circular orbit = 2 πr where r is radius or karṇa - distance of planet from centre of its orbit.

Here $\pi = \frac{600}{191}$, an approximation between

two values $\frac{22}{7}$ and $\frac{355}{113}$ $\left(\frac{600}{191} = \frac{355 \times 2 - 22 \times 5}{113 \times 2 - 2 \times 5}\right)$

Verses 19-21 :

Sphuṭa mandakarṇa in yojana

$$= \frac{\text{madhya karṇa} \times \text{sphuṭa karṇa in kalā}}{\text{Radius (3438)}}$$

As mandaphala correction in madhya graha, in śīghrocca of budha and śukra also this correction is done. (19)

Distance of 5 tārā graha is found by multiplying madhya karṇa by third manda karṇa in kalā and dividing by trijyā. Result is multiplied by 4th śīghra karṇa in kalā and divided by trijyā. Karṇa multiplied by 1200 and divided by 191 will give the orbit.

Notes : Madhya karṇa in kalā is the value of trijyā in kalā. Thus average distance of each planet is assumed 3438 kalā. Thus proportionate value of sphuṭa karṇa also is expressed in kalā.

$$\frac{\text{Sphuṭa karṇa kalā}}{\text{Madhya karṇa kalā (trijyā = 3438)}}$$

$$= \frac{\text{Sphuṭa karṇa yojana}}{\text{Madhya karṇa yojana}}$$

For tārā graha we obtain mandasphuṭa first, by the 3rd manda karṇa by this formula. Then the change due to śīghra orbit (śīghra karṇa) is found by treating manda sphuṭa as madhya karṇa = trijyā 3438.

Thus

$$\frac{\text{manda sphuṭa karṇa}}{\text{Trijyā (3438)}} = \frac{\text{Śīghra karna yojana}}{\text{Śīghra karna kalā}}$$

Verse 22 - Mandocca and pāta are based on bhagaṇa (revolution) only. Hence their orbit is taken as the nakśatra orbit itself.

Verses 23-27 : Linear motion of planets

Daily linear motion (yojanas)

$$= \frac{\text{Kakśā yojana} \times \text{Kalpa bhagaṇa}}{\text{Kakśa sāvana dīna}} \quad (23)$$

In a mean solar day, the yojana gati of planets in their orbits is stated.

Sun 130,868 yojana

Moon 11,200 yojana

Bha (Nakśatra) circle 2094 yojana

Budha (Śīghra kakśā around sun) (2,09,803)

Śukra (") (1,54,229)

Maṅgala (1,05,248) yojana

Guru (156,733)

Śani (41,008)

Notes (1) Formula of daily motion is obvious Movement in kalpa sāvana dīna is kalpa bhagaṇa = kalpa bhagaṇa X kakśā yojana

(as 1 bhagaṇa or revolution is length of orbit i.e. kakśā)

Hence motion in 1 sāvana dina

$$= \frac{\text{Kalpa bhagaṇa} \times \text{kakśā}}{\text{Kalpa sāvana dīna}}$$

(2) Evidently, in accepting $\frac{\text{Sun diameter}}{\text{Moon diameter}} = \frac{72,000}{444}$

= 163 the principle of equal linear motion has been dropped. As per this principle

$$\frac{\text{Sun diameter}}{\text{moon diameter}} = \frac{\text{Sun distance}}{\text{moon distance}},$$ as their angular diameters are almost same.

This is equal to $\frac{\text{Moon angular speed}}{\text{Sun angular speed}}$, for equal linear motion = 13.37 approx.

After rejecting this theory, he has not given any new principle to calculate the distance of the planets.

However, preceding discussion shows that, for inner planets

$$\frac{\text{Orbit length}}{\text{Sun orbit}} = \frac{\text{Śīghra kakśā}}{360^\circ}$$

and for outer planets

$$\frac{\text{Sun orbit}}{\text{Orbit length}} = \frac{\text{Śīghra kakśā}}{360^\circ}$$

Thus the dimension of śīghra paridhis are the basis of calculating their lengths. Then we can use the formula given above in note (1) in verse 23 to find linear speed.

Verses 28-29 : Linear motion in nakśatra kakśā

Linear speed in nakśatra kakśā (Ayanagati)

$$= \frac{\text{Linear speed in own orbit}}{\text{own orbit in kalā} \times 21{,}600}$$

Ayana gati in yojana is multiplied by graha kakśā yojana and divided by nakśatra kakśa yojana. Result will be added to graha gati, if nakśatras are moving towards east, otherwise substracted. This will give sphuṭa gati of graha from sāyana meṣa.

Notes : Motion of nakśatras is the motion relative to sāyana meṣa point due to precession of

equinox. Its motion towards east means that sāyana meṣa has moved west and ayanāmśa is added to find sayana position.

To find sāyana gati of graha, we have to find its motion in nakśatra kakśā because nakśatra gati is 2094 yojana in that orbit. Yojana gati of graha in bigger nakśatra kakśā is more in proportion to length of nakśatra kakśā. Here ratio of the two kakśās is expressed in kalās of graha orbit by

$$\frac{\text{graha kakśā in kalā}}{\text{Nakśatra kakśā 21,600 kalā}}$$

$$= \frac{\text{graha kakśā yojana}}{\text{Nakśatra kakśā yojana}}$$

$$\text{Here } \frac{\text{Sun orbit}}{\text{Nakśatra orbit}} = \frac{1}{360} = \frac{60}{21{,}600}$$

Thus sun orbit is 60 kalā. Other orbits in kalā can be found directly or in proportion to śīghra kakśā as explained in note (2) after verse 27.

Verses 30-31 : Kakśā and śara gati

Due to attraction by ucca, a graha goes up and down (or farther and nearer) in its orbit. But inspite of change in karṇa (distance from centre), its yojana gati in east always remains the same (mandasphuṭa gati only) (30)

North south motion due to repulsion of pāta is proportional to their mandakarṇa.

Notes : Principle of equal linear speed has been assumed for each planet separately i.e. it should remain constant in one orbit only. Speeds of different planets vary as shown in verses 23-27.

However, even this principle is not correct. The linear speed also reduces, when a graha goes farther in the orbit. According to Keplar's law, the areal speed (i.e. area covered by manda karṇa in unit time) is constant. It can be understood in another way. When an object is thrown upwards, its speed reduces slowly and finally it comes back. Thus the speed reduces with increase in distance in a gravitational field.

$$\frac{\text{Śara gati in yojna}}{\text{Karṇa or distance}} = \text{Śara angle in radian}$$

This multiplied by 3438 gives śara angle in kalā.

$$\text{Thus śara gati in yojana} = \frac{\text{Śara gati in kalā}}{3438} \times$$

Karṇa yojana

Here, eastward motion is only in their own mean orbit around sun. By adding śīghra gati it may change, and even go back wards also.

Verses 32-37 : Explaining with diagrams

Earlier astronomers have stated that around earth, successively larger orbits are of moon, mercury, venus, sun, maṅgala (mars), jupiter and ṣaturn, but this is not clearly observed. (32)

Kakśā values in yojana are divided by its kalpa bhagaṇa (fractional part). We get the distance covered by graha in its orbit in yojanas. (33)

Orbit of planet is drawn and its centre is marked as 'bhū' (earth). Position and east speed of each graha is indicated in their orbits. (34)

From this positon, graha is shifted forward (i.e. east) or backward according to śīghra phala.

We see the direction of tārā graha from earth, whether it moves backward (in cakrārdha i.e. 180° from śīghrocca) or not (for other positions). (35)

Tārā grahas with speeds slower than sun (i.e. mars, jupiter and saturn) are bent towards sun-planet direction from earth-planet direction in their retrograde motion. (36)

Angular diameter and śara of a planet decreases or increases according as its distance from earth (śīghra karṇa) increases or decreases. This demonstration shows that the tārā grahas revolve round madhyama sun. (37)

Notes : Retrograde motion of planets has already been explained with diagrams in spaṣṭādhikāra, chapter 5.

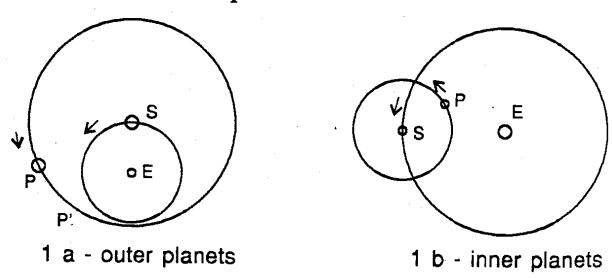



Fig 1 - Planets around mean sun

S = Sun, E = Earth, P = Planet

Mean sun S moves round the earth and around sun the planets move, inner planets in smaller orbit and outer planets in bigger orbit. This doesn't make any difference in calculations as, all planet orbits are around sun in both system. Sun has apparent motion around earth.

Verses 38-42 : Lower values of Laṁbana

If we assume that orbits of tārā grahas are as big as stated by sūrya siddhānta or Bhāskara II, then their horizontal laṁbana in east or west will be 1/15 of their daily motion. (38)

Thus madhyama laṁbana (horizontal) of maṅgala 31/26 ÷ 15 = 2 kalā approximately. This lambana in cakrārdha is found by multiplying it with trijyā and dividing by śīghra karṇa, and comes to about 6 kalā. (39)

When maṅgala is 24 kalā vakra from its true position, it rises in west and its disc is seen after sunset. (40)

At setting time, maṅgala will be seen with a particular star (indicating its position in ecliptic), 24 kalā west.

Next day at rising time it should be with same star due to laṁbana. But this does not happen. The author has not seen its lambana in nīca place (180° from sun) even to be 1 kalā (against 6 kalā calculated value). (42)

Notes : At nīca position maṅgala is vakrī with maximum speed (59/8 - 31/26) i.e. (sun - mars speed) = 27/42. In half day, maṅgala will move about 14 kalā west. However, laṁbana at setting time makes it 6 kalā west. The difference is 6+6 = 12 kalā east, between west horizon and east horizon place, which compensates the vakra gati towards west. Thus position of maṅgala should appear same at west setting and next rise in east.

Verses 43-46 : Higher sun/moon ratio

Due to very small real laṁbana of maṅgala. I have assumed bigger orbits for these planets compared to sūrya siddhānta. (43)

Bhujajyā of (moon-sun) is multiplied by moon distance and divided by sun distance according to sūrya siddhānta or Bhāskara II. (44)

The result is added or substracted from moon in bright or dark half. On Ist quarter of śukla 8th (when moon - sun is 84° to 87°) or last quarter of kṛṣṇa 8th (moon-sun, being 273° to 276°), half of moon disc should be bright, but it doesn't happen. (45)

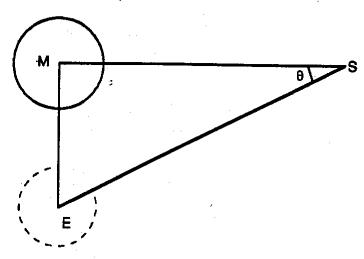



Fig 2 - Half bright moon

But at the end of śukla 8th 2nd quarter (when moon-sun=90°) or Kṛṣṇa 8th 2nd quarter (when moon-sun = 270°), half of moon disc looks bright. Hence I have taken sun distance more than those texts. (46)

Notes - Moon is half bright, when, sun and earth are at perpendicular distances from moon i.e. $\angle EMS = 90°$. But viewed from earth E, the angle between sun S and moon M is $\angle MES$ which is less than 90° for śukla pakśa and outer angle (reflex

angle marked with dotted line) is bigger than 270°. (Figure 2)

This difference θ = $\angle$MSE is given by

$$\frac{MS}{\sin MES} = \frac{ME}{\sin \theta}$$

$$\text{or } \sin\theta = \sin MES \times \frac{ME}{MS}$$

$$= \text{Bhuja jyā of (moon-sun)} \times \frac{\text{Moon distance}}{\text{sun distance}}$$

when $\angle$MES - 90°, then its jyā = 1

$$\text{or } \sin\theta = \frac{\text{Moon distance}}{\text{sun distance}} = \frac{1}{13.37}$$

More accurately, $\tan\theta = \frac{ME}{MS} = \frac{1}{13.27}$, = 4°17′ approx.

Hence the half bright position should be 4°17′ away from the position of 90° or 270° difference as seen from earth. But this appears negligible, hence sun distance should be much larger.

Verses 47-52 : Finding true distance of sun

The much larger value of distances and orbits of planets as ratio of moon in comparison to sūrya siddhānta, should be derived logically. Hence, I am stating the method, how these values have been found. (47)

Without authority of Vedas, only guess work is not appreciated. Hence, I have accepted the diameter of sun as stated in vedas. (48).

In Brahmavidyā upaniṣad, while explaining the importance of Praṇava (ॐ), diameter of sun has been stated to be 72,000 yojanas. (49)

In Atharva veda also, same diameter of Mahāpuruṣa (Sun) has been stated. In sūrya siddhānta, the diameter of earth and angular diameter of earth are stated, which are verified by observation. (50)

By observing through instruments also, mean bimba (angular diameter) is 32/32 kalā as stated by Bhāskara II. (51)

I have stated distance of sun from ratio of yojana diameter and angular diameter. Accordingly orbit extent has been stated.

Notes : This has been explained in chapter 8 of lunar eclipse. Extent of solar maṇḍala means its circumference as Kakśā length means the same. The ākāśa yojana = 5 earth yojanas. Hence true diameter according to vedas should be

$$\frac{72{,}000 \times 5}{\pi} \text{ yojana} = \frac{72{,}000 \times 5 \times 8}{\pi} \text{ kilo meters}$$

= 9,16,732 kms. which is the correct modern value. By assuming it to be diameter in earth yojanas it is only 63% of correct value. This interpratation of ākāśa yojana is based on distance of uṣā (dawn) from sunrise as 30 yojanas mentioned at several places in vedas.

$$\frac{\text{Diameter yojana}}{\text{Distance yojana}} = \text{angular diameter (radian)}$$

$$= \frac{\text{bimba kalā}}{3438}$$

$$\text{or Distance} = \frac{\text{Diameter} \times 3438}{\text{bimba kalā}}$$

This gives the revised distance of sun, derived from observed value of bimba.

Verses 53-54 : Revision of panetary orbits.

Śighra paridhi of other planets also is found by ratios.

$$\text{For inner planets, orbit} = \frac{\text{Sun orbit} \times \text{Śīghra paridhi}}{360^\circ}$$

$$\text{Orbit for outer planets} = \frac{\text{Sun orbit} \times 360^\circ}{\text{Śīghra orbit}}$$

Notes : This has been explained in verses 24-27 note(2).

Verses 55 - Moon orbit

Distance of moon has been decreased due to slight increase in moon's laṁbana. Thus its biṁba and diameter of earth's shadow have been decreased.

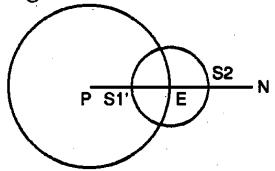
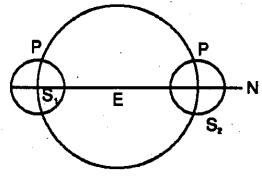
Notes : Laṁbana has been observed to be (moon speed / 14) instead of (moon speed / 15). Due to its higher value, it should have smaller orbit as lambana decreases with increase in distance.

Since angular diameter of moon is same as mentioned in sūrya siddhānta, linear diameter should be decreased slightly in same ratio, as

$$\text{angular diameter} = \frac{\text{Linear diameter}}{\text{distance}}$$

Verses 56-58 : Higher diameter of nakśatra kakśā

Maximum śīghra phala of grahas from sun (to stars) compared to that from earth is seen more for budha and śukra (56) and less for maṅgala, guru, sani. Then the difference between bhujaphalas (57) is seen 1/360 of the manda orbit. Hence the orbit of nakśatras have been assumed 360 times the sun.

Notes : Variation in śīghra paridhi is due to same season as variation in manda paridhi i.e. because the real orbits are elliptical. However, here it has been assumed that the distance of nakśatras in 360 times distance of sun (compared to 60 times of sun). Actually it is 4 lakh times for nearest star. Due to finite distance (360 times sun distance) the distance of śīghra paridhis are different from sun and from earth. Hence there is difference is of 1° i.e. 1/360 of suns orbit. This is explained by diagrams below -

3 a - outer planets

3 b - Inner planets

E is earth, S_1, S_2 are two positions of sun. P is position of planet. In fig 3(a), P is kept at centre to see the position of earth relative to planet. The positions S_1, S_2 are at distance of sun's orbit. Hence the same śāghra phala for those positions will substend different angles at Nakśatra circle at N.

Verses 59-64 : Distance and diameter

For sun and moon, from angular diameter, we know their distance (karṇa). For stars and other planets, angular diameter (biṁba) is known from their distance. (59)

We make a circular disc and find the distance of the disc from eyes at which it completely covers the disc of star, i.e. their angular diameters are same. (60)

Distance is found by multiplying distance between eye and disc by angular diameter and divided by disc diameter. (For sun and moon). By reverse process, we can find bimba from distance. (61)

Alternately, for sun, at sunrise or sunset time, east or west window (of a closed and dark room) is covered by palm leaf. (62)

A small hole is made in it. On the other side of the wall, we see the image in shape of disc due to light coming out of hole. (63)

Diameter of hole is substracted from diameter of light image to give hāra. Distance between wall and hole is multiplied by diameter of sun and divided by hāra to get the sphuṭa karṇa (current distance) of sun. (64)

Notes : (1)

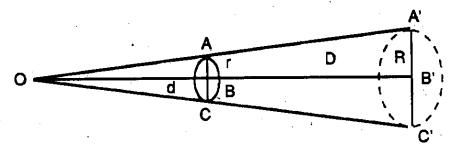



Figure 4 - diameter and distance

In figure 4, eye is kept at O, At distance d = OB, a disc ABC with radius r is kept. It completely covers the disc A'B'C' of the star or planet A'B'C' of radius R and distance OB' = D

Angular radius are same ∠ AOB = ∠A′OB′ = θ in this case. Diameter is 2θ , substended by AC = 2r or A′C′ = 2R.

$$\theta = \frac{r}{d} = \frac{R}{D} \text{ in radians}$$

$$\text{thus } D = \frac{R \times d}{r} = \frac{(2r) \times d}{(2r)}$$

Hence distance is found by distance of disc multiplied diameter of sun or moon and dividing by diameter of disc.

(2) Image from hole - This is called pin hole camera now in which the diameter of hole is ideally assumed to be zero.

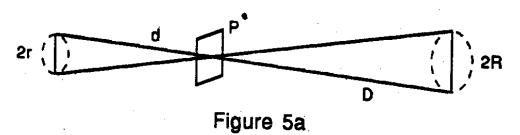
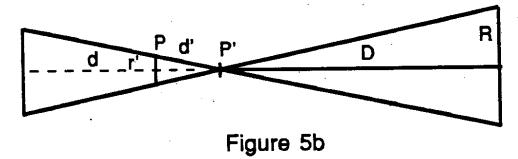



Figure 5a

In figure 5(a), hole in palm leaf at P, creates an image of diameter 2r at distance d on wall. Sun of diameter 2R is at distance D. Then in similar triangles.

$$\frac{d}{2r} = \frac{D}{2R} \quad \text{or} \quad D = \frac{2R \times d}{2r} \text{ as before.}$$

Correction for finite diameter of hole : In figure 5b we have image of radius r due to hole of finite diameter r′

Figure 5b

The rays from outer most parts of sun converge at point P′ out side the hole wall interval.

PP′ = d′

Then $\frac{r}{d+d'} = \frac{r'}{d'} = \frac{R}{D-d'} = \frac{R}{D}$ as d′ is small compared to D.

Here D is taken as distance of sun from P′ instead of P which is same

or $\frac{r-r'}{d} = \frac{R}{D}$

Thus we take (r-r′) instead of r in the above formula.

Verses 65-69 : Diameter of stars

By this method we can know the angular diameters of planets or stars which set with moon. (65)

For that we keep a palm leaf (circular) of such size at 36 hands (48 ft) which will just cover moon's disc completely. Then the circular leaf is kept at a place where it covers the star (at same distance from eyes). (66)

A hole of such size is made through which star or planet is just seen completely. (67)

1/4th of diameter of hole is multiplied by distance of planet and divided by distance of palm

leaf from eye. Result will be diameter of the planet. This is most accurate method for finding the diameter. (68)

1 aṅgula at this distance is seen as 1/15 degrees in sky. Hence the distance between eyes and palm leaf is 1/4th of trijya ($\frac{1}{4}$ X 3438 = 859 - $\frac{1}{2}$) angle = 36 hand.

Notes : 1 Hand = 24 aṅgulas.

Hence angle subtended by 1 angula at 36 hands is 1 / 26 X 24 radian = 3438 / 36 X 24 kalā = 4 kalā approx = 1/15°

Since angular diameter of hole and star are same, it is found by ratio, diameter aṅgula = diameter X 4 kalā. Hence diameter will be multiplied 4 times in stead of dividing by 4 to find the angular diameter in kalā. There appears to be an error in the print, otherwise Candraśekhara could not do such mistake.

36 hands = 36 X 24 aṅgula = 864

Thus $\frac{1}{4}$ radius = $\frac{1}{4}$ X 3438 = 859- $\frac{1}{2}$ is approximately equal to 36 hands.

Verses 70-75 : Height and distance.

From shadow of a śaṇku, height of lamp or a mountain can be found. This interesting method is being described. (70)

From lamp kept at a height we keep a śaṅku at a distance on level ground and measure its shadow. (71)

In same direciton, another śanku of same height is kept and its shadow also is measured. Distance between two śaṅku is measured. To this, we add the second śaṅku shadow and substract shadow of first śaṅku. (72)

Result is multiplied by first shadow and from quotient, shadow of first śaṅku is substracted. Remainder will be the distance of first śaṅku from base of lamp. (73)

The earlier quotient multiplied by height of śaṅku and divided by shadow of first śaṅku gives height of lamp. (74)

Or, distance between śaṅku and lamp is multiplied by śanku and divided by shadow. Result is added to height of śaṅku to give height of lamp. (75)

Notes : This is one of the problems of high school trigonometry, called heights and distances. Since much more complicated problem of spherical trigonometry were known to all astronomers, this simple example is intended only to popularise the astronomy to layman. Such simple and practical methods can be understood and verified by any body. This method is described in detail in Līlāvatī of Bāskara II, which has been described as necessary base alongwith his bīja gaṇita to understand astronomy. Pāṭīganita of Śrīdhara also describes it in chapter on Chāyā vyavahāra (shadow methods). Interestingly all the methods of astronomy are described in Nārada purāṇa pūrva bhāga, 2nd quarter, chapter 54. The method of

shadows has been called 'parikarma'. In these places methods for single śaṅku shadow has been described and the method of two śaṅkus has been posed as a problem (solved in Līlāvati). Solution is given in a single verse in Līlāvati -

छायाग्रयो रन्तर संगुणाभा छाया प्रमाणान्तर हृद्भवेद् भूः ।

भू शंकु घातः प्रभया विभक्तः प्रजायते दीप शिखौच्च्यमेवम् ॥

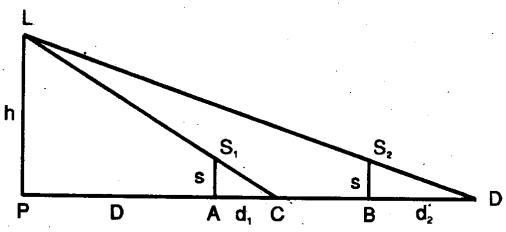



Figure 6 - Height and distance of lamp

In figure 6, a lamp is kept at L at height LP = h from plane land. Śanku of height s is kept at places AS_1 and BS_2 creating shadows AC = d_1 and BD = d_2

Distance of first śanku S_1 from lamp (from their bases) is PA = D.

In ΔLPC, S_1 A // LP and PC = D + d

$$\frac{LP}{SA} = \frac{PC}{AC} \text{ or } \frac{h}{s} = \frac{D + d'}{d'} = \frac{D}{d'} + 1$$

$$\text{or } D = \frac{h - s}{s} \times d_1 = PA \quad \text{- - - (1)}$$

$$\text{similarly } PB = \frac{h - s}{s} \times d_2 \quad \text{- - - (2)}$$

From (1) and (2), × = AB = PB - PA

$$= \frac{h - s}{s} (d_2 - d_1)$$

$$\text{or} \left(\frac{h}{s} - 1\right) (d_2 - d_1) = x$$

$$\text{or} \frac{h}{s} - 1 = \frac{x}{d_2 - d_1}$$

$$\text{or } h = \left(\frac{x}{d_2 - d_1} + 1\right) \times s \quad (3)$$

From (3) we find value of h. Then from (1) value of D is found.

$$\text{Alternatively } \frac{h - s}{s} = \frac{x}{d_2 - d_1}$$

$$\text{or } D = \frac{h - s}{s} \times d_1 = \frac{x\, d_1}{d_2 - d_1}$$

D can be found first. Then (1) will give h.

Verses 76 - 79 : Height of a hill

On a plane land where top of a hill is seen for a distance of 1 kosa, we place two śaṅku in a line from hill top at 100 yards distance minimum. (76)

Each śaṅku will be 5 hands high and stout. Then on earth's surface, we keep a mirror in line of hill top and śaṅku so that, tops of mountain and śaṅku are seen in one line. (77)

From location of mirror, we fix the positon of shadow ends and, as in previous method for height of lamp through two śaṅkus, we find height of hill and its distance. When hill is very far, its height is found by a single śaṅku. (78)

To find the visible and obstructed portions of a hill, oblique śaṅku is used.

Notes : Distance of minimum 100 hand is taken for accuracy, otherwise there will be very negligible difference.

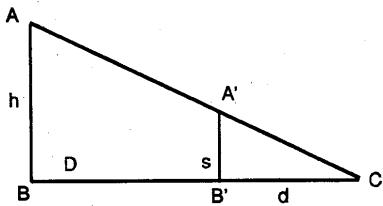



Figure 7 - height of a hill

A hill of height AB = h and a śanku of A′B′ = s height are viewed from mirror at C. Then AA′C are in one line and B′C = d is shadow of śaṅku. If BC = D, distance of hill base from shadow end, then in similar triangles ABC and A′B′C,

$$\frac{h}{s} = \frac{D}{d}$$

This will give height of hill if distance D of hill is known. In practice, D cannot be measured as, base of mountain is away from B due to its slope. Hence two śaṅku have to be used.

Since AB and A′B are parallel, A′B′ will be bent from vertical at B′, if we consider curvature of earth. This will be equal to angular distance of that place given by D x 360° / 5026 as 5026 yojana is circumference of earth equal to 360° arc.

First we have to approximate the distance D from straight śaṅku, then find the angular distance. A′B′ is bent towards hill by that angle. This will

give visible distance as line CB′ is horizontal line at B′. By method described in previous, chapter, we can find obstructed portion due to earth's curvature.

Verses 80-82 : Height of tree

Height of a tree upto 100 hands can be measured by keeping two śaṅkus of 10 hands height at a distance of over 10 hands. Heights or distances between śanku can be 5 hands also. (80)

Alternately, we can find the shadows of meru (top of hill or a tree) and śaṅku due to sun and moon. Śaṅku is multiplied by shadow of meru. (81)

Product is divided by shadow of śaṅku to give height of meru (tree or hill) (82).

Note - Angular height of sun is same from all places.

$$\text{Hence } \frac{\text{Tree height}}{\text{Tree shadow}} = \frac{\text{Śaṅku height}}{\text{Śanku shadow}}$$

Verses 83-85 - Height of cloud

Clouds are almost static when they are not moving, angular distance between cloud and sun is measured in kalā. Similataneously, distance of shadow from own place is measured in hands. (83)

Distance of cloud shadow is multiplied by trijyā (3438) and divided by angular distance from sun. Result will be distance of cloud from shadow end. That distance (karṇa) is multiplied by śaṅku shadow and divided by chāyā karṇa of śaṅku. This will be distance of cloud (its base point) in direction of sun.

Distance of cloud base is multiplied by śaṅku and divided by its shadow. That gives height of cloud similarly by trairāśika (Rule of 3). (85)

Notes : These calculations are based on equal proporiton of two sides of one triangle to the two sides of a similar triangle. Out of these four quantities, if 3 are known, fourth can be calculated. This is expressed by Bhāskara II in Līlāvati / Chāyā vyavahāra).

त्रैराशिके नैव यदेतदुक्तं व्याप्तं स्वभेदैर्हरिणेव विश्वं

i.e. all these calculations are full with trairāśika as world is full with (3) forms of lord viśṇu.

(Three forms Brahmā, Viṣṇu, Śiva of god will lead to understanding of fourth abstract and unknown form. Or three vedas, give the fourth Atharva veda of practical applications).

Shadow of cloud can be seen when it is almost in same direction as sun. Thus the angle between them is very small.

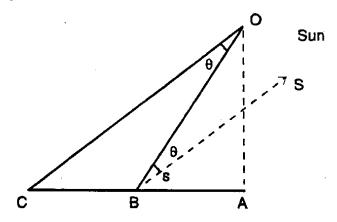



Figure 8 - height of clond

A cloud at O is seen from B. Sun is in direction of BS. Shadow of O is at C and A is its base point.

$\angle SBO = \angle BOC = \theta$ (say)

In ΔOBC,

$$\frac{\sin\theta}{BC} = \frac{\sin OBC}{OC}$$

$$\text{or } OC = BC \times \frac{\sin OBC}{\sin\theta}$$

Sin OBC is slightly less than 1, Sinθ is slightly less thanθ hence $\frac{\sin OBC}{\sin\theta} = \frac{1}{\theta}$ approx.

Hence $OC = \frac{BC}{\theta}$ where θ is in radians

$= \frac{BC \times R}{\theta}$ where θ is in kalā

i.e. chāyā karṇa of cloud $= \frac{\text{chāyā} \times \text{trijyā}}{\text{Angle}}$

For the cloud, shadow is AC, OA is height. This is similar to triangle of śaṅku height and its shadow. Hence

$$\frac{OC}{\text{śaṅku chāyā karṇa}} = \frac{AC}{\text{śaṅku shadow}} = \frac{OA}{\text{śaṅku height}}$$

Verses 86-102 : Vision limit for different heights.

In eighteenth chapter limiting distance of vision had been described. Now further information is given which will make the spherical shape of earth more clear. (86)

The circle around a śaṅku with radius equal to the limiting distance of vision, is called the visible horizon of śaṅku. (87)

The hill which is not visible due to great distance, becomes visible when we climb on a high place. (88)

From two hills at a distance, two circles of visible horizon are formed. In the common area of two circles, both the hill tops will be visible. (89)

On the boundary of horizon of one hill, the other hill will be seen upto some portion below the top. (90)

From horizon point of one hill, the angle of visible part of second hill is multiplied by difference of height of both hills and divided by distance between the two śaṅkus. This will be sphuṭa value of visible angle in miniutes (liptā or kalā). (91)

If the horizon circles of two hills do not meet, then from horizon of one hill, the other will not be visible. (92)

If the horizon circle of a small hill is completely within horizon circle of a big hill, then from horizon of big hill only big hill will be seen, smaller hill will be below the horizon. (93)

If the horizon circles of both hills touch each other, then from the contact point, bigger hill will be seen just above the small hill. (94)

If surface of earth had been plane like a mirror, then from small height also, small hills at distance would have been seen close to earth's surface. (95)

From the hill top, round shape of earth is seen within the horizon circle. The round shape of ecliptic also is clearly visible from hill top. (96)

A person at hill top sees the sunrise earlier by asus equal to deśāntara liptā of visibility limit

compared to person at the base. He sees the sunset, same time later. (97)

A person of 3-1/2 hands height sees the horizon on a plane surface upto 9000 hands. (98)

If the difference of time at hill top and base is less than 1 pala, it is neglected in calculation. Even though it is observable, it has no use in eclipse. (99)

Circumfernce of earth is (8,04,24,960) hands. On this the visible distance limit is being stated in units of 1/4 koṣa. First śaṅku is 4096 hands height. Next second to 11th śaṅkus are each half of the former saṅku (100);

To find the visible distance, height of śaṅku is divided by (4096) hands and multiplied by square of its visible distance ($162^2 = 26,244$). Square root of the result is the distance. (101)

Sl. No. of Śaṅku	Śanku height in hands	Visible distance (1/4 kosa units)
1	4096	162
2	2048	115
3	1024	81
4	512	57
5	256	40
6	128	29
7	64	20
8	32	14
9	16	10
10	8	7
11	4	5

Radius of earth is (1,28,00,000) hands. We find its square and square of radius added with 1 hand. Of the difference square root is taken. This will be limit of visible distance for śaṅku of 1 hand.

To find visible distance, the distance for śaṅku of 1 hand is squared and multiplied with height of given śaṅku. The square root of the product will give visible distance upto 8 lakh hands. (102)

Notes : (1) Visible horizon

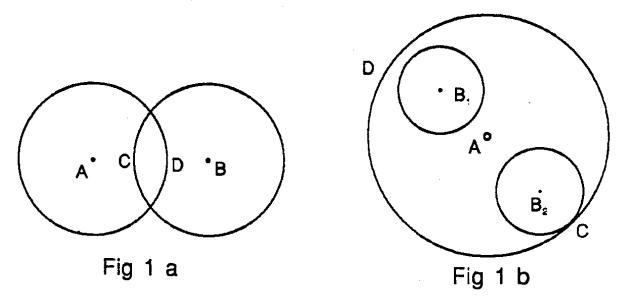



Fig 9 - Visibility circles

Figure 9(a) shows visibility circles of hills at A and B having common area CD. Since this area CD lies in both circles, both the hills will be visible from any point in it. From point C, B will be just visible, but a grater portion of A will be visible. Its sphuṭa angle is defined as

$$\text{Angle of visible poriton X } \frac{h_1 - h_2}{AB}$$

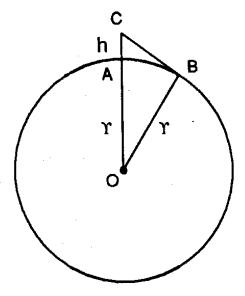
where h_1 and h_2 are heights of hills at A and B

In figure 9(b), visibility circle of B_1 is completely within circle of A. From point D, A will be visible, but B, is beyond limit of visibility, hence will be below horizon. Visibility circle of B_2 touches it at C. Hence from C both hills will be just visible.

(2) **Visible distance of desired śaṅku**-Śaṅku of height h is placed at A. Its visible distance is at point B. BC is tangent and perpendicular to radius OB. Hence

Fig 10 - Visibility limit

$OC^2 = OB^2 + BC^2$

or $(r+h)^2 = r^2 + d^2$

where r is radius of earth and d is distance BC from śanku top

or $r^2 + 2rh + h^2 = r^2 + d^2$

or $d^2 = 2rh + h^2 = (r+h)^2 - r^2$ - - - (1)

or 2rh approx. - - - (1a)

Thus from (1), we add the śaṅku height to radius and from its sqaure, we substract the square of radius. That is d^2 whose square root is the distance d.

From (1a), $d^2 = 2rh$

If n is distance for 1 hand śaṅku, then

$n^2 = 2r$

or $d^2 = n^2 h$ or $d = \sqrt{n^2 h}$

Hence square of visible distance of 1 hand śaṅku is multiplied by height of śaṅku and its square root is taken. In this formula, we neglect the height square as very small. Hence this will work only for d upto r/16 = 8 lakh hands. For greater accuracy, we find the distance in comparison to śaṅku of 4096 hand height from (1a).

Verses 103-108 : Lords of days etc.

To know the lords of days, years, hora and month, the traditional position of planets will be assumed. Slowest planet will be kept at top. (103)

It will be followed by next faster planets in order. According to Sūrya siddhānta their order is śani, guru, maṅgala, sūrya, śukra, budha and candra. (104)

In this sequence, fourth planet from śani will be owner of day and 3rd from guru will be lord of civil year. (105)

In 60 daṇḍas, there are 24 kāla horās. They start with mid night at Laṅkā. (106)

Beginning with śukra, 2nd and 3rd etc. planets upwards will be lords of civil months. At the beginning of creation, sun was lord of all - day, month, year and horā. (107)

Lords of year (solar and other), months (solar) and days etc are counted from beginning itself. (108)

Notes : This system of ownership was fixed by Varāhamhira in his Bṛhatsaṁhitā at the beginning of Vikrama erā. He has stated his time as 500 Śaka, which is actually Sudraka śaka in 500 years before Vikram era i.e. 557 BC. By misquoting as Śālivāhana śaka which started 135 years after his time, his time is taken as 578 AD. The same system was followed in Chaldea in around 300 BC.

The order of ownership is based on following scale of civil time -

24 horā = 1 day (origin of hours)

30 days = 2 month

12 months = 1 year

Order of planets with increasing speed is

(1) Saturn, (2) Jupiter (3) Mars (4) Sun (5) Venus (6) Mercury (7) Moon

Starting with horā lord with saturday, next day horā lord will be 25th i.e. (7X3) + 4th lord.

Similarly after 30 days we cross 3 lords each day i.e. 90 lords in 1 month = 12 X 7 + 6 i.e. 7th planet will be lord of next month. Thus the previous planet or moving upwards in the list we get the month lords.

After 12 months we cross 12 X 6 = 72 lords or 10 X 7 + 2 = 2 lords. Thus 3rd lord will be owner of successive year.

Thus Ist horā lord on saturday is saturn, but on next day Ist horā lord is sun, hence it is called sunday. Thus the weak day lords are found by counting 4th planet in this series each time - Sunday, Monday etc.

Verses 109-119 : Extent of spread of light

Scriptures have said that brightness of sun is hundred times the combined light of moon and stars. When diameter is 10 times, area of circle is 100 times. (109)

Spread of brightness is according to area. Hence the light from 1/10th of diameter will reach 1/10th of distance reached by light from full diameter. (110)

At 10 times distance, brightness becomes 1/100. Hence, at distance of 2000 times the sun's diameter, heat of sun light vanishes. Hence the heat of other planets and stars also vanishes at

distance of 200 times their diameter because their brightness is 1/100 of sun. (111-112)

Similarly, heat of earth's energy also vanishes at distance equal to 200 times its diameter. Brightness of ray however is upto 50 times, this distance. (113)

Visibility of ray is further 25 times the distance. This limit has been estimated by me from light of lamp, moon and sun. (114)

Diameter of sun is (72,000) yojanas. Hence limit of its heat is upto (14,40,00,000) yojanas. (115)

Brightness of its rays is upto (7,20,00,00,000) and its visibility limit is (1,80,00,00,00,000) yojanas. (116)

Human distance and time units multiplied by 360° give divine units. According to divine units 100 crore yojaṇas is the diameter of universe (Brahmāṇḍa). Hence, in human yojanas it is (3,60,00,00,00,000) yojanas (117-118).

The limit of visibility of sun (and stars) is the visibility orbit of sun. Orbit of that universe (circumference) is (11,30,97,60,00,000) yojanas. At this distance angular diameter is 1/12 vikalā. Other planets are 1/72 of this value. (119)

Notes : Light comes out of a sphere of radius r from its surface area 4 πr^2, hence its total out put is proportional to r^2. After unit time, it spreads to equal distance in all direction. At distance R, it is spread over surface of sphere 4 πR^2. So intensity of same amount of light is propotional to $1/R^2$, which is light per unit area. Thus if r is increased by K times, total light will increase K^2 times, and its $1/R^2$ intensity will be at KR distance.

Here, propogation of heat, brightness and visibility all are subjective words without any quantitative definition. Earth is at distance of 109 X diameter of sun. Heat limit has been assumed to be about 20 times this distance. According to this we should feel the heat of mars and venus at their farthest distance. We definitely receive some energy, but physically we cannot feel the heat. Similarly, brightness means very bright and visibility means just visible to human eye. This depends on eye sight. With telescope, visibility is more and light rays go almost to infinite distance. Telescopes can see objects upto 10 billion light years away.

Verses 120-122 : Criticism of Bhāskara II

Śrī Bhāskarācārya ! You have said that ākāśa Kakśā lies at the place where light of sun vanishes and blackness starts. According to you, the planets move exactly this distance during a day of Brahmā (1 kalpa). If a learned man like you tells like this, we can only feel hurt. (120)

If a planets moving in any orbit moves equal to length of ākāśa kakśā, then a man will move a distance equal to earth's circumference within his own house, why this doesn't happen ? (121)

You have stated the circumference of sky as (18,71,20,69,20,00,00,000) yojanas. Diameter of sun (6522 yojanas according to Bhāskara) is not even 1 part in 1 lakh parts. Then how the darkness upto sky sphere is removed by sun ?

Notes : These discussions are based on two cosmological assumptions

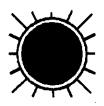
(i) Limit of sky is the distance till which sun rays reach i.e. sun is visible.

(ii) All planets move with same linear speed and move the distance of sky orbit in one kalpa. In a way this is the basis of calculation of kalpa and yuga, when all planets complete one revolution. Then from actual linear speed of moon, we can calculate length of sky orbit.

Verses 123-125 : Prayer and end

God is ocean of surprises. There are infinite worlds within his great body. Their number is not known to Brahmā also. hence the measurements of world in vedas and puraṇas cannot be false (must be true for some world). But I will discuss only the planets visible near earth. (123)

May lord Jagannātha, do good to the beings with bodies of 5 elements like sky, by whom Śankhāsura had been killed giving pleasure to gods, whose lotus hand looks more beautiful with sign of conch (śaṅkha), and who is lord of śaṅkha kśetra. (124)

Thus ends the 19th chapter describing stars and sky in siddhānta darpaṇa written as text book for correct calculation by Śrī Candraśekhara born in famous royal family of Orissa. (125)

Chapter - 20

INSTRUMENTS

Golādi Yantra Varṇana

Verses 1-5 : Scope and introduction

Without instruments, motion of planets and eclipse etc. cannot be known easily. Motion of time also cannot be known without instruments. Hence Srī Candraśekhara describes the process of constrc-tion of instruments according to his intellect. (1)

Types of instruments are classified on basis of sphere or time. Within those also, many sub types exist. Out of them some instruments can be constructed easily. Some are described for infor-mation of laymen. (2)

Here two types of gola yantras will be described. One is of one Kakśa (axis) and other is bahu Kakśa (multiple axes). One axis yantra has been described by earlier ācāryas. Bahu kakśa is being described according to my own research. (3)

After bath and well dressed, astronomer should go to a secret (or reserved) and clean place and he should worship various gods - Sun and other planets, nakśatras all around. (4)

Lokapālas such as Indra, guhyaka and guru (teacher) also are worshipped to remove obstruc-tions. After that, construction of golayantra should

be started for instrument knowledge of students. (5)

Notes : (1) Instruments are necessary to explain the model of universe, to make measurements of planetary positions, and to know the times from planetary motions or from automatic watches.

Models are necessary because, three. dimensional spherical locations cannot be drawn on a paper and it is difficult to comprehend their true motion without models. The gola yantras described here are basically models to explain planetary orbits and the sphere of earth. They can also be used to make measurements, but the method thereof has not been described.

Construction or name of other handy instruments has been mentioned only without their methods of use. They can be used for rough measurements.

The really accurate instruments can be constructed only at a great cost with govenment assistance. During thousand years of muslim and foreign rule this was not provided, rather suppressed. We neither know their theory nor have samples to see, so that they could be described by the author.

(2) Since last one thousand years, only obstruction has been feared in construction of astronomical instruments due to foreign rule. Hence it has been prescribed that the work should be done secretly. Various pujās are for praying divine help, as well as to make it look like other elaborate pūjās.

Only when Mughal empire declined after Aurangajeb, Sawai Jaisingh, could control a sizeable area in Rajsthan and Gujarat and after a lot of requests to Muhammad Shah he could reconstruct some of the instruments described in earlier texts. He collected informations about all practical construnctions in independant Islamic countries - motably observatories of Maragha, south from Tabriz in Iran founded by Halāgū Khān and Samarkanda, founded by Ulug beg. He also collected many books from Europe to know about their current state of development. He procured a telescope from European travellers and tried to learn the practical methods. Through Jesuit travellers to China he procured Chinese lists also. However, the attempts of revival lasted only for about 20 years from 1720 to 1743 till the time of his death. He named his observation tables Ziz-i-Muhammad Shāhī, so that his work would be allowed to continue. Yantra rāja-racanā and sūrya siddhānta vyākhyā are directly attributed to him. In addition, his astromomers like Jagannātha Samrāta wrote many books. Jagannātha directed the observations, wrote books based on Ptolemy and Euclid and independant works also like yantra prakāra. Kevalarāma wrote many sāraṇīs (charts) for calculating and pañcāṅgs. Nayanasukha Upadhyāya translated many books and wrote yantrarāja.

(3) Instruments mentioned in earlier Texts
Time Measuring Instruments

	Instrument	Author/Text	Type
1	Nāḍikā yantra	Vedāṅga jyotiṣa	Clepsydra, out flow of water clock

2	Ghaṭikā Yantra	Sūrya siddhānta	Sinking bowl clepsydra
3	Naḍī valaya	Bhāskarācārya II	Equinoctical sun dial
4	Phalaka yantra	Bhāskarācārya II	A kind of sun dial
5	Kartari yantra	Lalla and Śrīpati	Equinoctical sun dial
6	Dhruvabhrama yantra	Padmanābha	Measuring time by rotation of saptarṣi (Great bear)
7	Kapāla yantra	Varāhamihira	Hemispherical sundial
8	Pratoda yantra	Gaṇeśa Daivajña	Whip shaped gnomonic device
9	Yantrarāja	Mahendra Sūri	Astrolabe
10	Swayamvaha	Āryabhaṭa I	Automatic time measuring device like clock, nature not described

Coordinate Measuring devices

	Instrument	Author	Type
1.	Turīya yantra	Cakradhara	Quadrant
2.	Gola yantra	Cintāmaṇi Dīkśita	Armillary sphere
3	Yantrarāja	Mahendra Sūri etc.	Astrolabe
4	Yaṣṭi	Bhāskara II etc	Staff
5	Cakra yantra	Varāhamihira	wooden or metallic wheel
6	Cāpa yantra	Sūrya siddhānta and Bhāskara II	Half the structure of Cakra yantra

Kartarī yantra of Brahmagupta was formed of two semicircular plates, in planes of equator and meridian planes. They meet each other other like

two blades of a scissors - hence named kartarī. It was translated as usṭura - lava which became 'Astrolab' in European languages. Model for intersection of ecliptic and equator might be origin of the word 'astronomy'.

Accurate measurements, needed big masonery structure which can be constructed only with govt help. Jaisingh built the following instruments, which are a combination of above in much bigger size -

Low Precision instruments

	Instrument	Number	Location
1.	Dhruvadarśaka Paṭṭikā (North star indicator)	1	Jaipur
2.	Nāḍī valaya (Eqiunoctical dial)	5	Jaipur (2), Vārāṇasī Ujjain, Mathurā
3.	Palabhā (Horizontal sundial)	2	Jaipur, Ujjain
4.	Agrā (Amplitude inst)	5	Delhi, Ujjain, Māthurā, (2), Jaipur
5.	Śaṅku (Horizontal dial)	1	Mathurā
6.	Unknown instrument	1	Vārānasī

Medium Precision Instruments

	Instrument	Number	Location
1.	Jaiprakāśa (Hemispherical inst)	2	Delhi, Jaipur
2.	Rāma yantra (cylinderical inst)	2	Delhi, Jaipur
3.	Rāśi valaya (Ecliptic dial)	12	Jaipur
4.	Śara yantra (Celestial latitude dial)	1	Jaipur

5.	Digamśa (Azimuth circle)	3	Jaipur, Ujjain, Vārāṇasī
6.	Kapāla (Hemispherical dial)	2	Jaipur

High precision instruments

	Instruments	Number	Location
1.	Samrāṭa (Equinoctical sundial)	6	Delhi, Jaipur (2), Ujjain Vārāṇasī (2)
2.	Ṣaṣṭhaṁśa (60 deg) meridian chamber)	5	Delhi, Jaipur (5)
3.	Dakśiṇottara Bhitti '(Meridian dial)	6	Delhi, Jaipur, Ujjain, Vārāṇasī (2), Mathurā

Instruments added afterwards

1.	Miśra yantra (Composite instrument)	1	Delhi
2.	Śaṅku yantra (Vertical staff)	1	Ujjain
3.	Horizontal scale (Known as seat of Jaisingh)	1	Jaipur

(4) Most of the texts including the present book do not explain how the instruments are used for measurement. Their use as model is very clear. But they have not been in use for about 1000 years. Hence their principle of working is derived from their descriptions mainly during Jaisingh period. Reference may be made to three authorities in their field -

1. Article on 'Astronomical Instruments' in History of Astronomy in India - Indian National

Science Academy New Delhi-2, by Sri R.N. Rai, former head of Deptt. of Physics, NCERT, New Delhi.

2. Sawai Jai Singh and his Astronomy - by Sri Virendra Nath Sharma - Prof of Physics and Astronomy, University of Wisconsin, USA - Published by Motilal Banarasidass, New Delhi.

3. Sreeramula Rajeswara Sarma - His article in Indian Journal of history of Science, Vol. 29, No.4; Oct.94. and his many other papers referred in it. He is professor in Aligarh Muslim University.

Verses 6 - 44 - Golayantra of 2 or 1 axis -

Construct a wooden sphere (globe) of circumference 6 aṅgulas. Position of continents, oceans and mountains are marked in it (6).

Holes are made at pole points and a dhruvayaṣṭi (polar stick) called meru-daṇḍa (meru stick or back bone) also is inserted. This rod is round (cylindrical) 6 hands long, made of strong wood. (7)

To hold this merudaṇḍa, we fix two pillars of 5 hands length each firmly at north and south ends at a distance of 5-1/4 hands. (8)

These pillars are fixed in earth or in a heavy wooden frame. Height of pillars from wooden or earth base will be 5 hands each. At the height of 3 hands, we make holes in both pillars in which dhruva yaṣṭi can be inserted. (9)

Two holes in inclined direction are made at an angle with horizontal equal to latitude of the place. It is above the 3 hand height point in

northern pillar and below the 3 hand height in south pillar. (10)

Two circular strips of 360 aṅgula circumference each are constructed. One of them is called kśitija vṛtta (horizon circle). (11)

The other circle is called ūrdhvādhara vṛtta (meridian circle). This is joined to horizon circle at north and south points and two holes are made at joints. (12)

Dhruva yaṣṭi is inserted in the circles keeping globe in middle. Both circles will be loosely fitted and greased so that they are rotated easily. The dhruva yaṣṭi will be inserted in holes of the pillar (not specified whether horizontal or inclined holes). (13)

Another bamboo strip 360 aṅgula long will be folded into a circle and be kept in plane of equator of earth i.e. between horizontal and vertical circles. This is called nāḍi maṇḍala. Angles will marked (in time units) on this. (14)

On both sides of equator circle, we put 3 circles on each side at a distance of krāntis of 1, 2, 3 rāśis - i.e. total 6 diurnal circles of decreasing sizes. (15)

On both sides of dhruva yaṣṭi, one circular plate each will be fixed with yaṣṭi through its centre. Along the circumference 360° degree divisions are marked with one thin pointer at each division. (16)

The pointers will be fixed with 'vajralepa' (strong paste or welding) so that they do not fall. On this plate we give mark of north pole at 0° and south pole at 180°. (17)

On the rings of bamboo which start after dhruva signs at end of plate, lines are marked at the end of 1 aṅgula each, corresponding to 1° difference. These signs will be in different planes of the rings. (18)

On wooden globe of earth, we fix thin bamboo circles parallel to equator. These will be biggest at equator and become smaller as we proceed towards poles. (19)

In the bamboo circle, boundaries of rāśis are marked prominently. Then āyana corrected meṣa 0° is put in contact with equator circle. (20)

Maximum difference between this krānti vṛtta and equator circle will be at sāyana karka and makara beginning equal to 23°30′. (21)

On these circles 6 points are given - east, west, north south points of horizon circle and upper and lower points on vertical (meridian) circle. These two points (upper and lower) are called svastika. Upper point is called zenith (kha-svastika). (22)

Pātas of planets like moon is marked on ecliptic (krānti vṛtta). Moon vimaṇḍala (orbit) is fixed at that place on krānti vṛtta and again at 6 rāśi difference. (23)

At 3 rāśi difference from these sampāta points (of vimaṇḍala and krānti vṛtta), the distance between the circles will be equal to maximum śara of moon in north and south directions. (24)

Similarly vimaṇḍala (orbits) of other planets like mars also will be fixed to the krānti vṛtta. But

their pāta are corrected with śīghra phala. Their maximum śara (distance from ecliptic) is found in proportion to their distance (from earth). That will be maximum distance of vimaṇḍala in north and south. (25)

Horizontal circle at equator is called unmaṇḍala at other places. This circle measures the cara (difference in half day length). Intersection with diurnal circle and meridian circle is called khārddha. Perpendicular from khārddha on udaya-asta line on horizon plane is called 'hṛti'. In great circle (equator) hṛti is called 'antyajyā. (26)

Both base circles (kśitija and ūrdhvādhara), equator, ecliptic, have equal diameter. Still kśitija vṛtta is kept slightly bigger. Otherwise, the circles cannot be rotated. (27)

Dhruva yaṣṭi will be firmly fixed with earth. Bhagola (the four circles on its surface) is such that it can be rotated freely. That can be done by oiling the joints. (28)

This way bhagola is contructed and planets and stars are shown. Krānti of planets, and śara of planets other than sun is found and their sphuṭa position is calculated. (29)

These planets are placed in their orbits at their rāśis. By moving the gola we imagine the 'pravaha' rotation. (30)

This bhagola construction is according to Laṅkā position. Now it is explained as to how khagola will be fixed to it. In both pillars at the height of poles, holes are drilled by iron nail. (31)

Then khagola will be formed outside bhagola. 6 circles are tied firmly to it - kśitija vṛtta, pūrvāpara

vṛtta (east west), meridian, two angle circles and dik maṇḍala. These 6 circles of khagola being fixed, bhagola will move within it. (32-33).

To see the shadow and rising and setting of planets in own place, we keep the dhruva yaṣṭi in the inclined holes of pillar at angle of local akśāṁśa. (34)

In khagola and bhagola both - east west circle and horizon circle of the local place are tied. (35)

On khagola, we put nails on the points of intersection of yāmyottara and samamaṇḍala circles (upper and lower kha-svastikas are these points). On these nails, dṛgmaṇḍala is fixed so that it can move freely. (36)

Dṛg maṇḍala is kept outside bhagola like east west circle so that it can be moved around dhruva yaṣṭi as desired. (37)

Dṛgmaṇḍala will be kept on position of a planet. When it is kept on vitribha, it becomes dṛkkśepa circle. (38)

It is impossible for a man to enter within these spheres. Hence, we see the planet from outside towards earth's surface. This line will be same as line of sight of planet from earth's surface, but in opposite direction. (39)

In eight directions (4 cardinal and 4 angle directions) of horizon circle 8 small nails are given and the hemisphere below it is covered with cloth. Above horizon, a wooden horizon disc is kept, so that upper hemisphere is seen. (40)

Alternatively, sphere will be kept inside a trench so that lower half remains underground and

upper half remains visible outside. Then bhagola can be rotated and the visible hemisphere of local horizon can be seen according to position of earth etc. (41)

(Siddhānta Śiromaṇi) Bhagola is tied with dhruva yaṣṭi. Yaṣṭi is inserted inside khagola tube and bhagola is rotated. At end of yaṣṭi, two tubes or nails are used to keep khagola and dṛg gola fixed. (42)

One axis gola : This two axis gola made of bhūgola (earth), bhagola (nakśatras), dṛggola (visible circle) and ākāśa gola (sky) can be made one axis gola also. (43)

Bhagala (nakśatras) is seen in orbit of moon only, hence it can be attached to that only. Intelligent persons can change the sizes of these golas according to their convenience. (44)

Notes (1)

Fig 1 shows bha-bhū-gola as named in Sūrya siddhānta. It includes gola (spheres) of bha

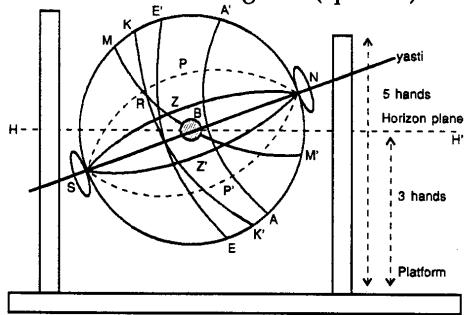



Figure 1 - Eka-Kakśa Yantra

(nakśatra) and bhū (earth). The figure shows yaṣṭi passing through NS in inclined position. Meridian circles through north and south poles N and S is NZSZ′ and horizon is NP SP′ perpendicular to it (it cannot be shown on paper) shown by dotted lines.

Small earth globe is at B. EE′ is equator, KK′ is ecliptic. One diurnal circle for north krānti is shown as AA′. Orbit of moon MM′ is inclined to ecliptic touching it at pāta R.

HH′ is horizontal plane passing through holes in pillars at 3 hands height above earth or wooden frame. Portion above HH′ is visible part of sky above horizon. Hemisphere below horizon HH′ is invisible. It is covered by cloth or kept underground.

Ecliptic, equator and diurnal circles are on bhagola sphere which is fixed by wires etc. (not stated in text) and is fixed to polar axis (yaṣṭi).

Khagola is outer sphere made of koṇa circles, east west and meridian circles, horizon and unmaṇḍala - six circles. It is attached to two hollow cylinders in which yaṣṭi in inserted.

Outer most sphere is dṛggola in which circles forming both inner spheres are mixed together. Hence it is called double sphere (dṛg = eyes = number two).

(2) This is called armillary sphere in western astronomy and Dhāt-al-Halaq in Arabic. A rough

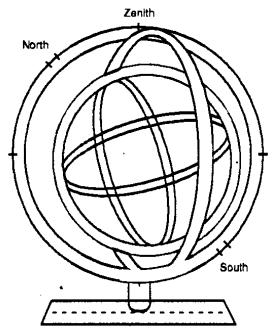



Figure 2 - Armillary sphere

diagram of armillary sphere is shown in figure 2. It is fixed on a stand like a globe. All circles are not shown. One metal sphere of 53 cms diameter of Jaisingh period is in Jaipur museum.

(3) **Use of the instrument** - (i) To find the time since sunrise and the lagna at that time, the sphere is set so that east point is exactly towards east and horizon is level as water. The position of sun on ecliptic is now obtained by calculation and the bhagola is rotated to bring this point of ecliptic on the eastern horizon and a pin is fixed here to mark the position of sun. A pin is also fixed to mark the point of equinoctical in the bhagola intersected by the eastern horizon i.e. east point. The bhagola is now rotated west ward till the sun throws its shadow on the centre of the earth. The distance between mark made on the equinoctical and new eastern point of the horizon will represent the time from sun rise. The point of the ecliptic, now cut by the horizon will be lagna (Siddhānta Śiromaṇi).

(ii) Sun and moon positions - When both sun and moon are visible, ecliptic ring is rotated so that shadow falls on itself. Then outer latitude ring is rotated so that its shadow falls on itself. The point of intersection of two rings (ecliptic and latitude) is spaṣṭa sūrya.

Next, at the same time, rotate the inner latitude (for sighting, two vanes are kept on a diameter) ring, so that moon is seen through the two sighting apertures on the vanes. The point of intersection of inner latitude ring and ecliptic facing moon indicates true moon. From the ecliptic, upto the place of sight, above or below ecliptic, the angular distance indicates the north or south latitude of moon (yantra prakāra).

(iii) Planets and stars in night - Calculate the sign and the angular distance of a star from 0° of ecliptic and it is marked on ecliptic ring. The outer latitude circle is aligned with that point and is clamped there. Then the planet or star is sighted with the latitude circle. Another person should observe the star with sights of inner circle. When this is done, the sign or latitude of the object is given by the intersection of latitude circle and ecliptic ring.

Celestial latitude is the angular distance of the point of intersection of latitude circle and ecliptic to the sighting aperture.

Yantra prakāra recommends two latitude circles (orbits) for any planet, one above and one within the ecliptic. At inner circle a sighting vane (holes in a diameter direction) is made. Outer circles are in dṛggola.

Verses 45-75 : Bahukakśa yantra (multiple axes).

Now I describe bahukakśa yantra (multi axis or chamber) by whose mere seeing; movement of planets, their disc, latitude, direct and retrograde motions can be directly observed from earth itself. (45)

Two solid wooden wheels of 1 hand diameter each are made. Two pillars are fixed in north south direction on an axle kept on two pillars, two wheels are kept within the pillars. (46)

In both wooden wheels, śara (wooden stick) is inserted through a hole and at the end of this śara, a 'nemi' (wooden, circuilar strip with divisions of 360° marked) is attached. Distance between nemi and axle will be 90 aṅgula. Wooden wheels are 3 hands from each other. (47)

To move the two wheels with nemi, one wooden stick 3 hands long will be inserted in these wheels. In the 3 hand distance between wheels, orbits of planets and earth will be placed. (48)

36 aṅgulas from south wheel towards north, one rod will be inserted. At the end of that, a small size earth will be fixed. Around earth will be moon's orbit. (49)

Moon's orbit will be movable round earth and will be 6 aṅgula length. At half aṅgula distance on circumference 1 rāśi will be marked. In moon's orbit, krānti, mandocca, pāta and candra position are shown. It will be fixed to south wheel through four rods or pins. (50)

12 rāśi will be marked in south wheel. In that scale only mandocca and pata of moon will be shown. In north wheel, orbit of other planets will be shown. (51)

Then a graha cakra yaṣṭi (rod containing orbits) will be made. This rod will be thin and at the end will lie sun. This will be wider at base where other planets can be fixed. This rod will be for fixing in the north wheel. (52)

At the end of planet rod (graha yaṣṭi), madhya sun will be on a thin pin. At 1/4 aṅgula from that sphuṭa sun will be placed in the direction of mandocca. This (sphuṭa sun) will be bigger than earth, smooth and perfectly round. (53)

At 1/4 hands (30 aṅgula) north of graha yaṣṭi, 12 spokes will be fitted with a central wheel. Each spoke (arā) will be 3 hands long. (54)

Madhyama sun will be at end of southern rod at centre of base wheel and around it will be orbit of other planets. (55)

In all these orbits, we show the ucca, krānti and pāta etc. It will be marked with rāśi and degrees. On its inner surface, the five star planets, starting with maṅgala will be located. Planets can be placed at their places in orbits made of wooden strips, which can be rotated. (56)

First orbit around madhyama sun will be of mercury with circumference 18 aṅgula. Next 2nd and 3rd orbits will be of śukra (36 aṅgulas) and maṅgala of 72 aṅgulas. (57)

Guru orbit will be of 246 aṅgula and śani of 441 aṅgula. These orbits called vimaṇḍala will be joined with unequal nails. (58)

On these south facing nails, vimaṇḍala is fixed. The nails will be equal in gola sandhi

(equinox position) where the orbit will be in east-west plane through medium sun. (59).

In south ayana end (max. south krānti), nails will be longer and at north ayana end they will be shorter. Their difference will be proportional to krānti difference in north and south ayanas. (60)

To rotate sun orbit, a hole in north wheel is made at 7/38 aṅgula distance from centre of wheel. This hole is bigger in uttarāyaṇa (north krānti) and smaller in south ayana. Graha cakra rod will be inserted into it. (61-62)

From outside of the north wheel, graha yaṣṭi is rotated in east direction. Planets will be fixed in their proper positions of rāśi etc, through pins. (63)

We set it so that sun at end of yaṣṭi is in east west circle and in gola sandhi from earth (meṣa 0° sāyana). At beginning of sāyana karka, sun will go (3/2/30) aṅgula north and in sāyana makara beginning same distance south. (64)

The circular hole (in north wheel) also will be similarly at more or less distance. Graha cakra yaṣṭi can also be rotated from south side by putting hands within its spokes. (65)

We find the manda sphuṭa graha (as seen from centre of sun) for the desired time. Sun will be rotated to its position as seen from earth. (66)

As seen from earth, graha will be seen at its sphuṭa rāśi etc. Then we come out of instrument and again rotate it westwards. (67)

Alternatively, an intelligent person can make an ecliptic with 27 nakśatras marked. It is fixed in

bhagola at proper place (inclined at 23°30′ to equator). In this nakśatra circle, we see the direct and retrograde motions of the planets easily. (68)

Or, we fix two pillars east and west from sun at distance of 344 aṅgulas distance. In that, circle of krānti will be fixed showing 1°=6 aṅgulas. (69)

Mean sun is put west from earth and maṅgala is kept at 6 rāśi difference from that in its orbit. (70)

The line from earth to maṅgala is extended upto sign in the east direction. In this direction, sun will be located in orbit of 48 aṅgula. (71)

Mean sun will move 1 aṅgula (7-1/2°) in half a fortnight (1/48 of year). mars will move 1 aṅgula in same period (equal to 5° in 72 aṅgula orbit) from sun. From the sun-mars line, the earth mars line will be upwards in the east pillar. (72)

As seen from earth, motion of mars will look retrograde. Similarly, the forward and retrograde motion of other planets also can be seen. Motion of moon and its eclipse should be seen from equator plane. (73)

Moon orbit is kept between śukra and maṅgala orbits, so that they don't collide. To move the planetary orbits to east or west, care has to be taken, so that they do not bend or break. (74)

This is description of my bahu kakśa (multi axis) yantra. In this, motion of planets can be seen as in sky. This can be seen separately for each planet with help of thread (to know the direction). (75)

Notes - O_1 is centre of north wheel W_1 with

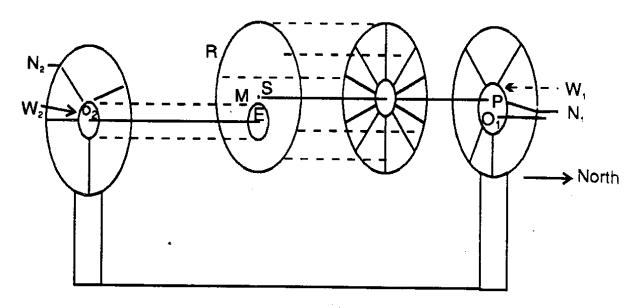



Figure 3 - Bahukakśa yantra

nemi N_1 around it. At P when OP = 7/38 aṅgula, yaṣṭi PS is inserted. In middle there are 12 spokes fixed around a wooden base. On nails from spokes, orbit of planet R is inserted. Earth is inserted through a rod fixed at centre O_2 of wheel W_2. W_2 also has N_2 of 90 aṅgula radius. Around earth E orbit of moon M is fixed through rods connected to wheel W_2.

Thus earth and moon are fixed in middle through south wheel. At same position, orbits of mean sun, and planet orbits around it, are fixed through north wheel, as shown in diagram.

Through the angles graduated on nemi of 90 aṅgula radius on each wheel, we can see the planetary position. To see them more clearly and to measure through thread, an ecliptic circle in middle is placed through pillars in east west direction.

The orbits in east west plane and ecliptic are shown below in figure 4.

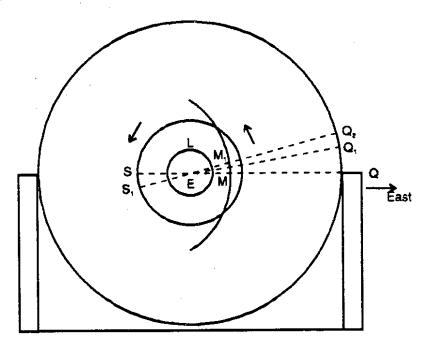



Figure 3 - Motion of Mars from earth and sun

E is earth with orbit L of Moon. S, S_1 are positions of sun around earth and M, M_1 the coresponding positions of mars at interval of 1/2 fortnight (7-1/2 days) S S_1 = M M_1 = 1 aṅgula.

In beginning S and M are on opposite directions from E with S in west and M in east. Position of M in ecliptic is Q. Position of M_1 after 7-1/2 days is Q_1 as seen from sun S_1 and Q_2 as seen from earth. Q_2 is above Q_1.

Verses 76-77 - Kāla yantra

Gola yantras of 1 and many axis have been described. From movement of shadow of any object or of own body, or from rotation of an instrument, time can be known. Still for the knowledge of laymen, I state about Kāla yantra (time instruments). (76)

Time can be known from cakra yantra, yaṣṭi, cāpa yantra (semi circular), śaṅku, quadrant, ghaṭī yantra, phalaka yantra described by Bhāskara, a glass pot, water, sand or thread yantra etc. Persons

knowing yantra can use any of them to find time. (77)

Verses 78-81 - Golārdha yantra

A hemispherical bowl is made in earth like lower half of a water pot. Its circular base is upwards on ground level. On that, direction points like east west are marked. The ground level circle is dividor of the sphere. (78)

On its internal surface, east-west and meridian circles (half portions) are drawn. Below the south point, north pole is marked at difference of latitude. Then we draw a circle with centre at pole and difference between sun and pole as radius. (79)

This will be the diurnal circle for that time. A straight śaṅku of height equal to radius of diurnal circle is kept. The diurnal circle also will be marked with daṇḍa pala etc. From position of śanku, shadow at that time can be known. Intelligent men can know the time during night also by observing moon. (80-81)

Notes : This has been called kapāla yantra by Varāhamihira etc. But kapāla yantras described by Āryabhaṭa and sūrya siddhānta are water instruments. Varāhamihira instrument is movable hemishphere with a śaṅku equal to radius at its centre. Its plane surface is raised so that north elevation is equal to latitude, i.e. it is in hoirzontal plane of equator.

However, the kapāla yantra here are fixed bowl excavated in earth as described in siddhānta samrāṭa of Jagannātha and yantraprakāra of Jaisingh. Jaisingh made two modifications, his Jayaprakāśa yantra contained cross wires in north

south and east west directions on ground level instead of śaṅku. Thus intersection of cross wires is tip of śanku. From that, we know the coordinates in both directions. Another hemisphere on its side is also used for transforming horizontal coordinates into equatorial coordinates and not for observation.

The main golārdha yantra may be called kapāla A, it is shown in figure 5. Its surface represents inverted images of two spherical coordinate system - horizon system and equatorial. In horizon system, the rim of hemisphere is the local horizon and bottom point, the zenith. Cardinal points are marked on rim and cross wires stretched between them.

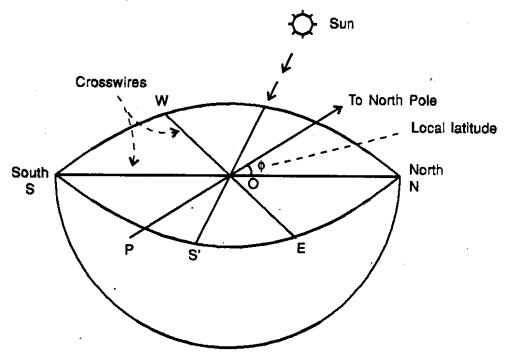



Figure 5 - Golārdha yantra

O is intersection of cross wires EW and NS or tip of śaṅku. N, W, S, E are cardinal points. P is position below S on great circle passing through N, S and bottom point (lower half of meridian). From bottom zenith point a number of equal azimuth lines are drawn upto the rim or horizon. Next, a number of equally spaced circles with their centres on vertical axis passing through

zenith are inscribed on the surface. These circles are parallel to rim and intersect the equal azimuth (angle with east west direction) lines at right angles.

For equatorial system, a point on meridian below south point S on rim represents the north celestial pole. At a distance of 90°, a great circle intersecting meridian at right angles represents the equator. On both sides of equator, a number of diurnal circles are drawn. From the pole, hour circles radiate out in all directions upto rim. These coordinates measure the time as stated in this book.

On a clear day, the shadow of the cross wire falling on the concave surface below indicates the coordinate of the sun. These coordinates may be read either in horizon or in equator system as desired. The time is read by shadow's angular distance from the meridian measured along a diurnal circle.

For determining ascendants, this has a set of 12 curves on its surface. The curves represent 12 ascendents. On a clear day, the shadow of cross wire falling on one of the curves, indicates the ascendant or sign emerging at the horizon at that very moment (In Jayaprakāśa, madhya lagna is indicated).

The ascendant lines are drawn from the fact that sāyana meṣa 0° always rises at east point on horizon. Then pole of ecliptic is exactly 90° away from the east point on its diurnal circle. With the location of pole as starting point (kadamba), divide the diurnal circle of pole into 12 equal parts. Then with these points of divisions as centres, draw arcs of radius 90° on the surface. These arcs represent path of the sun's shadow on the instrument's surface.

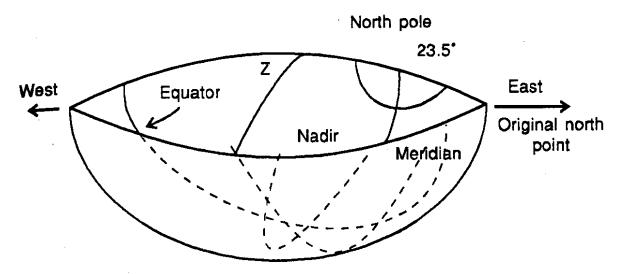



Figure 6 - Kapāla B for change of coordinates

In figure 6, Kapāla B has arcs representing two sets of coordinate systems - horizontal and equatorial. For the horizon system of coordinates, rim of instrument represents the meridian and north point Z of the rim represents the zenith. Another point at distance of latitude of place to the east of this zenith, designates north celestial pole. The north point of the original horizon is located at the east pint of the rim. The "original horizon" lies in the vertical plane passing through the east-west points on the rim. A great circle passing through the north and south points of the rim and bottom point of the rim represents the prime vertical (yāmyottara circle).

First, a point on the instrument's surface is plotted in given coordinates. Now angular distance of this plotted point from equator is found along hour circles. This is declination. Similarly, distance from rim of meridian gives the hour angle. Finally, by adding or substracting from hour angle, the angular distance between vernal equinox and meridian at time of observation, we get the right ascension (equatorial longitude).

The angular distance between any two stars may be determined by plotting two points on the instrument according to their coordinates and spanning the points with a dividor. Then from the graduated scale on rim, the separation in degrees in found.

Verses 82-92 - Māna yantra

Now I describe Māna yantra which can be used to find the angular difference between two planets or stars in the sky. We can also know the height and distance of a hill or a tree etc. (82)

A plane surface is made smooth by thick paste layer. A circle of 360 aṅgula circumference is made and marks at interval of 1 aṅgula = 1° are given. With same centre a wooden circular board is made and its centre is put on centre of ground circle. From the centre, lines are drawn to the 360 angle marks at circumference. (83)

At circumference of wooden board, even the degrees may be further subdivided. At centre of the board a strong and straight rod is inserted with length equal to the radius. (84)

Size of board may be 4, 5, 30, 15 or any desired part of the ground circle. (85)

Rod is held in hand and board is kept in plane of two planets or stars. By keeping the rod and an angle line in direction of planet, we know their angular difference. The difference in degrees is divided by 6 and multiplied by trijyā (3438). (86)

Result is divided by diameter of diurnal circle of that day, to get time difference in daṇḍa.

Height of Hill

$$= \frac{\text{Distance from base of hill to śaṅku base}}{\text{dṛgjyā}} \times \text{Śaṅku jyā}$$

Same way height of a tree is found. (87)

A dark cloud also can be seen through mānayantra to find its angle of elevation or depression. Its dṛgjyā is multiplied by (2500) hands (average height of dark clouds) and divided by śaṅku to find the distance of cloud base and own position. (88)

In finding height of trees or hill through two śaṅkus, we use māna yantra as a śaṅku. Its shadow at a near place is calculated. (89)

Then at a farther place the śaṅku and its shadow is calculated. Second śaṅku shadow is multiplied by first śaṅku and divided by first shadow and second śaṅku. it gives the ratio of distance of second shadow and first shadow from base of hill. Remaining method has been explained earlier. In this method height and distance can be calculated even when the ground is plane or oblique. (90)

For more correct value of tree or hill, yaṣṭi will be taken 5 hands long. Then the end of rod is kept in line of hill top. Then another śaṅku is kept between observer and hill, so that its top and hill top are in one line. Then we proceed as per earlier method. (91)

As we observe the north pole (in meridian circle), by same method we observe hill top (in any other direction). By seeing the jyā of elevation (unnatajyā = proportional to height) and jyā of

depession (dṛgjyā proportional to horizontal distance) at two places we can find the height and distance of the hill. Drg jyā gives distance and unnatajyā gives height. (Text describes east west difference jyā, this means distance in any direction, not in north only). (92)

Notes (1) : Construction of māna yantra has been described very clearly. But purpose of such construction is not clear. Oriyā translation by Pandita Vira Hanumāna Śastrī terms it 'māpa yantra' whose meaning is same. However, Bhāskara and Lalla have termed it yaṣṭi yantra, as mentioned in this chapter verse 77 also. Bhāskara II has mentioned that 'dhī' (i.e. intelligence) is main instrument (yantra) for using yaṣṭi or other instruments. But Sri S.D. Sharma has described a yantra named 'dhī yantra' on that basis. Yaṣṭi has to be used with a plumb line, hence Sri Sharma has considered dhī yantra as a stick with plumb line at its end.

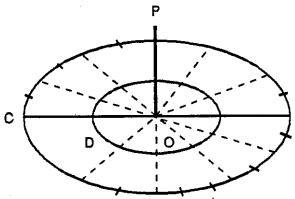



Figure 7 - Construction of māna yantra

Figure 7 shows construction of māna yantra. A circle C of 360 aṅgula circumference (i.e. 57/18 aṅgula radius) is drawn at plane ground. Its circumference is C and centre O. With same centre O, a wooden circle D is kept. C is divided into 360 divisions of angle. From O lines are drawn to each mark so that D also is divided into 360 parts.

This appears to be purpose of drawing bigger circle - for accurate division of angles on wooden board. Then the degrees can be subdivided. A pole OP equal to radius of C (57/18 angula) is kept. According to Bhāskara, D should be of radius equal to dyujyā. But this circle is made on the ground. When wooden circle is formed this is not important, only angle divisions are necessary as it is clear from verse 85. Length of rod equal to radius (57/18) makes it easy to calculate the jyā of bhūja and koti directly if it is kept in bent position.

(2) Uses : Verse 92 very clearly says that elevation of hill will be seen by same method as used for seeing the elevation of pole star. Siddhānta Śiromaṇi yantrā-dhyāya verses 40-42, describe method for finding elevation of pole star and bamboo etc, by using 'dhī yantra' (instrument of intellect). Commentary by Srī Kedāradatta Jośī gives the following method as agreed by others.

Fig-8b-dṛgjyā & unnatajyā

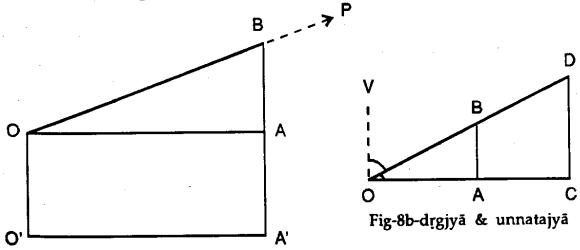



Figure 8 - Śaṅku and chāya from yaṣṭi or māna yantra

OB is yaṣṭi of length equal to radius (57/18 aṅgulas i.e. 1 aṅgula = 1° = 60 kalā). Eye is kept at O and OB is directed towards north pole in direction OP. Perpendiculars are drawn on earth form O and B through plumb lins OO′ and BA′.

Perpendiculars from both ends of yaṣṭi have to be drawn as eye is above the ground level.

Then OA = O′A′ is the bhuja or bhuja jyā because we have taken OB = R. This is proportional to distance of an object or dṛgjyā. In Fig 8b, natāmśa of object D is ∠VOB or ∠VOD and its jyā is OA = drgjyā.

AB = A′B - OO′ = Koṭi is the jyā of unnatāmśa which is ∠BOA in fig 8 B.

DC is height of object (proportional to unnatajyā) and OC is its distance (proportional to drgjyā) becuase OAB and OCD are similar triangles and

$$\frac{OA}{OC} = \frac{AB}{CD}$$

(3) Elevation of sun with yaṣṭi

To find the elevation of sun, we keep the yaṣṭi at centre of circle and point the other end towards sun, so that no shadow is formed. Since yaṣṭi = R, its perpendicular on ground is unnata jyā of sun and distance of base of perp is dṛgjyā or jyā of natāmśa.

In 60 daṇḍa sun moves 360° , hence in 1 daṇḍa it moves 6°. We see its movement along diurnal circle, hence to convert its elevation in equator, we multiply it by radius (3438) of equator and divide by semidiameter of diurnal circle (dyujyā) - (Verses 86-87)

The distance between śaṅku and the east west line in known as bhuja. Difference of bhuja of two

śaṅkus (in same direction) is multiplied by 12 and divided by difference of śankus is gives equinoctical shadow (palabhā).

If we observe śaṅku at three times, in morning, evening and at mid day we know declination and from that position of sun can be known (indicating the north). The base of morning and evening śaṅkus is on rising setting line which is parallel to east west line through centre of circle. Then midday shadow gives the north south incline of sun whose sum or difference with akśāmśa (or palabhā) gives declination as explained in chapter 5.

(4) Difference in two planets : Lalla describes the method of finding angular distance between sun and moon from which tithi of a lunar month can be known from formula

$$\frac{\text{Moon} - \text{sun}}{12^{\circ}} = \text{Tithi}$$

He described this as a śakaṭa yantra because two hollow tubes joined at angle are used, like two bamboo joined at angle to form chassis of bullockcart (śakaṭa). One tube is kept in direction of sun and through other, moon is seen (when both are visible). This position of tubes is clamped and put on the circle with point of intersection on the centre. Angle difference read on the ground is the angle between sun and moon. Similarly angular difference between any two planets can be known.

This is described under use of yaṣṭi by Sri R.N. Rai. Here no yaṣṭi is used, however, the graduated circle on ground is put to some use (with 'dhī' yantra i.e. intellect).

(5) **Use of circular disc :** By bending yaṣṭi in any direction and using a plumb line elevation of any object on earth or in sky can be known. More conveniently it can be done with śakata yantra or inclined tubes (Lalla). Siddhānta Darpaṇa refers to those methods only. Then what is the use of fixing rod at centre of a graduated circular disc ?

The clue is given by śalākā yantra (needles) mentioned by Lalla. He uses long pins to be used as śanku and hypotenus of shadows, but when disc is available, we can use pins only to read its angle measurements. Rod can be taken as a permanent pin at the centre. For that purpose it should have one sharp end, exactly perpendicular to the central point. We have to set the disc in plane in which difference of angle is to be measured and kept fixed in that position. For height of tree or hill, it will be vertical with edge of disc in direction of the tree etc. Then both points (base and top, or two objects in sky) are seen. In direction of rod and one of the points, pin is given at circumference of the disc. In line with other point and rod a another pin is given. The angular distance between the two points gives the angular distance between two objects (or elevation of top from base).

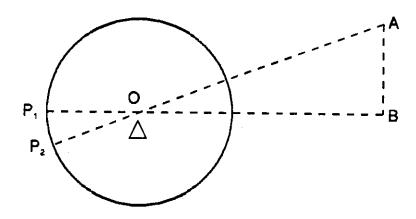



Figure 9 - elevation through māna yantra

Disc of māna yantra is kept in plane of observer and points A and B whose angular difference is to be found. Edge of the rod (yaṣṭi) passes through the centre O of disc. In line with point O and B, we put pin p_1. Similarly pin P_2 is put in line OA. Thus $\angle P_1 OP_2 = \angle AOB$ $\angle P_1OP$ is read from distance P_1 P_2 as O is at centre of the circle.

(6) Use of yaṣṭi as in double śaṅku method: In double śaṅku method we use śaṅku of same height. But here, we use the yaṣṭi as karṇa, hence śaṅku unnatajyā has different lengths. Thus we have to divide the śaṅku shadow by their śaṅku to know the proportionate length of shadow as mentioned in verses (88-89). It will be more clear from figure 10.

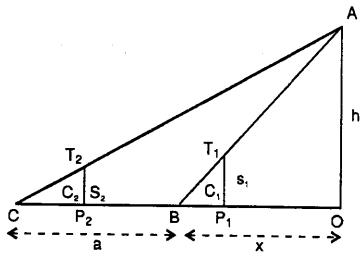



Figure 10 - Height of hill through yasti

We observe the elevation of hill AO of height h from point B and C which are at distance x and y = x+a from O. Yaṣṭi is CT_2 = BT_1 = Radius R (3438′). Thus śaṅkus at B and C are not equal

i.e. $s_1 \# s_2$

as in two śaṅku method.

Bhuja or chāyā of śaṅkus are c_1, c_2 given by $c_1 = BP_1$, $c_2 = CP_2$

Then in triangles BT_1P_1 and BAO

$$\frac{s_1}{h} = \frac{c_1}{x} \quad - - - (1)$$

Similarly, in triangles $CT_2\,P_2$ and CAO

$$\frac{s_2}{h} = \frac{c_2}{y} \quad - - - (2)$$

Dividing these two equations (1) and (2)

$$\frac{s_1}{s_2} = \frac{c_1}{c_2} \cdot \frac{y}{x}$$

$$\text{or } \frac{s_1 c_2}{s_2 c_1} = \frac{y}{x} \quad - - - (3)$$

This is the formula given

Now y = x + a where a is distance between shadow ends (where yaṣṭi is kept on ground).

a is measured, then

$$\frac{y}{x} = \frac{x + a}{x} = 1 + \frac{a}{x}$$

Hence $\frac{a}{x} = \frac{y}{x} - 1$ is known from (3)

and then x is known from value of a. Then from, (1) h can be calculated.

We keep bigger śanku and rods for more accuracy.

Verse 93 : Cakra yantra is circular. Its half i.e. semicircular instrument is called cāpa yantra. Half again of cāpa i.e. 1 quadrant is called turīya yantra. Cakra can give any time in day or night, cāpa yantra gives time in day, and turīya during half day. Turīya yantra has been called yaṣṭi here. (93)

Notes : (1) Cakra yantra -

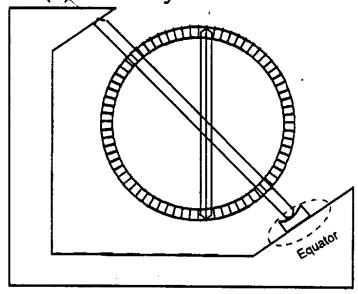



Figure 11 - Cakra yantra

The cakra yantra has been described by Varāhamihira as follows -

Take a circular hoop, on whose circumference, the 360° degrees are evenly marked, whose diameter is 1 hasta, and which is half an aṅgula broad. Through a small hole in the circumference, allow a ray of sun at noon to enter in oblique direction. We get the zenith distance of sun at mid moon. (The oblique direction can be seen by a pointer of diameter length).

It is not explained how the time will be determined. Other astronomers recommend cakra

yantra of metal or seasoned wood and of 3 meters diameter. A needle should be kept at centre whose shadow at lower end gives the height of sun.

Cakra yantra built in observatories of Jai Singh at Jaipur and Vārāṇasī is shown in figure 11. Jaipur piece is 6 ft. in diameter, one inch. thick and two inches broad, faced with brass on which degrees and minutes are marked. They are mounted on pillars and fixed so as to revolve round an axis parallel to earth's axis. At southern extremity of the axis, and on pillar which supports the instrument, there is a graduated circle in the plane of equator. This axis carries a pointer, which indicates the hour angle on the fixed circle. The main movable circle carries an index and a sighter through which heavenly bodies can be observed.

There are certain prominent stars whch are very near the ecliptic. Their celestial longitude λ and latitude β are -

Puṣya (δ cancri; λ = 128°1′ β = 0°.5′)

Maghā (αLeonis, Regulus, λ = 149°8′, β = 0°28′)

Śatabhiṣaj (λ Aquarii,λ = 340°′53′, β = 0°23′)

Revatī (ζ Piscicum;λ = 19°11′, β = 0°13′)

The circle should be held such that the above stars should appear to touch its circumference, then the circle will be in ecliptic plane. While observing any of the stars, one should observe a planet and determine the distance between the star and the planet. This distance is added to longitude of star in west or substracted from it, when star is east of planet, to get longitude of the planet.

(2) **Cāpa yantra** - A large semicircle is divided into 180° and subdivisions of a degree. At its centre, a fine hole is made through which a needle is inserted. It is held in such a way that, chord in horizontal. Then it is rotated so that both sides are equally illuminated by rays of sun. The number of degrees between horizon point (bottom of circumference) and shadow of the needle gives the zenith distance of sun. The distance from base of yantra is elevation. From them time after sunrise or before sunset can be calculated as explained earlier.

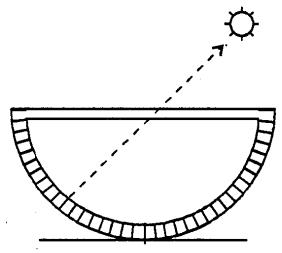



Figure 12 - Capa yantra

Cāpa yantra of Āryabhaṭa is slightly different, in which chord end was kept on the ground and gnomon or needle was moved on the circumference so that its shadow fell on the centre.

(3) Turīya yantra -

This is formed of fourth part of a circle, hence called 'Turīya yantra'. It is made of flat plate of metal and each arm (radius) is about 1.5 meters in length. It is graduated into 90° and degrees are further subdivided. A small nail is fixed at the centre and quadrant is kept in meridian (north south vertical) plane. One arm of quadrant is

vertical and other horizontal (this side will be upper most facing away from sun). At mid day, shadow of nail will give zenith distance. The difference of mid day zenith distance at its least value (for placės north of 23-1/2°N) for sun in sāyana cancer and greatest value for sun entering capricorn is double the angle of inclination of ecliptic. Time can be found as with cāpa yantra.

In another form, a small tube is fixed at the centre pointing towards point on circumference along one of the arms. Another small tube is fixed at this end pointing towards the centre. This end is known as the horizontal point end and point on circumference is known as sky point. A plumb line is suspended from the centre. The quadrant is now held in such a way that rays from sun entering the tube at the centre fall on the tube at the horizontal point. The degrees between this point and plumb line gives zenith distance of sun. Degrees between plumb line and other side give height of sun. Moon, planets, starts can be observed by placing eye at horizontal point and observing bodies by directing instrument at them, so that they will be observed by the light entering both the tubes.

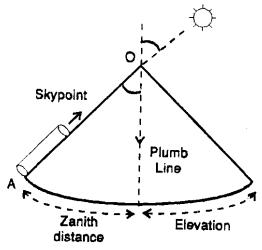



Figure 13 - Turiya yantra

The yantra cintāmaṇi recommends that each arm should be divided inot 30 parts and half chords parallel to the other arm should be drawn from each point. Also an index rod should be fixed to the tube at the centre rather loosely so that it will revolve freely along the circumference.

Verses 94-97 : Measure of time intervals

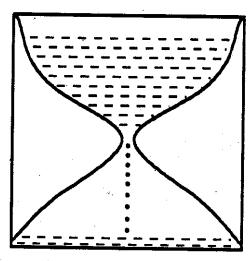



Figure 14 - water or sand clock

(1) **Water or sand clock** : One upper and one lower glass pots are joined through a thin hole in the middle through which water from upper pot falls into the lower pot (figure 14). (94)

When the first pot (filled with water) is kept up, it takes a fixed time for water to enter lower pot completely. Same time is taken when second pot is kept up. In this sand also can be used instead of water.

(2) **Nara yantra** : A hallow idol in human, monkey or peacock shape is made by a good artisan. A tube with thread inside it is fitted in it. It is kept in a water pot. Due to attraction like mercury, water enters it drop by drop. It fills up in 2 ghaṭi (1 muhūrtta) and then water comes out through jewelled mouth.

Notes : Fig. 15(a) shows an out flow type water clock in which the water comes out drop by drop from mouth of bird like pot. The water comes out through a thin tube which can be adjusted by inserting a thin wire to reduce the rate of water flow.

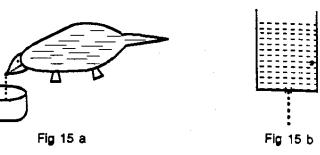



Figure 15 - Out flow type water clocks

Common water flow instrument is a simple cylinder shown in figure 15(b) in which water comes out through a hole at bottom in 1 ghaṭi or muhūrtta. Since this was in shape of Nali (tube or cylinder) it was called nālikā yantra. Hence Nālī or nāḍī was used for this instrument as well as the time unit of 1 ghaṭi measured by it. This was used in vedic age and in ancient Egypt.

(3) **Kapāla yantra** - A hemispherical bowl will be made with a small hole at bottom. It is kept on water so that water enters slowly through the hole. Size of hole is adjusted so that bowl is filled in 1 ghaṭī. This is called Kapāla yantra.

Note : This was called kapāla yantra by Āryabhaṭa. But it was normally called ghaṭī yantra due to its shape like a water pot (ghaṭa). The time unit measured by it (24

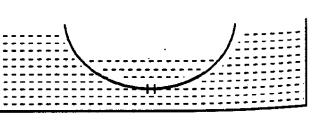



Figure 16 - Kapala yantra

minutes) and pot both are called ghaṭi due to that.

In every town people were kept to maintain this instrument who struck a bell (ghaṇṭā) at end of each ghaṭi. hence hour is called ghaḍi or ghaṭi or ghaṇṭā both. The people were called 'ghaḍiyālis'.

Due to heavy weight of bowl, it drowns with audible sound in water before it is full completely. Weight of the bowl and hole are adjusted so that sinking time is exactly 1 ghaṭī.

Verses 98 : Bhāskara has stated about phalaka yantra in detail, so it is not described here again.

Notes : Falaka yantra as stated in Siddhānta Śiromaṇi and explained by Munīśvara in his 'marīci' commentary is described here. Proof given by Munīśvara and explained by Sri R.N. Rai is also given.

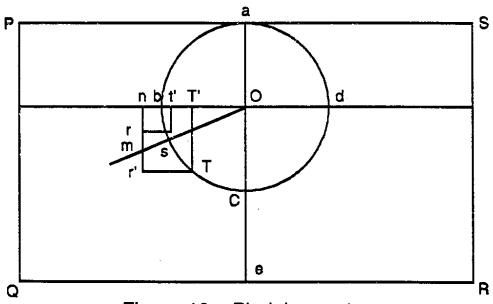



Figure 16 - Phalaka yantra

A phalaka or board of metal or good seasoned wood of rectangular form is made, 90 aṅgula high and double i.e. 180 aṅgula in length. At middle point of the length we should attach a chain by which it can be held in a vertical plane. From this middle point, a line is drawn which is perpen-

dicular to the edge and is called the lamba rekhā. This is divided into 90 equal parts, each of one aṅgula. Through each of the dividing points we should draw lines parallel to top and bottom edges. These horizontal lines are called sines.

With the point of intersection of 30th sine from the top and lamba rekhā as the centre, a circle of radius of 30 aṅgulas is drawn. This circle will cut the lamba ṛekhā at the 60th sine and its diameter is equal to 60 aṅgulas. Now circumference of circle is marked with 60 ghaṭīs and 360° and each degree is divided into 10 parts of 1 pala each. A pin is inserted at the centre of circle through a hole which is considered as the axis.

A thin paṭṭikā or index arm of copper or bamboo is taken 60 aṅgulas in length, divided into 60 parts. It is half an aṅgula broad except at beginning where it is 1 aṅgula broad and where a hole is bored for suspending it near mid point of board. Graduated side of pattikā coincides with lamba rekhā.

In fig. 17, PQRS is a board 18 units long and 90 units high aOCe is lamba rekhā, and aO = 30 units. With O as centre and Oa as radius, circle abcd is drawn. Index arm is of length Oe and is inserted at O. The hole is so adjusted that one side of index arm coincides with Oe when it is suspended at O.

The rough ascensional differences in pala are determined by khaṇḍakas or parts divided by 19, will here become the sines of ascensional differences adapted to this instrument.

R sine (asc. diff) = R tan ϕ tan δ, or Carajyā and the rough values of arcs corresponding to the first, second and third signs at a place are 10, 8, 3-1/3 palas when equinoctical shadow is 1 aṅgula. These are the values when R = 3438. When the radius is 30, the arcs will be multiplied by 30 and divided by 3438. To make them asus, they will be further multiplied by 6. Thus the arms for three signs are

$$(10,8,3-\frac{1}{3}) \times \frac{30 \times 6}{3438} = (10,8,3\frac{1}{3})/19$$

Since the arcs are small, the jyā of these values are approximately same.

The numbers 4, 11, 17, 18, 13, 5 multiplied severally by the equinotical hypotenus and divided by 12 will be khaṇḍakas or portions at the given place. These are for each 15 degrees of bhuja of sun's longitude.

For sāyana longitude of sun we make the bhuja, and add the khaṇḍakas for completed 15° parts or fractions. Sum is divided by 60 and added to equinoctical hypotenus. The result is multiplied by 10 and divided by 4. The quotient here is called yaṣṭi in aṅgulas (digits). This number of aṅgula is marked off on paṭṭikā counting from hole at O.

Proof of formula

In figure 18, let P, S, Z be north pole, sun and zenith. If A is altitude of sun,

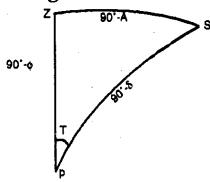



Figure 18 - Phalaka Yantra

are ZS = 90° - A. If δ is north declination of sun, then arc PS = 90° - δ . If ϕ is latitude of the place, arc PZ = 90° - ϕ.

$$\cos (90°-A) = \cos (90 - \delta) \cos (90 - \phi) + \sin (90 - \delta) \sin (90 - \phi) \cos T.$$

where T is hour angle of sun.

$$\text{or } \sin A = \sin \delta \sin\phi + \cos \delta \cos \phi \cos T$$

$$\text{or } \cos T = \frac{\sin A - \sin \phi \cdot \sin \delta}{\cos \phi \cdot \cos \delta}$$

where T is the time from mid day or to midday. If the sun is in southern hemisphere, δ is negative. hence

$$R \cos T = \frac{R \sin A}{\cos \phi \cdot \cos \delta} \mp R \sin (\text{asc. diff})$$

according as declination is north or south

Now, $\frac{1}{\cos \phi} = \frac{h}{12}$, where h is equinoctical hypotenus of śaṅku of 12 aṅgulas. Hence

$$R \cos T = \pm \frac{h}{12} \frac{R \sin A}{\cos \delta} \mp R \sin (\text{asc diff})$$

$$= y \sin A \mp R \sin (\text{asc diff})$$

where $y = \frac{h}{12} \frac{R}{\cos \delta}$ and is called that yaṣṭi

$$\text{Now } y = \frac{h}{12} \frac{R}{\cos \delta} = \frac{R}{12} \frac{h}{12} \left(\frac{12}{\cos \delta}\right)$$

$$= \frac{R}{12} \cdot \frac{h}{12} \left[12 + \frac{12 (1 - \cos \delta)}{\cos \delta}\right]$$

$$= \frac{R}{12} \left[h + \frac{h}{12} \frac{12 (1 - \cos \delta)}{\cos \delta}\right]$$

When the bjuja of sun's longitude is 15, 30, 45, 60, 75, 90, the value of 12 (1-cosδ) /cos δ is 4, 15, 32, 50, 63, 68 sixtieths respectively. The differences of these values are 4, 11, 17, 18, 13, 5 which have been given above. On multiplying these difference by h, equinoctical hypotenus and dividing by 12, the quotients found are called khaṇḍa for a place. By assuming bhuja of sun's longitude as an argument, we can find the result through khaṇḍas. Let r be this result, then

$$y = \frac{R}{12}\left(h + \frac{r}{60}\right) = \frac{10}{4}\left(h + \frac{r}{60}\right)$$

because in the instrument R = 30

$$\text{Thus R cos T} = \frac{10}{4}\left(h + \frac{r}{60}\right) \text{ Sin A}$$

$$\mp \text{R sin (asc diff)}$$

It is evident that the value of the yaṣṭi, y will always be greater than 30 because h is always greater than 12 except at equator where h is 12. At equator, yaṣṭi will be equal to 30 only if δ = 0. If on holding the instrument so that rays of the sun illuminate both its side (so that it is in a vertical plane), shadow of the axis at O cuts the circumference of the circle abcd in s. The angle sob is equal to angular height of the sun.

Now the index arm is put on the axis and putting it over the place where the shadow cuts the circle and measuring along index arm a length equal to yaṣṭi found above, let m be the point so obtained. Then

mn = y sin A

If the place is on the equator, we have to find T such that R cos T = mn. Then T gives value of time in degrees to or after midday.

At any other place, the correction for asc. diff. has to be applied. For sun having north krānti, R sin (asc. diff) is substracted from mn. Let the amount to be substracted be mr. Then R cos T = tt′ and the angle is given by the arc ct. If the sun is in southern hemisphere, correction mr′ is to be added and R cosT′ = TT′ and the angle is given by arc CT.

Once table of values of the yaṣṭi and correction for asc. diff for different bhujas of sun's longitude has been constructed for a place, the instrument will give time very quickly.

Verses 99-108 - Svayaṁvaha yantra.

For svayaṁvaha yantra (automatic rotating instrument) we make a disc of 60 aṅgula circumference. At each aṅgula of the circumference one line is given corresponding to 60 daṇḍas in a day. At 5 aṅgula intervals, bigger marks are given (5 ghaṭī = 2 hour interval). (99)

A smooth axle is fited at centre of this disc and is fitted on two pillars in north south direction 3 hands high. Disc should have uniform weight so that it moves freely. (100)

The disc can rotate in east west plane. Sun and moon signs are given at east and west horizons. Below west horizon a copper tube is placed vertically. (101)

This tube will be 60 aṅgula long. At bottom a small hole will be made, so that water comes out in 60 daṇḍas. Below it a pot will be kept to keep the outflown water; otherwise it will spread on ground. (102)

Then a thread is tied round the circumference passing from lower, eastern and top portion and coming back near the tube. A weight is suspended through the thread. (103)

The thread is made smooth through wax or grease (pārada). When weight is put in water, it will slowly turn the wheel once in a day-night. (A light weight - hollow cover of hard fruit is recommended which will float on water). (104)

In east horizon a pointer will be kept which will be near sun mark at sunrise time. Starting from sun rise time, the mark on circumference touching the pointer will indicate the time lapsed in day-night. Next day also wheel will be set in same way. (105)

Lower half will be kept covered so that it is not seen by visitors, otherwise their curiosity will be lost. This secret method should be shown only to good students. (106)

The gola yantra stated earlier can also be rotated like this svayaṁvaha yantra. Another view about this is being stated. Two circles are made of thin wood (circular disc with a wooden groove) to be fixed near dhruva. (107)

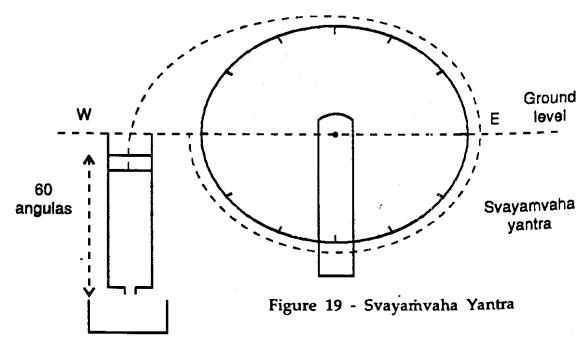
In hole of dhruva cakra, we put dhruva yaṣṭi after oiling it properly. Then west of the sphere, two nails are fixed and two tubes are fixed below them. (108)

Two weights are suspended from the nails into the tubes. Thread is tied around gola upto

the weights. Then gola yantra will move in equator circle like east west circle.

Notes : This is a crude and unnecessary instrument when accurate clocks on spring and pendulum action have already been made in Europe. First mechanical clock was made by Giovanni de Dondi between 1348 and 1362. This clock indicated motions of sun, moon and five planets with a series of gears. The use of the pendulum as a controlling device was suggested by Galileo and independantly discovered by Huygens (1629-95). Thomas Tompion in 1676 built two spring winding clocks with 13 foot pandulums, beating two seconds for Greenwich observatory. These clocks could go without winding for almost one year.

In stead of pendulum, eccentric free wheel was used in table clocks and wrist watches for keeping correct time. The rotation was made with electric power also in electric clocks. The electric movements in 20th century were controlled with electronic oscillations stabilised by oscillations of quartz crystal. These are most popular now and very accurate.

Figure 19 - Svayaṁvaha Yantra

More accurate watches for scientific purpose are made on basis of oscillations of Cs atom or Ammonia laser. These give error of less than 1 second in 10,000 years. Even at the time of Candraśakhara accurate clocks and watches were available in India. However, he has imagined a svayaṁvaha yantra mentioned by Āryabhaṭa and sūrya siddhānta whose mechanism has not been mentioned. This mechanism in principle can work, but it will be very rough. Eye estimate of time may be more accurate than this. Scheme of construction is shown in figure 19.

Verses 110-113 - Conclusion

(Sūrya siddhānta) By acquiring knowledge of graha, nakśatra and gola, a person becomes rich and goes to graha loka after his death. (110)

(Siddhānta Śiromaṇi) - Graha knowledge is divine and beyond human perception. It was given originally by the creator (Brahmā), then it was spread on earth by sages like Vaśiṣṭha. This sacred knoweldge should not be given to violent, treacherous, wicked and men of unsteady wit. By not obeying this injunction of sages, one loses his longevity and result of holy deeds. (111)

May the brightness of Supreme lord always come before my vision for giving pleasure, whose mahāprasāda stops re-birth and purifies down trodden like me by taking it in hands, mouth or stomach. (112)

Thus ends the twentieth chapter describing instruments in Siddhānta Darpaṇa written as a text book for accurate calculation by Śrī Candraśekhara born in respected royal family of Orissa. (113)

Chapter - 21

REMAINING EXPLAINATIONS

Vāsanā Śeṣa Rahasya Varṇana

Verse 1 - Scope - After completing description of graha and gola, the rationale of mathematical methods is being explained as answer to questions posed earlier.

Veses 2-6 - Difference in day lengths

Horizon of equator is called unmaṇḍala for other places (on same longitude). Sunrise occurs in own horizon for any place. Between local horizon and unmaṇḍala (horizon of eqautor).(2) the portion of diurnal circle intercepted is cara. This is the difference between sunrise times at equator and local place. It is day time when sun is above horizon in its diurnal circle. (3)

When sun is below horizon, it is night. Equator is the largest diurnal circle. It is bisected by horizon of all places. Hence, when sun is moving on equator, day and night are equal at all places. At equator, day and night are always equal. (4)

When sun is north of equator, then in north hemisphere, sunrise is before equator rise and sunset is after equator sunset. In south, hemispehre, it is reverse. (5)

Reason is that in north hemisphere unmaṇḍala is above horizon. Before reaching unmaṇḍala

(sunrise at equator), sun comes at local horizon and local sunrise occurs. In the south hemisphere unmaṇḍala is below horizon (hence opposite occurs). (6)

Notes : Already it has been explained in chapter 5.

Verses 7-13 - Day-night at poles

Special event occurs at places whose latitude is more than 66-1/2° which is being told here. (7)

On north polar region (66-1/2°N to 90°N), it is day as long as sun has north krānti and it becomes night when sun has south krānti. (8)

Equator is horizon for deva (in north pole) and asura (in south pole). Hence north pole is the zenith of deva and south pole, that of asuras. (9)

When sun is in north hemisphere, devas see it moving above south horizon. When sun is in south horizon, it is seen above north horizon by asuras. (10)

Half portion of the ecliptic (sāyana meṣa beginning to sāyana tulā beginning) is north of equator. Thus when sun is in north hemisphere for first half of ecliptic, it is day time for deva and night for asura. It is reverse when sun is in south gola. (11)

However, smṛtis has stated that for uttarāyaṇa (i.e. north ward movement) of sun, it is day of deva and night of asura. Dakśināyana (southward movement) of sun is night of deva. After midday, sun starts going down in the direct on of night and after midnight, it starts going up in direction of day. (12)

Hence from mid night itself start of day is assumed. Thus day of deva (gods) starts with uttarāyaṇa. By celebration of day starting rituals then by men, gods become happy. (13)

Notes : Sāyaṇa makara sankrānti is the start of uttarāyaṇa (just before christmas). Bhīṣma waited till christmas for his death after being mortally injured in Mahābhārata war. Now we are following nirayana system, hence christmas falls on 25th December (winter solstice on 23rd december) and makara saṁkrānti on 14th January. Thus makara saṅkrānti is celebrated as a festival.

Sāyana makara saṅkrānti is start of grand day, hence christmas is called badā dina. In civil calender it falls on mārgśirṣa month. Twilight period is 1/12th of day time i.e. 1/24 of day night. In grand day of 1 year it is 15 1/4 days approximately. Hence 15 1/4 days before start of mārgaśirṣa, it is baḍa Oṣā (grand twilight).

Verses 14-17 - Other day-nights

Brahmā is very far from earth, hence he always sees the sun and his day continues. Only when sun is destroyed in pralaya, his night starts and he sleeps. This is stated in Purāṇas. (14)

Day night of others (deva-asura, and men) is due to one sun only. At the end of Brahmā's night, another sun rises (because previous one has been destroyed). (15)

When sun is visible, it is day and when it is obstructed, it is night. Thus for pitṛs living on moon's back surface, day-night is equal to a lunar month. (116)

At the end of amāvasyā, it is mid day of pitars and at end of pūrṇimā, it is mid night. Bright half 8th day is their evening and dark half 8th day is their sun rise. (117)

Notes : Brahmā's day is when creation of sun and planets is on. When creation is destroyed, it is pralaya. In modern cosmology, it is expansion of universe, when matter is in different forms. At the time of contraction, there will be infinite rise in temperatures corresponding to pralaya.

Pitara live on the other side of moon. Hence when we see bright moon, it is dark for them.

Verses 18-22 - Different rising times of rāśis

The portion of ecliptic touching east horizon is lagna at that time. The portion of ecliptic touching west horizon is the asta (setting) lagna for that time. (18)

Between east and west horizons, the lower and upper directions cut the ecliptic in points called 4th and tenth lagnas. These can be known from tripraśnādhikāra. (19)

Oblique portion of ecliptic rises and sets quicker, but straight portion being longer (on equator) takes more time to rise or set. (20)

Hence the rising times of rāśis of ecliptic are not equal. At equator 90° part of ecliptic rises in 15 ghaṭī. 180° (half) of ecliptic rises in 30 ghaṭis. (21)

Whatever is stated or ommitted here, should be explained by the instructor, by rotating the model of globe (earth) and ecliptic. (22)

Verses 23-26 : Ecliptic parts visible at places

At akśāmśa 69°48′, dhanu and makara rāśis are not visible, but mithuna and karka always rise (they never set). (23)

For place of akśāmśa 78°30′, 4 rāśis starting with vṛścika are always below horizon and 4 rāśis starting with vṛṣa are always above horizon. (24)

At akśāmśa 90° north (on sumeru), 6 rāśis starting with tulā are never visible, but other 6 rāśis are always visible. (25)

Thus in deva part (north hemisphere) and asura part (south hemisphere), rāśis are visible or invisible acording to latitude (akśāmśa) of the place. The rāśi which is always above horizon in deva part, is always below horizon at that latitude in asura part. (26)

Notes : Condition for a star being circumpolar has already been explained in chapter 8 on lunar eclipse. The north krānti of a star being more than colatitude of the place, the star will never set. Similarly south krānti being more than colatitude for north hemispehre, the star wil not rise. The south krānti of (270°±30°) i.e. dhanu, makara or north krānti of mithuna karka (90° ± 30°) is 20°12′ i.e. sinδ = Sin 60° X Sin 23°30′, thenδ = 20°12′. Hence at 69°48′ = 90° - 20°12′, dhanu, makara never rise and mithuna, karka never set. Similarly krānti for vṛsa beginning 30° or simha end 150° is 11°30′ hence at 78°30′ = 90°-11°30′, the rāśis between them never set. Here, the rāśis are sāyana.

Verses 27-28 : Unequal linear speeds -

If yojana speed of planets is considred same, then according to their angular velocity, orbit has to be made bigger. (27)

Then the śīghra paridhi of planets doesn't come to observed value. Hence, the planets starting from budha sukra with successively lower speeds in angle must have lower linear speeds also. (28)

Notes : Relation with śīghra paridhi has already been explained in chapter 19 verses 53-54.

Verses 29-31 - Heliocentric motion

In old texts also the diameter of planets (angular), śara (latitude) and śīghra phala are stated to change. Thus they also accept the heliocentric orbits, through do not state it specifically. I have stated it clearly. (29)

Sun and moon rotate around earth and other planets rotate around mean sun. They move in east direction as seen from mean sun. (30)

This is the natural motion of planets, which is due to attraction of mandocca. When planet is far or near earth, its angular speed changes. (31)

Notes : Change in śīghra phala and distance have been explained in chapter 5 and 17, which indicates heliocentric orbit. This should have included earth also, but assumption of solar orbit around earth makes no diference in relative speed, hence there is no contradiction.

Linear speed of a planet around mean sun or earth has been assumed constant. Actually linear speed also is reduced when it is far from centre of orbit. Its angular speed is further reduced due to increase in distance.

Verses 32-41 : Calculation from kalpa beginning

Different astronomers have given different numbers of revolutions of graha and their ucca.

Hence some astronomers (Āryabhaṭa) tell that time periods of kalpa etc are not correct. (32)

But this is against scriptures and hypocricy. If we assume the kalpa bhagaṇas of mean sun and moon, then present mean positions can be known from lapsed civil days in the kalpa. (33)

We see that 1400 years after Āryabhaṭa, even 1 kalā difference has not occured in positions of mean sun and moon. Thus the theory of ancient scriptures cannot be considred wrong. (34)

Due to almost fixed position of sun mandocca, madhyama sun speed can be ascertained for 5-6 years, only by seeing earth shadow etc. (35)

But mandocca of moon moves faster, hence its ucca motion or mean motion cannot be known without siddhānta. (36)

By repeated observations of moon only its variation in motion can be known. Since speed of śani is very slow, we cannot find its mandocca and mean motion in less than hundred years. (37)

Due to these reasons, revolution of sun and moon, years months and days etc should be counted from kalpa beginning only. Thus the assertion of siddhāntas also are correct. (38)

The difference in bhagaṇa (revolutions) of ucca and pāta of moon and of maṅgala can be observed in a yuga only. The revolutions numbers in a yuga are for rough calculations. Accuracy can be achieved by revolution numbers in a kalpa only. (39)

Learend Bhāskarācārya found errors in calculations from yuga revolution numbers. Hence he assumed complete revolutions only in a kalpa. (40)

He ignored the creation period and made calculations from kalpa beginning itself. According to him, kalpa started from mean sunrise at Lankā, not from mid night. (41)

Notes : Bhāskarācārya has explained 'bhagaṇotpatti' (origin of revolution numbers) in detail in golādhyāya.

Moon's motion among the stars can be seen most easily, at the rate of 1 nakśatra per day.

Tropical revolution of sun can be found very easily with change of seasons and interval between equinoxes.

Sidereal revolution of sun could have been by following method - (1) Repeatition of solar or lunar eclipses and comparison of lunation and sidereal revolutions of sun. (ii) Observing the star which was rising at sun set. Motion of sun within stars will be seen towards east within a month. From sidereal and synodic motions of moon, the difference speed of sun could be determined (iii) The sidereal revolution can be checked after one year by heliacal rising of a particular star.

Among tarā grahas, average sidereal periods of mercury and venus coincided with sun, thus they were found oscillating around sun in smaller orbits. Other 3 tārā grahas had very longer periods of direct motion. Their sidereal periods could be found by periods between heliacal rising and settings. Let x° be angular speed of a planet in a day, and a° be the speed of sun. Then during one day, sun over takes the planet by (a-x)°. Thus their synodic period S between consecutive heliacal rising or settings is

$$S = \frac{360}{a - x}$$

From the value of x, we can calculate sidereal period 360/x as a is already known.

As the inner planets appear to oscillate around sun, similarly for outer planets mars, jupiter, saturn, earth will appear to oscillate round sun. Seeing in reverse, the average of sidereal period round earth and round sun will be same. Hence bhagaṇas given in our siddhāntas tallies with heliocentric revolution period given in modern astronomy. This result has been proved by D. Aṛka Somayājī in 'A critical study of Ancient Hindu Astronmy' published by Karnāṭaka university, Dhārawar.

The revolution of planets and pāta and ucca of moon make complete revolutions within a period of 43,20,000 years. This is the minimum period of complete revolutions and has given the definition of a yuga.

Mandocca and pāta of other planets move very slowly and minimum period of their complete revolution is 1000 yuga or a kalpa.

Verses 42-54 : Reasons of correcting bhagaṇas

Bhāskarācārya found that positions of sun and moon were more, when calculated from number of days lapsed since kali beginning. (42)

Hence, he deducted the difference kalās from the revolutions of sun and moon and changed their revolution numbers. (Sun correction was made by

ommitting creation years, start of kalpa from sunrise etc.) (43)

Similarly, he found bhagaṇas of other planets by making correction (bīja) from calculated positions to tally with observation. (44)

By his bījā corrections in sun and moon, it became clear that bhagaṇas are not fixed (otherwise corrections will not be needed). (45)

Calculation of sun and moon by present siddhānta (i.e. siddhānta darpaṇa) confirms well with the observations. But there is more error in Bhāskara method. (46)

Had the correction in bhagaṇas been done before Bhaskara to calculate the observed position of planets, (47) then the error of Bhāskara values would have been less. 516 years before him Brahmagupta had stated revolutions in a kalpa in which he found errors. (48)

Thus he revised the revolution numbers which is his great achievement. But his defect is that he started motion of planets from kalpa beginning (not deducting years of creation) and from Laṅkā sunrise (instead of mid night). (49)

Still, by recording the correct planetary position of his time he has done a great favour to me (author made further corrections on that basis) with lesser intellect. (50)

Āryabhaṭa, Śatānanda, Bhāskara II etc. wrote about planet positions of their times and calculations according to them were done by Kāma (Kamalākara) Bhatta etc. (51)

I have compared their calculations with present position of planets and accordingly I have found the correct bhagaṇas. (52)

Thus I feel that my calculations will hold true upto 10,000 years in future. There is not likely to be any difference in tithi, nakśatras etc or true positions of maṅgala etc. (53)

I do not know past, present and future. Hence my methods cannot remain valid for ever. However, after 10,000 years, corrections will be needed in bhagaṇa values to correct the true position of planet as observed then. (54)

Verses 55-70 : Corrections after ten thousand years.

If the planets calculated after 10,000 years according to kalpa bhagaṇas are less or more than the true position, the following numbers will be added or deducted to kalpa bhagaṇas to get correct values. The corrections can be some multiple of given values also. (55-60)

Planets	Addition to Bhagaṇa	Multiples of these	Deductions from Bhagaṇa	Multiples of these
Mars	254	2	455	2,3,4
Mercury	709	2, 3	963	2,3,4,5
Jupiter	1164	2, 3	1217	2,3
Venus	1672	2,3,4,5,6,7,8,9,10	2381	2,3,4,5
Saturn	2635	2,3	3090	2, 3

Thus there are 33 atternative multiples of corrections. If there is difference of 30 kalā between Bhāskara time (1124 AD) and my time (1869 AD) i.e. in 745 years, then this substraction or division will have to be made. (61)

For tithi and nakśatra or for eclipse, we need mandocca and pāta of moon. Hence their corrections are being stated. (62)

Kalpa revolutions of candra mandocca have been stated to be (48, 81, 17, 940) (63)

After long period, (how much ?) if the observed values are more than values calculated from this figure then this revolution number will be corrected. (64)

We add or substract 1672, 2381 or 4053 (65)

If in the revolutions of moon pāta, there is difference of 1° in observation and calculation after 10,000 years, (66)

we add or substract 1672 to pāta bhagaṇa of moon. Correction to revolution of krānti pāta will be (3090) or (67).

(5170), (6180) or (8360), double or triple of (8360) i.e. (16720), (25,080) or (19,810) or (28,170). If krānti pāta doesn't become correct even after that, then krānti will be found from shadow of sun. (68-69)

Sun will be calculated from calculation as well as from shadow (by method explained in tripraśnādhikara).

Notes : It is not given here whether the corrections are for 1° error or not. Same correction is recommended to Bhāskara figures for errors upto 30′ i.e. 1/2°. Thus the minimum error which can be measured with eye is 1/2° First let us see the correction in Bhāskara values.

Values of Bhāskara - If error is seen in shorter period, correction is more.

Planet	Revolution	Correction for 745 year	Correction for 10,000	Multiples of given values
Mars	2,29,68,28,522	+42,590	+3172	12.49
Mercury	17,93,69,98,984	-31,843	-2372	2.46
Jupiter	36,42,26,455	-71,250	-5308	4.36
Venus	7,02,23,89,492	-1,31,632	-9806.6	4.12
Sāturn	14,65,67,298	+72,418	+5395	2.05

Thus the figures are not corrected from Bhāskara by this formula. Actually all the figures for these planets are quoted from Brahma sphuṭa siddhānta. Brahmagupta mentioned revolution of moon in a yuga in that book, hence moon's revolution for kalpa were quoted from his other work Khaṇḍa Khādyaka. Brahmagupta was the first to give kalpa revolutions, who claimed the origin in Brahma siddhānta and Viṣṇu dharmottara purāṇa (those parts are not available now).

Let us see the corrections of Brahmagupta figures by Bhāskara in 516 years. That also doesn't follow these figures or their multiples.

It is clear from the figures that the correction for outer planets are proportional to (sun-planet) speed, i.e. the comparison is made between synodic periods. Figures of mercury and venus are propertional to their periods of revolutions. Except for Jupiter there is too much difference in positive and negative corrections and there are two many optional multiples of correction for venus. They do not fit into any consistant theory.

1° error in 10,000 years becomes 4,32,000 degrees error or 1200 bhagaṇas in 1 kalpa. This figure approximately tallies only with figures of Jupiter. Corrections are reduced so that change at present time is complete bhagaṇa (to be found by indeterminate equations)

Verses 71-78 : Guru years

According to Bṛhatsamhitā of Varāhamihira, when creation started, Jupiter was at beginning of meṣa. Hence it must have entered kumbha rāśi after 22 years. (71)

Then guru year named 'prabhava' started. Again when guru entered meṣa, the year was śukla. (72)

Hence to find the guru year starting with 'prabhava', 2 rāśis are added to mean Jupiter. If bhagaṇas of Jupiter are changed in future, then... (73)

Guru revolution will be divided by 5 (because guru varṣas are 60 in 5 revolutions of 12 years each). Since beginning of creation viśvāvasu (39), 51st kapila or piṅgala - - - (74)

3rd (śukla) or 15th vṛṣa or 27th vijaya samvatsara will be counted (corresponding to full revolution, samvatsara number will change by 12 or its multiple). The name of saṁvatsara current in a country should be written. (75)

· For those assuming kalpa revolution (36,42,57,840) of Jupiter, saṁvatsara should be started with śukla only. (76) The mean Jupiter calculated from this revolution number should be reduced by 15 kalā to see its real mean position. (77)

After 1142 years (19X60+2), we should calculate guru according to above value and traditional value and see whether same value comes. Then corrections should be made. (78)

Notes : Traditional guru year is as given by sūrya siddhānta which assumes 3, 64, 220 revolutions in a yuga instead of (3, 64, 257.84) assumed here.

Verses 79-83 : Calculation of padakas

When there is error in astronomical calculations, god as protector of world order creates a

man to correct this subject. Only that man is able to understand this subject. All cannot know the secrets. (79)

Motion of planets in 1 day has been given in kalā etc in 10 sub-divisions. To find the motion in days which are multiples of 10, the daily motion will be multiplied by 10. (80)

Smallest division value is divided by 60. Remainder is kept there and quotient is added to next higher division. Number at that division also is divided by 60 and so on. Product is again multiplied by 2, 3 - - - 9 and divided by 60 starting with smallest place, to know motion for 20, 30, - - - 90 days. (81)

Results are kept till viparā (=1/60X60 vikalā) position. Starting from rāśi we keep the total for 5 places in chart (upto parā only). (82)

These will be written in columns. These numbers are called padakas. It is easier to calculate planets from ahargaṇa with them. (83)

Verses 84-86 : Geocentric corrections

Finding true positions of planets through mandaphala and śīghraphala is applicable only for observing from earth. Same position will not be observed from other planets. (84)

The positions observed from other planets will be found from the distance of that planet from earth and other planets from sun. This is the meaning of mandaphala and śīghraphala. (85)

(Siddhānta śiromaṇi). Earth centre is also the centre of nakśatra maṇḍala, but it is not the centre of planetary orbits. Hence a planet will not be seen at its mean position, when observed from earth.

Hence, astronomers correct the mean planets by bhujaphala (manda or śīghra) to find true position. (86)

Verses 87-112 - Eccentric circle for moon

To understand the process of finding true planets and see their real position (for sun or moon), we make kakśā vṛtta on plane ground with scale 1 aṅgula = 1° (i.e. radius = 3438′ = 57°18′ = 57/18 aṅgula). (87)

Circumference of kakśā vṛtta will be 360 aṅgula. In this, 12 rāśis from meṣa are marked starting from east point. (88)

At the centre, earth is formed of 9 aṅgula diameter. Mandocca of moon is imagined in meṣa at some distance from earth. (89)

From centre of earth at a distance of twice the jyā of antya phala (300′X2=600 = 10 aṅgulas), a point is given in the direction of mandocca. (90)

From mid point of line joining this point and centre, we draw a perpendicular equal to trijyā. The end point is at distance from centre or antya jyā given by square root of sum of squares of trijyā and antya jyā. (91)

Twice the distance (115/2) is divided by 60 to make it aṅgula. With this diameter, we draw a circle from mid point of centre-antyā. ((2)

This circle will be called 'prativṛtta' (Eccentric circle) of radius (57/31). This is formed due to attraction of mandocca. (93)

Pratimaṇḍala (Eccentric circle) meets the kakśā vṛtta at the end of odd quadrants and is equal to that. At end of even quadrants, its width is 26 kalā more than kakśā vṛtta. (94)

Manda kendra of the graha is its distance from mandocca. This kendra moves in opposite direction of the planet. Hence in pratimaṇḍala rāśi's are written in opposite direction. (95)

When manda kendra is equal to 12 rāśis, the difference between kakśā vṛtta and prati vṛtta is maximum equal to 5/13 aṅgula. (96)

For manda kendra in 6 rāśi, this maximum difference is (4/47) aṅgula. At these places, the attraction due to ucca and nīca are maximum. (97)

When mandocca is in east, pratimaṇḍala is attracted towards east. When moon is at the end of mithuna. (98)

..it will be at end of dhanu in prativṛtta (same points measured in different directions). When it is at end of dhanu rāśi in kakśā, it will be at mithuna end in prativrtta. (99)

Since planet is attracted towards ucca in east, true planet will be less than mean planet in north part of orbit in six rāśis from meṣa beginning. (100)

In south part of orbit in six rāśis starting with tulā, true planet is more then mean planet. At beginning of meṣa and tulā, true and mean planets are same. (101)

We draw lines from earth centre to true and mean planets. These lines will cut both the circles. (102)

The difference between two lines in prativṛtta in kalā is the manda bhujaphala. This is oblique almost in direction of mandocca. (103)

Between the two circles, portion of line from earth centre to true planet is koṭi or koṭiphala. Dohphala and koṭi phala are squared and added. Square of sum is karṇa (true distance). (104)

This karṇa line is inclined towards mandocca (from line to mean planet). The difference between two planets (mean and true) is antyaphala or antyajyā. From diagram of this true moon, we can find observed values of karṇa and biṁba and eclipse also. (105)

Biṁba (angular diameter) of graha is smallest at ucca and largest at end of 6 rāśis. For kendra of 6 and 12 rāśis, there is no bāhu (bhuja) phala and koṭiphala is maximum. (106)

For manda kendra at end of odd quadrants, mandaphala is maximum and koṭiphala is zero. At these places biṁba of planet has mean value (half the sum of ucca and nīca biṁbas). (107)

Planet is slowest at ucca where it is farthest and is fastest at nīca due to nearest position. Its variation is same as bīmba value. (108)

Though motion is same, gatiphala is negative iṅ 6 rāśis starting with makara and positive for manda kendra in other six rāśis (starting with karka). (109)

(Siddhānta Śiromaṇi) In oil mill, bullock moves in one direction, but the crusher of oil seed moves in opposite direction. Śimilarly in nīcocca vṛtta, planet moves in the opposite direction. (110)

When sun and moon in ucca kakśa are in odd quadrants, their distance is more than trijyā and it is attracted towards ucca. (111)

Notes : Elliptical motion of a planet is approximated by two equivalent methods - one by nīcocca vṛtta or manda paridhi and the other by eccentric circles. Both the methods give approximate mandaphala from which true mandaphala is obtained by a geometrical construction. It can also be done by continuously varying

mandaparidhi which makes the orbit completely equivalent to ellipse as proved in chapter 5. Here this has been done by unequal eccentric circles which is a new idea in siddhānta jyotiṣa.

Figure 1 (a) and 1 (b) show epicycle (mandaparidhi) and eccentric circle methods with fixed manda paridhi

Figure 1a - Mandaparidhi

Figure 1b - Equal pratimandala

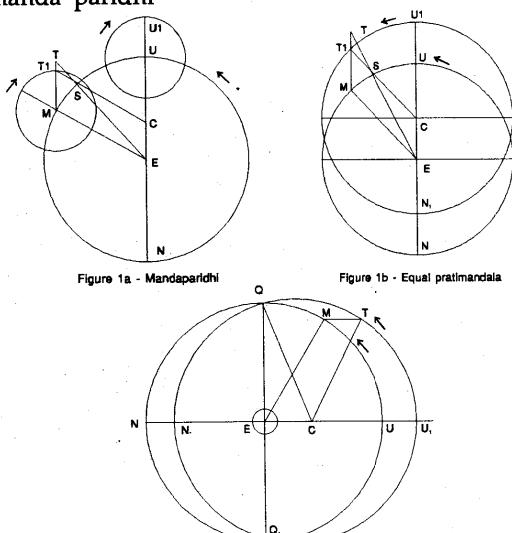



Figure 1c - unequal pratimandala

Fig 1 (c) is of unequal pratimaṇḍala described here. In all the figures E is centre of earth, UMN is kakśa vṛtta of sun or moon. U is mandoca and N is nīca in that circle. Mean planet is moving on this circle in anti clockwise (positive) direction, and its distance from U is manda kendra.

C is on line EU such that EC is parama manda phala at U (mandakendra = 0° or 360°). M is a

position of mean planet. At U position, mean and true positions both start from same direction, U_1 is the true position. In 1(a), true planet is on manda circle with centre at M and radius EC, but is moving in opposite direction. In (b) and (c), true planet is moving in a circle with centre at C in same direction. Thus $CT_1 \parallel EM$ in (b) and $MT_1 \parallel EC$ and equal. Thus in all figures $CEMT_1$ is parallelogram.

In 1(a) and 1(b) T_1 on second circle is the approximate true position. T_1C meets kakśā vṛtta at S. If ES meets MT_1 extended at T_1 then T is the true position of planet. This can also be done by continuously varying the mandaparidhi, due to which path of T becomes an ellipse. If mandaparidhi is r and kakśā radius is R, then change in manda radius is T_1T. In similar triangles

TT_1 S and SCE,

$$\frac{T_1 T}{TS} = \frac{CE}{ES} = \frac{r}{R}$$

$$\text{or } T_1T = \frac{r}{R} \times TS = \frac{r}{R} \times (ET - ES)$$

$$= \frac{r}{R}(K - R) = \frac{Kr}{R} - r = r\left(\frac{K}{R} - 1\right)$$

Here K = manda karṇa

Thus T gives the correct position of true planet, as per changing r. In fig. 1(c), the elliptical motion is obtained by increasing the radius of pratimaṇḍala so that, for mandakendra at Q or Q′, at end of Ist and 3rd quadrants, true and mean planets are same. Then radius of pratimaṇḍala CQ is given by

$CQ^2 = EC^2 + EQ^2$ = (parama mandaphala)^2 + (Trijyā)^2

Here CT II EM, but CT is slightly bigger then EM Hence MT is approximately in direction of EU but not exactly parallel to it.

For moon EC = 300 kalā = 5 aṅgula

CQ = $\sqrt{3438^2 + 300^2}$ Kalā = 57/31 aṅgula

R = 57/18 aṅgula

$UU_1 = EU_1 - EU$

$= (EC + CU_1) - R$

= 5 + 57/31 - 57/18 = 5/13 aṅgula

$NN_1 = CN - CN_1 = CE + EN - CN_1$

= 5 + 57/18 - 57/31 = 4/47 aṅgula.

This explains the varying parama manda phalas.

Similarly we can explain variations in biṁba and gatiphala also.

Verse 113-114 : Ucca kakśā

Manḍa karṇa of sun and moon multiplied by 53 and 23 respectively give their ucca karṇa. Accordingly their ucca kakśa should be imagined.

Notes : For sun, ucca radius (karṇa)

= mean distance of sun (manda karṇa) X 53

= 76,08, 294 X 53 = 40,32,39,582 yojana.

Ucca karṇa of moon = 48,705 X 23 = 11, 20, 215 yojana

This has no physical relevance. Ucca of these planets move with much slower speed than suggested by these orbits.

Verses 115 : Corections for moon

There is no logic behind pāta of moon and three other corrections of its motion. These are based only on observed results.

Notes : Reasons and natures of these corrections have been explained in chapter 6. Though Candraśekhara didn't understand the reasons behind them, he explained the deviations observed by Bhāskara II and himself through these empirical equations. He could understand the period of these variations and their maximum values.

Verses 116-129 : Direct and retrograde motions

As in bahu kakśa yantra (chapter 20), kakśā of a planet is made with mean sun at centre and earth is kept at centre of mean sun orbit. (116)

In prati maṇḍala, śīghra kendra is given in opposite direction. Moiton of śīghrocca is much more then mean motion of planet. Hence motion of sphuṭa graha is corrected with śīghra phala. Similarly mean gati is corrected with śīghra gati phala. (117)

At nīca position or near it, budha and śukra have more speed than sun and they are between sun and earth. Hence they are seen moving in reverse direction. (118)

Again, the planets slower than sun i.e. maṅgala, guru and śani are seen moving in reverse direction, when they are in direction opposite to sun, i.e. in 180° position and nearer to earth. (119)

Now direct and retrograde motions are explained in detail with logic. Sun orbit is made in proportion to its mean distance from earth. (120)

With scale of 1 aṅgula = 13,000 yojana, mean distance of sun is $\frac{76,08,294}{13,000}$ = 58 approximately. Thus radius of sun orbit will be 58 aṅgulas. (121)

The centre point of this orbit will be earth; and mean sun is shown west from that with mean sun as centre, a circle of radius 87 aṅgula is made, showing maṅgala orbit (with same scale 1 aṅgula = 13,000 yojana). (122)

Maṅgala will be kept in eastern point of its orbit. A line through earth, sun and maṅgala is drawn and extended to east. (123)

In east direciton; a star will be assumed at big distance. Maṅgala will be kept close to earth in direction of the star. This will be on right side of the orbit. Motion will be assumed upward this side. (124)

Then sun at left side will be moved down wards with 1 aṅgula daily speed (orbit is 360 aṅgula = 360°) is moved same distance with sun, but within it maṅgala is moved upwards with 1/5th less speed i.e. 0/48 aṅgulas per day (its linear speed). (25)

Mangala is moving north (0/48) aṅgula, but its orbit is moving south by 1 aṅgula. Thus its net south movement is 0/12 aṅgula in its orbit. (126)

Distance of maṅgala is half the distance of sun, hence the retrograde motion will be 24 kalā for 0/12 angula linear speed. (127)

When sun and mars are in same direction from earth, south motion of maṅgala will be 107 sub-divisions of aṅgula. It is 107 kalā in sun orbit of 360 aṅgula circumference, hence it is multiplied

by radius. (58) of sun orbit and divided by distance of mars (radius of sun orbit + mars orbit = 58+87 = 145). (128)

We get 107 X 58 / 145 = 43 kalā as direct motion of mars per day for this position. This is clearly seen in the above diagram. Similarly we can see the direct and retrograde motions of other planets also. (129)

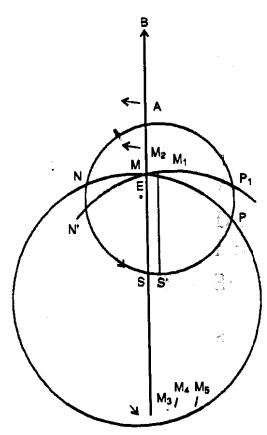



Figure 2 - Direct and retrograde motion

Notes - E is centre of earth, SPN is orbit of sun round earth of radius 57/18 aṅgula (i.e. 34°38′ where 1 aṅgula = 1°) Thus 57/18 or 58 approx. angula = 76,08,294 yojana, mean distance of sun.

i.e. 1 aṅgula = 13,000 yojan approx.

In same scale Mars orbit around S will be of 87 aṅgulas radius.

Orbit of mean sun S is 360 aṅgula where 1 angula = 1°. Here the diagram in fig 2 has been

drawn with 1 cm = 10 aṅgula = 10° = 1,30,000 yojāna.

When mean sun is at S, mars is at M in its orbit PMN. SEM are in one line towards a star B.

When S goes to S′, orbit of mars goes to P_1 M_1 N_1 with centre at S′ all south wards by 1 aṅgula (instead of 1 mm, sun is moved by 4.5 mm for clarity of figure).

Corresponding position of mars is M_1, but mars in same anticlockwise movement goes up to M_2 in its orbit. Average speed of 31 kalā in mars

$$\text{orbit} = \frac{31 \times 87}{58} = 48 \text{ kalā}$$

Hence, compared to earth at E mars goes from M to M_2 where MM_2 = 1 - 0/48 = 0/12 aṅgula.

At 58 aṅgula distance at point A it will make 12 kāla angle. But it is at distance

EM = SM - SE = 87 - 58 = 29 aṅgula

$$\text{Hence angle is } 0/12 \times \frac{58}{29} = 0/24 \text{ degrees.}$$

Similarly, when mars is in same direction as sun at M_3, when sun is at S′, position M_3 shifts to M_4 due to shift of orbit. Due to own motion of mars, it moves further south to M_5. Here M_3 M_4 = 1 aṅgula, M_4 M_5 = 0/48 aṅgula. Distance from earth E is EM_3 = ES + SM_3 = 58+87 = 145 aṅgula

Since the angle at 58 aṅgula is M_3M_5 = 1/48 (1 aṅgula = 1°) angle x at 145 aṅgula is given by

x X 145 = 58 X 1/48

$$\text{or } x = \frac{1/48 \times 58}{145} = 43 \text{ kalā}$$

Verses 130-142 : Śīghra and mandagatis

We take the sun orbit as before (mean sun moving in circle around earth of radius 57/18 aṅgula). Earth is asumed as centre of mārs orbit also (mean mars). It will be a circle of 109 angula radius around earth. (130)

Sun, earth and mars will be kept in same line towards some star of nakśatra orbit. Mean sun is sighrocca of mars. Due to its attraction mars will be seen at its nīca place at 72 aṅgulas west in cakrādha i.e. 180° away from śīghrocca sun. (131)

Daily motions of sun and mars will be shown towards south and north in their orbits as before. (132)

There east motion of mars will be seen 13 kalā from sun i.e. about 1/6th of sun's attraction (72 aṅgulas). (133)

Mars at nīca position will be seen moving retrograde from earth. On earth sun line, śīghrocca and nīca are in opposite directions. At one place mars will be moving east wards (direct) and at nīca, it will move towards west. (134)

Due to attraction of śīghrocca, maṅgala and other planets have śīghra prati vṛtta (eccentric). Śīghra kendra measured from sun as śīghra, proves the orbit of planets around mean sun. (135)

When orbits of mārs etc are shown around earth as centre, then they are affected by mandocca attraction also as in case of moon. (136)

In absence of mandocca attraction, their correction from mean place would have been only due to śīghrocca attraction. Then we could get correction by multiplying the śara (deflection) by

trijyā and dividing by 4th karṇa (true distance). (137)

In the orbit around mean sun, a planet is deflected by two attractions - mandocca in its own orbit and śighrocca duẹ to change in sun position (sighrocca). (138)

Thus the movement of planets being corrected by sighra and manda both, their orbits around mean sun is proved. (139)

The orbit of 5 star planets is seen centred at sun, hence their direction of manda attraction is different from śīghra attraction. (140)

Since the 5 star planets move in sun centred orbits, their manda sphuṭa positions are calculated at first by half śīghra phala of mean planet and then half manda phala of corrected planet. (141)

Then sphuṭa mandaphala correction is made. Then the planets being farther from earth, and to see the manda spasṭa planet in nakśatra, fourth śīghra phala correciton is made.

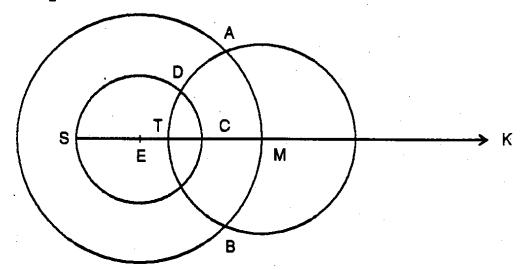



Figure 3 - Mean orbit of mars

Note : (1) Retrograde motion from mean orbit - Fig. 3 shows mean orbit of mars. E is centre of earth. CDS is orbit of sun with radius 57/18 aṅgula (1.9 cms where 1 mm = 4 aṅgula). Orbit of mean

mars is also with E as centre but with radius EM = 109 aṅgula (= 2.7 cm). Mean sun S, Earth E and mean mars M are in one line towards star K. True position of mars is seen deflected west from mean position M to true position T where MT = 72 aṅgulas (1.8 cm). Thus true planet is attracted in direction of sun, which can be considered its śīghrocca. True planet can be considered moving on śīghraparidhi ATB of radius 72 aṅgulas. 31 kalā movement at M is in radius 109 aṅgula hence it is 31 X 109 / 58 parts of aṅgula. Similarly in śīghra paridhi of 72 aṅgulas it is 72X31 / 58. Difference is (31/58) (109-72) = (31/58) X 37

As seen from sun at S at distance 58+37 = 95, it is 31 X 37 / 95 = 13 kalā approx.

(2) Manda and śīghra by epicycle method - Planets have two fold inequalities (1) inequality of apsis or mandocca; and (2) the inequality of apex of quick motion or śīghrocca. These are mandaphala and śīghra phala. Śīghra phala is elongation of inferior planet and annual parallax for superior planet.

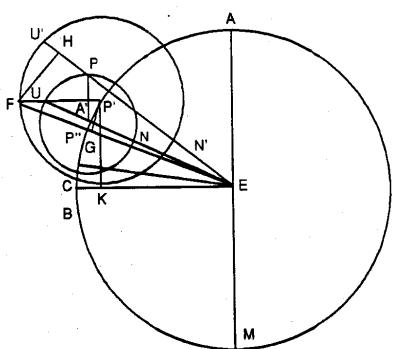



Figure 4 - Epicyclic method for manda and śīghra parallax of superior planets

Calculation of both is shown by epicyclic method in figure 4. ABM is orbit of a planet with centre E, the earth. Its radius is called trijyā. AEM is apseline (ucca nīca line) and EC is direction of śīghrocca. A is centre of first epicycle of planet and UPN epicycle or mandaparidhi. U is its apojee or mandocca and N is mandanīca or perigee. UA′ is radius or manda - antyaphalajyā.

As for sun or moon, when planet on epicycle is at P, arc UP = arc AA′ (angles are equal). EP cuts the orbit at P′ which is position of planet after manda correction which is equal to arc A′P′.

Now with P′ as centre, second epicycle - śīghra paridhi is drawn whose radius is called śīghra antyaphala jyā. EP′P cuts it at U′ and N′ which are śīghrocca and śīghra nīca. P′F is drawn parallel to EC, then F is position of planet on epicycle. EF is joined cutting orbit in P″, which is true position of planet. The correction P′P″ is called śīghraphala.

To find P′P″, we draw P′G, P′K and FH perpendiculars on EF, EC and EU′

Arc P′C is distance between śīghrocca and corrected planet and is called śīghra kendra.

P′K is śīghra kendra jyā and EK, śīghra kendra koṭijyā. From similar triangles FP′H and P′EK

$$\text{FH or Dohphala} = \frac{P'K \times FP'}{P'E}$$

$$= \frac{\text{Śīghra kendrajyā} \times \text{radius of śīghra paridhi}}{\text{radius of orbit}}$$

$$= \frac{\text{Śīghra kendrajyā} \times \text{śīghra paridhi}}{360^\circ}$$

From same triangles

$$HP' \text{ or koṭi phala} = \frac{EK \times FP'}{P'E}$$

$$= \frac{\text{Śīghra kendra koṭijyā} \times \text{śīghra paridhi}}{360^\circ}$$

Hence, HE or sphuṭa koṭi = P'E + HP'

= Trijyā + koṭiphala

In second and third quadrants, sphuṭa koṭi = Trijyā - koṭi phala

So, FE or karṇa = $\sqrt{FH^2 + HE^2}$

$$= \sqrt{\text{Doh phala}^2 + \text{sphuṭa koṭi}^2}$$

Now, from similar right angled triangles P'GE and FHE

$$P'G = \frac{FH \times P'E}{FE}$$

$$\text{or Jyā arc } PP' = \frac{FH \times P'E}{PE}$$

$$\text{or Śīghra phala jyā} = \frac{\text{Doh phala} \times \text{trijyā}}{\text{karṇa}}$$

When this correction in śighra phala or arc PP' is applied to planet corrected with mandaphala, we get the true planet.

(3) **Epicentric method** - In figure 5 (a), APB is orbit of mean planet with centre at E, the earth. AEP is the apse line (ucca-nica line). EF is taken equal to radius of first epicycle (mandaparidhi). With centre at F, another circle A'B'P' is drawn equal to circle ABP. Then A'B'P' is manda-prati vṛtta of planet. Let A' be apogee or mandoca and P' be perigee or manda nīca on the eccentric.

As sun or moon, when mean planet is at M, then planet on eccentric is at M_1 so that arc AM = arc A′M′. EM_1 cuts the concentric in M_2. Then M_2 is the planet corrected by mandaphala. Correction i.e. mandaphala is equal to arc MM_2 which can be found like moon.

Now from ES, EG is cut off equal to radius of śīghra paridhi or śīghra antya phalajyā. With G as centre another circle equal to concentric is drawn. This is called śīghra prativṛtta. ES is produced to meet it in S′. Then S′ is śīghrocca in eccentric. M_3 is planet in second eccentric such that arc $S'M_3$ = arc SM_2. If E M_3 meets the concentric in M_4, then M_4 is true position of planet. Śīghra phala is arc M_2M_4.

To find the arc M_2M_4, this part of the diagram is drawn separately in figure 5(b).

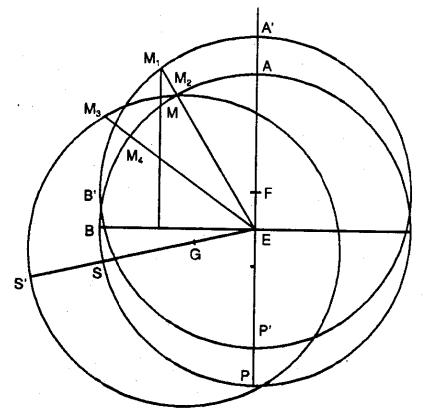



Figure 5a - Eccentric method

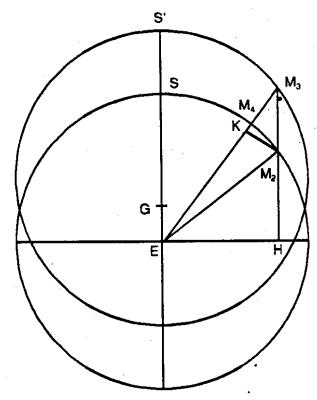



Figure 5b - Śighra Eccentric

EM_2 and M_2M_3 are joined. M_3M_2 produced meets perpenducular line to ES at H. M_2 K is drawn perpendicular to EM_3.

Now M_3M_2 = GE = Śīghra antya phalajyā. Arc SM_2 is angle between Śīghrocca and once corrected planet and SM = Śīghra kendra.

Thus its jyā EH is śīghra kendrajyā and M_2 H, its koṭijyā is śīghra kendra koṭijyā.

M_3H, or sphuṭa koṭi = $M_3M_2 + M_2H$

= śīghra antya phalajyā + śīghra kendra koṭijyā (for śīghra kendra between 3 and 9 rāśis)

sphuṭa koṭi = śighra antya phalajyā + śīghra kendra kotijyā M_3E or karṇa = $\sqrt{EH^2 + M_3 H^2}$

= $\sqrt{\text{Śīghra kendrajyā}^2 + \text{sphuṭa koṭi}^2}$

From right angled similar triangles M_3 M_2K and M_3EH,

$$M_2 K = \frac{EH \times M_3 M_2}{M_3 E}$$

or Jyā arc $M_2 M_4$, or Śīghraphala jyā

$$= \frac{\text{Trijyā}}{\text{Trijyā}} \times \frac{\text{Śīghra kendrajyā} \times \text{Śīghra antya phalajyā}}{\text{Karṇa}}$$

$$= \frac{\text{Dohphala} \times \text{Trijyā}}{\text{Karṇa}} \text{ (as before)}$$

This is same as that obtained by epicyclic method

Verses 143-146 - Revised methods

Thus the tārā grahas are made true by half śīghra phala, half mandaphala, mandaphala and śīghra phala. But in case of maṅgala budha and śani, some errors were noticed. For correcting that error, (143)

I have done parocca sanskāra in these corrections. The corrections in moon like tungāntara etc also should all be considered on the authority of sages. (144)

To find the observed positions of planets, whatever corrections are made to traditional values, all are called bīja samskāra. (145)

(From Vārttika) Good astronomers have done bīja corrections to planet positions found as per texts. From these corrected planets only, tithi, eclipse etc are decided and good or bad results are told. (146)

Verse 147-151 : Manda and sīghra according to Bhāskara (Siddhānta Śiromaṇi) Manda sphuṭa planet moves in manda prativṛtta and sphuṭa moves in śīghra prativṛtta. Hence manda sphuṭa planet should be considered mean planet before doing śīghra corrections. (147)

The planets are moving in their prativṛtta (eccentric), then their position in nakśatra orbit is

the sphuṭa (true) planet at that time. To know the position of this true planet, ancient scholars have assumed ucca. (148)

The point on eccentric farthest from earth is called ucca. Since this position also is moving, scientists have calculated its motion also. (149)

At 6 rāśi from ucca, the point of eccentric nearest to earth is called nīca. At these positions, mean and true planets are in same line with earth, hence mean and true planets are same and there is no bhujaphala. (150)

Planet is farthest at ucca position, hence it looks smallest there. At nīca it is nearest and looks largest. (151)

Verse 152 : Ecliptic direction

From shadow of śaṅku, observation through instruments and āyana considerations, I have found the diffenence between maximum north and south krānti to be 47° only. Hence, I have asumed 23°30′ maximum krānti instead of 24° assumed in old texts.

Verses 153-163 - Nati and lambana

Lunar eclipse has already been explained fully (in chapter 8). Now reason of lambana and its variation is being explained. (153)

When moon and sun are 6 rāśi from each other and their degrees minutes etc are equal, then, if its śara is less than mānaikyārdha (sum of semidiameters of covered and coverer - earth shadow) - - - (154)

...earth shadow covers disc of moon. Thus moon and shadow of earth both are in same orbit

(moon's orbit). Hence there is no nati on lambaṇa between them - they will be seen together from all directions. (155)

In solar eclipse sun is covered and moon is coverer - both are not in same place (i.e. sun is not in moon orbit). Due to their positions far from each other, their angular separation varies and only in that line they are seen together. This is reason of lambana or nati. (56)

Dṛg maṇḍala has centre at earth's centre. Hence on earth's surface, a planet in dṛg maṇḍala is seen in lesser than half area of dṛg maṇḍala by radius of earth. (157)

When at the end of amāvasyā (sun and moon in same direction), sun is on east horizon, then from earth centre, sun and moon are in one line. (158)

But from earth's surface, moon is seen lower. Its deviation is maximum as it is in perpendicular direction to sun's direction. Hence, this apparent deviation is parama lambana or nati (maximum parallax). (159)

When planet is in zenith, it is seen in same line from earth's centre or surface. Hence, there is no nati. At any other place between zenith and horizon, the difference between direction of planet seen from surface and direction from centre is called nati. (160)

When sun is in zenith, dṛg maṇḍala will be ecliptic, hence there will be no nati. Maximum lambana will be at horizon. (161)

When a planet is in dṛkkśepa vṛtta (north south direction) the deviation is called nāti. It is

always less than parama lambana (as planet is never on south point of horizon). Hence, in dṛk-maṇḍala and ecliptic... (162)

if there is difference of krānti of sun and moon, their east west difference also increases. At zenith, ecliptic and dṛkmaṇḍala are same, hence there is no nati or laṁbana. (163)

Verses 164-173 : Vitribha lagna

Perpendicular to ecliptic at tribhona lagna (i.e. lagna - 90°) passes through zenith. At mid day also, moon is away from ecliptic at distance of its śara. (164)

Hence due to laṁbana, moon doesn't remain in same line with sun at end of amāvasyā (moon-sun=0°). Hence, vitribha lagna is needed to calculate laṁbana or nati. (165)

Natyajyā of vitribha is in meridian line. If vitribha is inclined towards east or west from meridian, it increases. (166)

Natāṁśa of vitribha is in its dṛk maṇḍala which is great circle through kadamba (pole of ecliptic), and always perpendicular to ecliptic. Natyajyā of vitribha thus increases in proportion to its krānti (distance from equator). (167)

When north krānti of vitribha is equal to north latitude of a place, then at the time of rising of sāyana meṣa 0°.... (168)

... madhya lagna (ecliptic point at meridian) and vitribha lagna are same. They are different otherwise. The jyā of difference between vitribha and madhyalagna natāmśa.... (169)

...is considered as śara (north south distance) between them. At śara distance from ecliptic, moon has sphuṭa (changed) zenith angle. (170)

Hence to find correct dṛkkśepa (north south distance from zenith), sum or difference of śara and akśāmśa is taken and it is further corrected with krānti of vitribha. (171)

When sphuṭa parvānta (sun - moon = 0° or 180°) is corrected with lambana, moon and sun are exactly in north south circle as seen from surface of earth. (172)

With increase in krānti of tribhona lagna, change in grāsa and sparśa time of eclipse due to śara of moon will increase. (173)

Notes : This has already been explaiend in chapter 9 on solar eclipse.

Verses 174-189 : Eclipse duration through diagram.

In diagram of eclipse (chapter 10), the 'chādaka' (coverer) will be moved on grāhaka path as explained earlier. Then duration of eclipse will become clear. (174)

When at the time of sphuṭa amānta (sun-moon = 0°), sāyana meṣa 0° is rising, then this itself will be mid time of eclipse. No addition or substraction will be needed. (175)

This is because lagna will not have krānti then (sāyana meṣa 0° is at equator). At other times, mean amānta time will be different from sphuṭa amānta. When meṣa 0° is rising, sparṣa and mokṣa times also will remain same. Their mean times need correction for krānti of lagna and direction of śara. (176)

When moon is on horizon, its distance is same from earth's furface and earth's centre. (177)

As moon rises above horizon, its distance from earth surface gets smaller compared to distance from earth center. Hence, apparent diameter of moon (angular) increases and its shadow cone (śaṅku means cone here, not gnomon) becomes wider. (178)

As sun and moon are in different orbits, the shadow cones will be bigger on earth surface and the time of eclipse and total covering will increase. (179)

In lunar eclipse, covered planet (moon) and coverer (earth shadow) both are in one orbit, hence even with increase in śaṅku, time of eclipse and maximum covering doesn't increase on being viewed from surface. (180)

At the end of pūrṇimā, moon motion in vimaṇḍala is oblique to ecliptic. (It is more oblique for relative motion of moon). Hence motion of śara also is oblique (inclined to perpendicular to ecliptic.) Oblique speed is 1/12 of śara. (181)

Hence the time of sparśa, mid eclipse etc. will before or after the calculated time by the time arising out of 1/12th of sara. (182)

(In lunar eclipse), śara of covered moon is in different direction from coverer (earth shadow). Hence direction of eclipse and śara are different. (183)

In solar eclipse, moon is coverer whose śara is in direciton of coverer. Hence eclipse and śara are in same direction. (184)

In both eclipses, only moon has valana (due to its śara). Hence in lunar eclipse, at the time of contact (sparṣa) true valana will be given in its own direction from east point. (185)

At the time of mokśa, it will be given in opposite direciton from west point. In solar eclipse, mokśa is in east, hence valana at mokśa time will be in its own direction and at sparṣa time in opposite direction from west point. (186)

At equator, planet is in its own direction at time of rising, midday or setting. At other places valana due to akśāmśa occurs. (187)

As the disc of planet appears shifted towards pole, the eastern point bends towards north. (188)

The disc in west sky is deviated towards south. Since disc direction is changed due to ākśa valana, the true āyana valana is changed. (189)

Verses 190-192 : Āyana valana

(Siddhānta Śiromaṇi) Yaṣṭi is square root of the difference of squares of trijyā and āyana valana jyā. Yaṣṭi is multiplied by śara (latudude) of planet (moon) and divided by trijyā to give śara for krānti correction. Otherwise 3 rāśis are added to graha and its bhujajyā is found. That is multiplied by śara and divided by trijyā to give the śara for krānti. (190)

In this, Bhāskaracārya has taken śara along dhruva prota less than kadamba prota which cannot be proper for āyana karma. (191)

After āyana dṛkkarma, graha, nakśatra and their conjunctions are in dhruva prota line. After drkkarma correction, the planet in krānti vṛtta is called bhuja. (192)

Mean śara in kadamba prota and koṭi of śara—squares of both are added and their square root is karṇa. This will be the sphuṭa śara in direction of dhruva. This karṇa will be greater than śara (in kadamba direction). (193)

Notes : These formula have already been discussed. Dhruva prota śara has been approximately considered equal to kadamba prota śara, which is not strictly correct (see chapter 8).

Verses 194-196 : Rising of planets at poles

There is night for six months in meru (polar region). Moon and other planets rise and set there (north pole) when they are in north krānti. (194)

When planets are within two rāśis of sun, their heliacal rising and setting will be considered. (195)

When the difference in krānti of sun and another planet is equal to the kālāṁśa for rising and setting, their rising setting will start. (106)

Verse 197 : As the north akśāmśa increase, krānti vṛtta will be more inclined towards south, from dṛk maṇḍala of moon. Hence, north horn of moon will be upper. (197)

Verse 198 - Vaidhṛti and vyatīpāta are called mahāpāta. Even when there is big difference in orbits of sun and moon, their rays meet due to equal krānti.

Verses 199-206 - Importance of star circle

A straight line from centre of earth to moon and other planets crosses krānti vṛtta and goes upto stars circle. (199)

This line is called 'bhagaṇa' line (revolution is measured from its point of intersection with nakśatra circle). Calculation of planets starts from bhagaṇa only. Hence this line is very useful to the world. All seasons occur due to sun and there is development of moving and non-moving beings (200)

Sun removes darkness also, thus it has many virtues. Similarly moon has many virtues like 'āhlād' (happines) (201)

From work (result) we know the reason (origin). Similarly people compare the terrible and comfortable qualities of the creator by seeing sun and moon. (202)

(In astrology), we know about results of deeds in past life from position of planets. (203)

From motion of planets; we also know about length of orbit and their liṇear diameters. From their śara, we know that earth is fixed. (204)

From star circle, we can imagine the great job done by the Creator. It also gives some light. (205)

Sailors know direction from stars and do not lose their way. Thus star circle (bha-cakra) is very useful to people. (206)

Verses 207-213 : Reason of seasons

Time has three main indicators - summer (hot), rains and cold. Each has two parts, thus making six seasons. (207)

In Bhārata varṣa (India) all six seasons have their full duration. With change of place, the effect of seasons vary. (208)

First half of hot season is vasanta (spring) which is pleasant. Second half is very hot. First half of rains is varṣā (rainy season) and second half is śarat (autumn). (209)

First and second halves of cold season are 'hemanta' (winter) and śiśira (cold winter). Due to change of place (akśāmśa) and change in krānti of sun, seasons change. (210)

When sun rays fall oblique on earth, heat is reduced and cold increases. (211)

Where sun rays fall perpendicular, heat is more and it starts summer. (212)

In summer, straight rays of sun, evaporate the water of oceans. The vapour goes up and cools. Then due to attraction of earth, it falls. Hence rains start after summer. (213)

Verses 214-224 - Season zones

Region from equator to 8° latitude is very hot and hence there is no hemanta or śiśira there. Summer and rains are more (214). After rains it is śarat. In beginning of summer, it is spring. Between sarat and spring, there is mild cold due to heavy rains. (215)

From 8° to 16° latitude, cold (śīta) is for two months pauṣa and māgha (Dec. January) only and in remaining ten months there are 4 seasons only. (216)

From latitude 16-24° (most part of India), cold is for four months from mārgaśīra (around 15th November) and in remaining eight months starting with caitra (about 15th March) there are 4 seasons. (217)

Upto 24° akśāmśa (from equator), it is tropics. At 24° latitude, all six seasons are of 2 months each. Region from 24° to 40° latitude is called sama maṇḍala (sub-tropical) (218)

From 24 to 32° latitude, it is cold for six months starting with kārttika (15th Oct.). Remaining six months are hot and have other 4 seasons. (219)

From 32° to 40° latitude, it is cold for eight months starting from āśvina (15th Sept.). Remaining 4 seasons are in 4 months starting with Jyeṣṭha. (220)

From 40 to 50° latitude, it is temperate region (cold area). From 40 to 48° latitude, it is cold for the months from bhādrapada (15th August) and for very short period of two months other seasons come. Thus the season zones change from equator at intervals of 8° latitude each. (221-222)

From 48° north to meru (90° north) - 42° latitude zone is mostly covered with ice. This is for north hemispehre. In south hemisphere, also there are similar zones, but seasons are corresponding to opposite rāśis. (223)

For counting of seasons, we should count the months from vaiśākha when sāyana sun enters meṣa. (224)

Verses 225-231 : In spring time, forest land is covered with fall of dry leaves. Due to friction among bamboos, the spark ignites dry leaves and whole forest burns. (225)

Due to that, dense black smoke rises upwards with hot surface air. This layer of smoke covers the sky and hangs in air. (226)

In summer season, due to hot sun rays, water from oceans is evaporated and goes up. This mixes with smoke and forms cloud. Thus cloud contains smoke, water, air and lightning. (227)

As an elephant draws water with its trunk, or a man in his cloth bag, similarly sun also draws up ocean water through its rays and mixes it with clouds. Thus cloud looks dense and heavy. (228)

Clouds are situated 1/4 to 1/2 kosa high from surface and are attracted by earth. When they become heavy, are not distant from surface, they fall in form of water. (229)

Fate in controlled by the creator, who acts through time and place. Same fate gets rains from clouds. Varāhamihira has written about clouds, thunder and lighting. This is not discussed here. (230)

Ocean contains salt water, but due to sun rays, only small particles of water evaporate and rise. Salt particles do not rise. Hence rain water is sweet. Now I describe some pāṭīgaṇita (arithmetics) for recreation. (231)

Verses 232-239 - Cube root method

Product of three equal numbers is called cube (ghana). To find the root of this cube, formula is being stated. We give a point at the last digit towards right (unit place). Then on fourth digit from unit, another point is given. (232)

Thus points are given on every fourth digit towards left. The number before left end dot up to left end is taken. From that, we substract the maximum possible cube root. The number, whose

cube was deducted, is treated as labdhi (quotient). (233)

To find second labdhi, first labdhi is squared and multiplied by 300. Result is kept separately. (234)

By that we divide the number remaining upto second point from left. Result is taken as second labdhi. (235)

If labdhi is more than 9, it is taken as 9 only. Below the result kept separately, we write the result found by multiplying square of second labdhi, with first labdhi and 30. (236)

Below it we write cube of second labdhi. Sum of these three digits is substracted from dividend. If remainder is very small, second labdhi is reduced by one and again the three numbers are written one below the other. (237)

Sum of these three digits is substracted from dividend. Digits till next (third) point are joined with the remainder. The two labdhi numbers obtained so far, are treated as first labdhi. From that, we find next labdhi as before, and sum of 3 numbers is substracted from dividend. This process is continued till digits remain in dividend. This labdhi is cube root. (238)

Cube root has same number of digits, as there are points on the cube. When cube is less than 1000, its cube root is of single digit and found in one step without above process. (239)

Verses 240-242 - Method for cube

These are stated by Bhāskarācārya II in Līlāvati. Product of three equal numbers is called

ghana (cube) (second method). Cube of last digit is written of number whose cube is to be found. Then square of last digit is multiplied by 3 and by the remaining digits. Then the square of remaining digits is multiplied by 3 and last digit. Then cube of remaining digits. (240)

These three numbers are written one below other and shifting them to one place left. Their sum will be cube of whole number.

(Third method) The number is divided into two parts and same method is used. (241)

Fourth method : The number whose cube is to be found is divided into sum of two parts. Three times the number is multiplied by both parts. To that, we add cubes of both parts. Result is cube of the number.

Fifth method : If cube of a square number is to be found, then cube of its square root is found. Then we find square of the result. (242)

Verses 243-244 : Two verses about cube root (Līlāvatī) (Same method is given by Āryabhaṭa).

(1) Unit place is called 'ghana', 10 place is 'first aghana' and 10^2 place the 'second aghana'. We take groups of 3 digits starting from night. (ghana is marked with sign (I) and aghana places with other marked with -)

(2) Greatest possible cube is substracted from last ghana place.

(3) Second aghana place (right to last ghana place) is divided by thrice the square of cube root already obtained in (2) (243)

(4) From first aghana place (right to second aghana place) substract square of quotient multiplied by 3 times previous cube root.

(5) From the ghana place (right of first aghana place) we substract the cube of quotient.

(6) The process is repeated till all digits are exhausted.

Notes (1) Method for cube

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab\,(a+b)$

Method5

$(a^2)^3 = a^6 = (a^3)^2$

Example 1-$12^3 = (10+2)^3$

$2^3 = 8$					8
$3.2^2\,1=12$			1	2-	
$3.2.1^2 = 6$			6	-	
$1^3 = 1$		1	-		
12^3	=	1	7	2	8

Example 2 - $123^3 = (12 \times 10+3)^3$

$3^3 = 27$							2	7
$3.3^2.12 = 324$					3	2	4-	
$3.3.12^2 = 1296$			1	2	9	6		
$12^3 =$ 1728		1	7	2	8			
123^3	=	1	8	6	0	8	6	7

Example 3 - $(125)^3 = (120+5)^3$

$120^3 = 1728000$

$5^3 = 125$

$3 \times 5 \times 120 \times 125 = 225000$

$1235 = 1953125$

Example 4 (Śrīdhara)

$$\sum_{k=1}^{n} \left\{3k(k-1)+1\right\} = n^3$$

$k^3 - (k-1)^3 = k^3 - (k^3 - 3k^2 + 3k - 1) = 3k(k-1)+1$

Putting k = 1, 2-3 ...... n and adding

$1^3 - 0 = 1$

$2^3 - 1^3 = 3(1.2) + 1$

$3^3 - 2^3 = 3(2.3) + 1$

$n^3 - (n-1)^3 = 3(n-1)n + 1$

we get $n^3 = 3[1.2 + 2.3 + ...+(n-1)n] + n$, which proves the result.

(2) Cube root

Principle is explained by a two digit number (ab)

$(ab) = 10a + b$

$(ab)^3 = (10a + b)^3$

$$= \frac{10^3 a^3 +}{1} \frac{3.10^2 . a^2 b \ +}{2} \frac{3.10a . b \ +}{3} \frac{b^3}{4}$$

b^3 is number + remainders at unit place.

At 3rd place we have number in 10s - Ist aghana. At 2nd place we have number in 10^2 -2nd aghana. At Ist place we have ghana number in 10^3. i.e. after ommitting 3 digits to its right.

From Ist place we substract $a^3 \times 10^3$ then we get the numbers at 2, 3, 4 places.

Approximately this is largest number at place $2 = 3 \times 10^2.a^2 b$. To find b we have to divide it by $3 \times 10^2 a^2 = 300a^2$

We may ommit two digits to right at this step. Taking 2nd aghana only, the digits are $3a^2b$ only. So in Aryabhaṭa method this is divided by $3a^2$ only. However, the remaining numbers also,

are multiplies of b and if we don't have sufficient remainder equal to those multiples, cube of result will be more than given number. Hence b is reduced by 1.

Thus the remainder is equal to $(300a^2 b + 30 ab^2 + b^3)$ or in decimal notation $3a^2b$ at second aghana, 3 ab^3 at Ist aghana and b^3 at ghana as recommended by Āryabhaṭa.

Examples : Cube root of 1953125

Āryabhaṭa Method

```
                    1 - -, 1 - - 1
                    1 9 5 3 12 5 (1
                    1^3
                  ----------
3.1^2      =      3) 09 (2 --- (A)
                    6
                  ----------
                    35
3.2^2.1-           =12
                  ----------
                    233
2^3        =        8
--------------------------------
3.12^2 = 432)       2251 (5
                    2160
                  ----------
                    912
3.5^2.12 =          900
                  ----------
                    125
5^3        =        125
                  ----------
                    x
```

Siddhānta Darpaṇa Method

1	$\dot{1}$ 95 $\dot{3}$ 1$\dot{2}$5 (125
	1^3
$300 \times 1^2 = 300 \times 2$	953 ......(A)
$30 \times 1 \times 2^2 = 120$	728
$2^3 = 8$	
$300 \times 12^2 = 43200 \times 5$	225 125
$30 \times 12 \times 5^2 = 9000$	225 125
$5^3 = 125$	
	xx

At (A) we could get 3 as quotient, but next remainder would have been very small, hence we have reduced it to 2.

Verses 245-248 - Cube root of component numbers

To find the cube root of a number expressed in two components (of degree or minute or other sexagesimal components), we substract the maximum cube from first components (degrees), cube of the first root is substracted from cube of next higher integer. Result is called 'antya-hāra' or divisor. (245)

If root of first component is more than 30, then remainder of first components is multiplied by 60 and added to second component. Sum will be divided by antya hāra (divisor) to get the minute component of the cube root. (246)

If root of first components is less than 30, then we correct the divisor by 'guṇya' and 'guṇaka' (first and second factors). From divisor we substract

the remainder of first component to get guṇya. Guṇaka is found by dividing the divisor (antya hāra) by first component of root increased by one. (247)

Guṇya and guṇaka are multiplied together and divided by antya hāra. Result is substracted from antya hāra to get sphuṭa antya hāra (correct divisor). Remainder of first components is multiplied by 60 and added with second component as before. Sum is divided by revised divisor to get the second component of cube root in minutes.

Note : (1) Examples will clarify the method. First we take a number bigger than 30 degrees say $(32° \ 20')^3$ = 33,802° 42′ approx.

3^3	33,802° 27	42′ (32° 19′ 6
300×3^2 =2700 x 2 =5400 $30 \times 3 \times 2^2$ = 360 23 = 8	6802 5768	
Divisor D =	1034 =	R remainder
	x60+42	
$33^3 - 32^3$ = 3169	62082	
x1	3169	
	3039 2	
x9	28467	
	19250	
x6=	19014	
	236	

Thus the root 32° 19′ .6 is approximately correct. accuracy will increase if root is much bigger than 30°

We take a small number 2°20′ to explain second method

$(2°20')^3 = 12° \ 42'$

	12° 42′ (2° 20′.1
23=	8
$D = 3^3 - 2^3 = 19$	4 = R
	x60 + 42
D-r = D-5 = 14	282
14x20=	280
14x1=	20
	14
	6

Gunya = D-R = 19 - 4 = 15

$$\textbf{Guṇaka} = \frac{D}{2+1} = \frac{19}{3}$$

$$\textbf{correction r} = \frac{\text{Guṇya} \times \text{guṇaka}}{D} = \frac{19 \times 15}{3 \times 19} = 5$$

Thus the answer is 2° 20′.1 which is approximately correct.

(2) **Proof of the formula** : The formula for approximate roots is based on linear interpolation. Correction of divisor is based on second order interpolation. This is based on concept of differential coefficient which is proportional to rate of change of dependant variable (Rolle's therem in

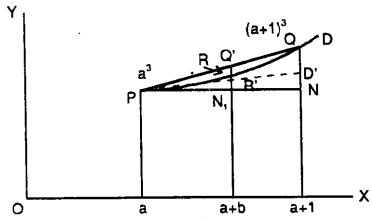



Figure 6 - Approximate cube root

Differential calculus). Second order correction is based on Taylor's theorem upto second differential.

We consider variation of function $y = x^3$ in figure 6. Point P is the cube of x= a, i.e. y coordinate is a^3. The next cube is $(a+1)^3$ at point Q. Real value lies between a and a+1 which may be written as a+b where b is less than 1

Thus the real root is a° 60b′

Increase in root	Increase in cube
1	$(a+1)^3 - a^3$ = QN = D
b	D x b = R = Q′M

Thus extra value b is given by

$$b = \frac{R}{D} = \frac{R}{(a+1)^3 - a^3}$$

However, cube increases more rapidly. Then linear ratio i.e. rate of increase also increases. Real value of increased root is R′ corresponding to reduced value D′ of D

Increase in D is D-R, this is for length (a+1) from origin. For unit distance from a to (a+1) the change is DD′

$$= \frac{D - R}{a + 1}$$

or reduction DD′ = $(D-R) \times \frac{D}{a+1} \times \frac{1}{D}$

$$= \frac{\text{Guṇya} \times \text{guṇaka}}{\text{Divisor}}$$

Correct divisor D′ = D - DD′

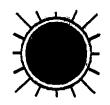
When a is big, interval of 1 is considered small, and the first formula can be used.

Verses 249-252 - Conclusion and end

Method to find square root has already been stated. Method for cube root has been stated, because it is needed in calculations. These methods have been stated in a general way. Other techniques can be known by scholars by themselves (from other books on mathematics). (249)

(Siddhānta Śiromaṇi) : Due to fear of bulk, I give only a little bit of logic and proceed. Intelligent men will understand from that only. The subject can be understood only by knowledge of gola (spherics). (250)

Yogīs win over their enemies like moha (illusion), krodha (anger) and kāma (desires) etc. for control over mana (mind), prāṇa (life force or breath). With that they are free from five troubles like vidyā (informaiton), asmitā (pride), anurāga (result of rāga - obsession), dveṣa (enemity) and abhiniveṣa (fixation). Even then yogīs are unable to see the supreme truth even for a moment. That supreme truth or realisation is given by Lord Jagannātha very easily to his devotees. May that Lord free us from our troubles. (251)

Thus ends the twenty-first chapter in siddhānta darpaṇa describing rationale of astronomy, written as a text book and accurate calculations by Śrī Candraśekhara, born in a famous royal family of Orissa. (252)

Chapter - 22

SAṀVATSARAS

Kalādhikāra - Samvatsaras etc.

Verse 1 - Scope : Lord Kṛṣṇa is Kāla (time or death) himself and the rod of death god merges into him. He is shining like blue lotus. His feet are worshipped by Śiva who is subject of Pārvatī's plays. He is in form of Jagannātha at Nīlācala. He controlled the tyrant Kaliya but took mercy on him after hearing weeping of women. After worshipping him I start Kālādhikāra.

Notes : This is the fifth part (adhīkāra), describing time (kāla). This starts with chapter 22 which describes units of time like saṁvatsara (different types of years).

Verses 2-15 - Kāla as god.

Kāla is of two types - 'nitya' (eternal) and 'janya' (changing) Eternal Kāla is god himself. God is eternal hence his other name is kāla. (2)

Since god measures time (through periodic events), he is 'janya' kāla also. Hence for welfare, people remember god before starting all works. (3)

(Kūrma purāṇa) God is without beginning and end. he doesn't diminish. He is within every point and beyond every thing. He is great lord, all pervading, independent and soul of everybody. (4)

Kāla itself is the supereme Brahmā, Vāsudeo and Śaṅkara. World is created by Kāla only, and is destroyed by that. (5)

(Smṛti) At every place and in every event, god only is the lord. He is in all forms and without beginning. May he increase my happiness (6). I pray Acyuta (indestructible) by whose memory, all defects are destroyed in tapa (penance), yajana (worship) etc.

If Supreme lord is without form, how he can be imagined? Answering this, yoga vāśiṣṭha has stated about his appearing in human body, which is being quoted here. (8)

(Vāśiṣṭha Rāmāyaṇa) Kāla is of two types -nitya (eternal) and janya (created or changing). First (nitya) kāla is supreme lord himself. He cannot be imagined by speech or mind. But to grace the devotees he takes body (9) God holds sword (khadga) and noose (pāśa) and is adorned with armour (kavaca) and rings (kuṇḍala). He has 6 mouths in form of six seasons (10). Valour of his 12 hands are 12 months. He is attended by persons in his own form. (11)

Lord of Lakśmī (wealth goddess) is thus a form of kāla but takes various forms for the sake of devotees. Hence, devotees meditate on any of his forms, according to their desire. (12)

From god as eternal time, the other type of changing time has also been created. This is told in vedas and by Svāyambhuva manu also (in Manusmṛti) (13)

(Manusmṛti) God as form of time for the purpose of creation, has created divisions of time, nakśatra, planets and this world. (14)

In horā skandha (Bṛhat-samhitā of Varāhamihira), Sun has been called soul of kāla. It is not correct to interprate this as opposite to siddhānta version. (15)

Verses 16-24 - Time units from sun and moon

Sun is origin of creation and in that sense, he is epitom of kāla. Main division of time from truṭi till pralaya is year. Components of year are - ayana, month, pakśa and day etc. Another name of year is saṁvatsara. (16-17)

Sun moves northward for 3 seasons (6 months) and then moves south wards for other 3 seasons. These periods are called 'ayana'. This is according to krānti motion, 'ay' verb means 'to move'. (18)

In two months period when aśoka tree has special signs like flowers etc, first season vasanta (spring) occurs. (19)

The time, which is measured is called māsa (month, māsa = to measure). The time, which measures changes in moon phase is called lunar month. Measure of sun's passage in one rāśi is called solar month. (20)

Periods 30 civil days and 30 sidereal days are called civil (sāyana) month and nākśatra (sidereal) month respectively. (21)

Periods of increasing or decreasing phase of moon are called śukla pakśa (bright half) and kṛṣṇa

pakśa respectively. These pakśas are used for pitṛ functions. (22)

Measure of fractional increase of decrease of moon's phase is indicated by tithi. (23)

Similarly time has other divisions like 'yāma' muhūrtta etc. Due to lack of space derivation of all have not been stated. (24)

Verses 25-26 - Nine fold divison

Time is divided in nine ways - Cāndra, nākśatra, sāvana, bārhaspatya, saura, mānava, paitra, daiva and brāhma. (25)

Among these, only the first five are used by human beings. Others starting with mānava are used in their context only. (26)

Verses 27-34 - Cāndra divisions

Movement of moon 12° ahead of sun as measured from earth's centre is called cāndra māna (tithi). (27)

When difference between moon and sun becomes one full revolution, it is called cāndra māsa (lunar month). Half the lunar month is called pakśa. $1/15^{th}$ part of pakśa is tithi and half of tithi is called karaṇa. (28)

Extra or lapsed tithi or month, fast, festivals, and sacred ceremonies, auspicious or bad times, and rites for deceased - all are decided according to cāndra māna (tithi) only. (29)

Months (lunar) are named according to the names of nakśatras joined by moon on pūrṇimā of that month. Similarly, at start of jovian year (Bārhaspatya year), the nakśatra in which jupiter enters, is name of that jovian year, acording to

sūrya siddhānta. But due to fear of lengthening, all details are not described. (30-31)

Kṛttikā and rohiṇī, both nakśatra can be assumed Kṛttikā and due to conjunction with any of them at pūrṇimā, month is named Kārttika. (32)

Then we could as well tell the kārttika and mārgaśīrṣa months as rohini and bharaṇī respectively, when moon joins their neighbouring nakśatras bharaṇī and rohiṇī at pūrṇimā. (33)

Due to this difficulty, kārttika and mārgaśīrṣa etc. are counted as per the old tradition (without frequent changing of names. The said conjunction at pūrṇimā also occurs frequently. (34)

Verses 35-36 - Nākśatra time

Time taken by nakśatra in one complete revolution in west direction is called 1 nākśatra day. Ghaṭī and pala are subdivisions of this time only (60 divisions at each step). Life period of Brahmā also is as per this time only. (35)

Life in nakśatra units is multiplied by kalpa solar days (·15,52,00,00,00,000) and divided by kalpa nakśatra days (15,82,23,78,28,28,000) to get sphuṭa solar years etc. (36)

Verses 37-46 - Solar time

As stated before, period between one rising time of a graha or nakśatra to its next rising time is called its sāvana dina. (37)

Still for calculation of mean planet, period taken by mean sun in crossing (21,600) kalā i.e. one revolution with respect to earth, is called sāvana dina whose value is (21,659/8) asu. (38)

Difference between two sphuṭa sun rises is sphuṭa sāvana dina. Generally this is used as a sāvaṇa dina. According to this sāvana dina only, yajna, purification, counting of days, lords of year and month etc are decided. (39)

According to Jovian years, saṁvatsaras starting with prabhava, lost years, extra years, years stated in svara śāstra and horā etc are counted. This is described in Bṛhatsamhitā. (40)

The time taken by sun to move 1° is called 1 solar day. 30 such days make one solar month 12 solar months make one solar year. (41)

According to solar months, we observe saṅkrānti, ayana, seasons etc. Day and night of deva and asura, yuga and manvantara etc are counted by solar time only. (42)

In some places, marriage, festivals, sacred thread ceremony, house construction etc are done by solar time also. (43)

(Sūrya siddhānta) From tulā beginning to 86°, movement of sun is called "saḍaśīti mukha' period (i.e. eighty six day period). There are 4 such periods in a year. This ends with duplicate rāśis only (3,6,9,12 rāśis). (44)

When sun enters 27° of dhanu, 23° of mīna, 19° of mithuna and 15° of kanyā, the periods ending with that are called śaḍaśītī mukha as they come after intervals of 86° each (i.e. 86 solar days). (45-46)

Notes : These are periods of 86 solar days each, which are slightly bigger than civil days. They are unequal depending on speed of sun. Solar days from 15° of kanya to tulā 0° i.e. 16

solar days are out of these 4 periods. 4 divisions of year approximately correspond ot equinox and solstice days but reason for deducting 4 solar days from each is not understood.

Verses 47-60 : Sankrāntis -

Entry of sun in two movable (cara) rāśis at beginning of odd quadrants - i.e. meṣa and tulā are called viṣuva sankrānti. Entry of sun in two cara rāśis at beginning of even quadrants (karka and makara) are called south and north ayana sankrāntis. (47)

Entry of sun in 4 rāśis between moving and double (i.e. 4 fixed rāśis) - vṛscika, kumbha, vṛṣa and siṁha is called viṣṇupadī saṅkrānti. (48)

Last point of a rāśi is beginning point of next rāśi. When disc of sun touches that point, saṅkrānti (crossing over from one rāśi to next) starts. From beginning of sankrānti till the end, when last point of sun's disc is in contact with border point, it takes 33 daṇḍas (as diameter of sun is 33 kalā). Hence 33 daṇḍa is the sacred period of saṅkrānti. (49)

In ten sankrāntis, the whole sankrānti period of 33 daṇḍa from beginning to end is sacred period. In dakśiṇāyana (karka) saṃkrānti, last 16/30 daṇḍas (2nd half) and in uttarāyana (makara) sanskrānti first half of 16/30 daṇḍa are sacred. (50)

Sankrānti is the period when parts of sun disc are in both the rāśis. When saṅkrānti falls in day time, bath, charities etc are done. (51)

When saṅkrānti falls in first half of night, 2nd half of previous day is observed. When it (mid

point) falls is second half of night, then first half of next day is observed as saṅkrānti day. (52)

When saṅkrānti falls exactly at mid night, the day on which its greater portion falls (when sun is slower) is observed as sankrānti. This is not considered in ayana sankrānti. (53)

South or karka ayana, falling in night makes second half of previous day as saṅkrānti. North or makara saṅkrānti falling in night makes the first half of next day as sacred. (54)

When saṅkrānti is in day time, whole day is sacred. Though this is smārtta view, the period near saṅkrānti is definitely very fruitful. (55)

As saṅkrānti is a sacred day, above good works are prescribed but non-vegetarian food is prohibited. Restriction of non veg food is from 30 daṇḍa before sankrānti and upto 30 daṇḍas after it, i.e. for total of 60 daṇḍas. (56)

Last degree of a rāśi is māsānta and first degree is called niramśa. When sun is in two degrees, it is māsānta or niramśa kāla. In this period auspicious works like journey, marriage should not be done. (57)

Saṅkrānti (crossing) period of centre of sun (puruṣa) is thousandth part of a truṭi. It is not possible for human beings to know this. (58)

(Sūrya siddhānta) Two equinoxes (viṣuva saṁkrānti) and two ayana saṅkrāntis (solstice) are 'nābhi' i.e. dividing points of ecliptic - hence they are very important, sāyana karka and makara also are equidistant from equator, as sāyana meṣa and sāyana tulā are equidistant from meru. (59)

From north ayana (i.e. makara sāyana saṅkrāntis) there are 3 seasons starting with śita for two months each similarly from sāyana karka saṅkrānti (south ayana) 3 seasons starting with varṣā (rains) 2 months each. (60)

Verses 61-70 - Measure of different years

Number of mean sāvana days in five types of years is being stated according to mean sun motion. (61)

Candra varṣa	354/22 days
Nākśatra varṣa	359/1 days
Sāvana varṣa	360/0 days
Bārhaspatya varṣa	361/15 days
Saura (solar) varṣa	365/15/31/31/24 days
12 lunar revolutions	327/52 days (62-65)

Manus starting with svāyambhuva, rule for their periods called manu (or manvantara). For that, there is no separate count of days, months etc. (66)

One lunar month is day night of pitars. 360 such days (i.e. 360 lunar months) constitute a pitar year. There are (10,631) sāvana days in a pitṛ varṣa. (67)

Time scale for deva and asuras is same, but their day and night are in reverse order. When one has day time (for 6 months), the other will have night. (68)

360 daiva (or āsura) days i.e. 360 solar years make divya varṣa, which contains (1,31,493/9) sāvana days. (69)

In one year of Brahmā there are (31,10,40,00,00,00,000) solar years (43,20,000 years of yuga X1000 yuga in a kalpa X 720 day nights in a year) Day of Brahmā (kalpa) has already been described. (70)

Verses 71-74 - Other opinions

Harivamśa purāṇa has given different value of manu which cannot be derived from solar motion. That is mentioned here out of respect for declaration of sages. (71)

(Harivaṃśa purāṇa) 10 divya varṣa make one day-night of manu. 10 day-night of manu make 1 pakśa. (72)

10 manu pakśas are 1 manu month end 12 manu months is one manu season - as stated by seers who know the truth. (73)

For vaidika and smārtta functions there are many types of time scales. This arrangement of time has been discussed in smṛtis. Hence proper time for marriage etc are not being discussed here. (74)

Notes : Manvantara of Harivamśa

Manu year = 10 seasons (suppose)

= 10 X 10 manu months = 10^3 manu pakśa

= 10^4 manu days = 10^5 divya years

= 360 X 10^5 years (solar)

1 manvantara = 43.2 X 10^5 years of yuga X 71 yugas

$$\frac{1 \text{ manvantara}}{1 \text{ manu year}} = \frac{43.2 \times 71}{360} = 8.6 \text{ years approx}$$

i.e. 1 manu = 86 seasons approx.

Verses 75-76 : Moon motion

According to motion of moon, ocean water rises. Hence waters of Gaṅgā and other rivers also must be rising. But this doesn't appear reasonable, hence it is not being described. (75)

Hemisphere of moon facing sun is lighted with sun rays and opposite part remains in shadow. In front of half shadow, people on earth see, that phase of moon (bright portion). This increases as moon moves away from sun. In other direction when moon approaches sun, its phases decreases. Do the pitars living on moon see the west ward motion of sun for a month ? (76)

Note : It is held that soles of deceased (pitars) live on moon surface opposite to earth. For them sun will rise in east and set in west after 15 days.

Verses 76-77 - End

May Lord Kṛṣṇa as Jagannātha destroy my attachment to greed of world, who is shining black like cloud, bees, yamunā river water, blue lotus, black spot in sun and blue emerald, but wears yellow dress shining like lightning, campā flower, kuṃkuma, turmeric and gold. (76)

Thus ends the twenty second chapter describing kāla in siddhānta darpaṇa written as text book for accurate calculation by Śrī Candraśekhara, born in famous royal family of Orissa. (77)

Chapter - 23

PURUŚOTTAMA STAVA

(Prayers to Lord Jagannātha)

Though the knowledge of astronomy is by grace of god, the beauty of sanskrit verses having double meanings for each word, cannot be expressed in english. We may be content with our inner devotion devoid with word marvel of sanskṛta prayers.

CHAPTER - 24

Upasañhāra Varṇana
(concluding chapter)
Kautuka Panjikā Vidhāna
(Easy calculation of calendar)

Verses 1-2 - Scope

I had started this text with prayer of my most respected Lord Jagannātha. Again with respect to Him, I am closing this book. (1)

Before closing this book, I. will describe many methods for easy calculation of calendar. My father had prepared pañcāñgas with great labour. In my young age, I had prepared this easy pañjikā on that basis. (2)

Kautuka Pañjikā

Verse 3 : Object

I am describing the method to prepare new pañcāṅga by seeing old ones only, without knowing spherical mathematics etc.

Verses 4-13 - Tithi, nakśatra and yoga

When sun is in odd quadrants from mandocca, sāvana dinas in an year will be 358/18/16. In this period, there would be exactly 364 tithis. (4)

In 358/13/3 sāvana dinas there are 354 nākśatra dinas. Hence manda kendra from sun is not needed. (5)

In 358/14/30 sāvana dinas there are exactly 380 1/2 yogas. This happens when sun is at end of odd quadrants from mandakendra. (6)

Karaṇa in a particular tithi ardha will be repeated after one year in same tithi ardha. If nakśatra (13°20′) is divided into 4 quadrants (3°20′ each), there are nine quadrants in a rāśi. (7)

In these sāvana days (358/18/16), moon will complete its 12 lunar months (synodic) and there will be exactly 4 tithis more. There are 3 nakśatras more than a complete revolution. (8)

From complete cycle of yogas, there are 2-1/2 yogas more. After this additions to tithi etc of previous year, the tithis etc of present year are called iṣṭa or accepted tithi etc. In iṣṭa tithi (starting time) we add 18/16 nāḍi (daṇḍa), in iṣṭa nakśatra 13/3 daṇḍa and in iṣṭa yoga 14/30 daṇḍas are added. (9)

If after these additions, the sum is less than 60 daṇḍa, then in iṣṭa days 1 vāra is added. If result is more than 60 then 2 vāras are added. (10)

In tithis of previous (grāhya) year, we add 4 tithis, in grāhya nakśatra 4 nakśatras and in grāhya yoga 2-1/2 yogas are added. Then we get iṣṭa tithi etc. (11)

The questions such as these can be raised - what will be iṣṭa tithi in a given month and pakśa? What will be nakśatra or yoga on that iṣṭa tithi ? (12)

Since tithi is the main thing in a pañcāṅga, the day on which iṣṭa tithi is over or the nakśatra or yoga will be completed on that tithi, all will be taken as iṣṭa tithi etc. According to this iṣṭa tithi, we add or deduct 1 day to grāḥya day. (13)

Notes : (1) Selection of this period for new calender.

A synodic lunar year ends after 354.367 days after which tithis are exactly repeated.

However, lunar nakśatras are repeated after 355.816 days after completing 13 exact revolutions round earth. Since we count the days from tithi, convenient tithi after completion of lunar years are searched, when other elements are slightly different, from previous year.

Calculations are based on these modern figures

Sidereal period of moon = 27.321161±3-7/2 hours

Synodic period of moon = 29.5305881 ± 7 hours

Tropical solar year = 365.2421 9879 days

Sidereal year = 365.256362 days = 365/15/22/54/12

Indian year = 365.2587564 days = 365/15/31/31/24

Daily speed of moon = 13°.176358° = 13°/10/34/53

Daily speed of sun = 0°.9856091 = 0/59/8/11.57

(Sun + moon) speed = 14°.161967

1 cycle of 27 yogas = 25.420197 days

14 cycles of 27 yogas = 355.88275 days

	Tithi	Nakśatra	Yoga	day
Tithi	1	0.970187	1.045528	0.98435
Nakśatra	1.030729	1	-	1.014601
Yoga	0.95645	-	1	0.94149

Karaṇa is entirely decided by tithi, hence we consider other pañcāṇga elements for each tithi following completion of lunar year. When fraction of day and addition remains within 1 day, vāra (week day) is same as day number, for more than 1 day, next week day is taken. After 354 day of lunar year, week day number is 354 ÷ 7 = remainder 4 i.e. week day will be added 4 or substracted 3, we write - 3.

Value in days on completion of lunar year

Tithi completed	Days	Weak day	Nakśatras (after 13 cycles)	Yoga (after 14 cycles)
0	354.36705	-3	-0.80282	-1.60990
	(+0.98435)		(+0.97019)	(+1.04553)
1	355.35140	-2	0.16737	-0.56437
2	356.33575	-1	1.13756	0.48116
3	357.32010	0	2.10775	1.52669
4	358.30445	1	3.07794	2.57222
5	359.28880	2	4.04813	3.61775
6	360.27315	3	5.01832	4.66328

We see that on completion of 3rd and 4th tithis, we have most convenient times to calculate nakśatra and yogas. After 3rd tithi, week day will be same, nakśatra will be 2.108 more and yoga will

be 1.527 more. After 4th tithi, week day wil be 1 more, nakśatra will be 3.078 more and yoga will be 2.572 more. Before that, week days and nakśatras deviate more from round numbers. After 4 tithi the addition to nakśatra and yoga will be more and yoga will be neither complete number nor round number.

(2) **Correction terms :** Third and fourth tithis are both convenient, rather third is slightly better. However, Candraśekhara has selected the completion of 4th tithi after end of a lunar year for correction of previous year elements.

At the end of 4th tithi, completed days are 358.30445 = 358/18/16 as given. (From above chart)

Week day is 1 more, if after adding 18/16 daṇḍa extra at end of tithi, the tithi ending is within 60 daṇḍa. If it is more than 60 daṇḍa, 2 is added to week day number.

Nakśatras after 13 complete cycles, are 3.07794 more. Fraction of 0.07794 nakśatras = .079078 = 4/45 daṇḍa. Here 5/3 daṇḍa have been deducted so that after 358/13/3 days 13 X 27+3 = 354 nakśatras are completed.

2.57222 yogas are completed after 4th tithi. Thus after 14 cycle of 27 yogas and approximately 2.5 yogas, the lapsed yogas are 14 X 27+2-1/2 = 380-1/2 yogas. Fraction above 2.5 yogas is 0.07222 yoga = 0.07222 X 0.94149 days = .06799 days = 4/5 daṇḍa.

Here 3/16 daṇḍa have been substracted from 358/18/16 to get 358/14/30 days for 380-1/2 yogas.

(The slight difference is due to difference in sidereal solar year and siddhānta year as written above.

(3) Example : Suppose we have to calculate new year elements iṣṭa from old year (grāhya)

Old Year Elements (Grāhya)	New Year elements (Iṣṭa)	Additional time to be added (daṇḍa/pala)
Tithi = t	t + 4	18/16
Nakśatra = n	n + 3	13/3
Yoga = y	y + 2-1/2	14/30

If grāhya element is for vaiśākha sukla 5, in next year's calender we get the value for vaiśākha sukla 9. That tithi will end 18/16 daṇḍas after 5th tithi had ended previous year. Nakśatra on that day will be 3 more and this 4th iṣṭa nakśatra will end 13/3 daṇḍa after previous year value. Similarly 2-1/2 will be added to yoga. That will end 14/10 yoga after grāhya ,yoga had ended previous year. Then we add 1/2 duration of that yoga, then y+3 yoga will end.

Verses 14-16 : Correction in tithi

In solar months from meṣa to simha, we add to iṣṭa tithi 38, 68, 78, 68, 38 palas respectively. (14)

But these will be substracted from iṣṭa tithi in five solar months starting with tulā. When sun is in 18° of the rāśi, this addition or deduction amount will be calculated in proportion to the difference of sun degrees. These palas are not added or substracted for endings of nakśatras. (15)

The times of increase or decrease in tithi duration for solar months starting with dhanu are

dhanu (15), makara (14), kumbha (13), mīna (12) mena (11), vṛṣa (10), mithuna (9), karka (10), siṁha (11) kanyā (12), tulā (13), vṛścika (14) . . . (16)

Verse 17 : These amounts will be reduced or added to the time of tithi endings. These amounts reduced by 1/12 will be reduced or added to nakśatra times.

Notes : Compared to lunar year, this calendar period is 4 tithis more. Compared to solar year, it is about seven days less. Its value being between both, minor adjustments in both types of values wil be needed.

Difference from solar year is

365.256362 (Siddhānta value 365.258756)

- 358.30445 days

= 6.951912 days (6.954306)

= 6.85187 degrees (6.85423)

These periods for calculation are calculated with reference to mandocca 78° of sun. Present year position of 78° for sun will not have any correction for mandaphala. But last years position will be corresponding to 78°+7° = 85° position of mean sun. For that true sun will be less by r sin 7° = 16 kalā approximately, where r is parama mandaphala. Thus tithi, calculated from (moon-sun) will be more corresponding to 16 kalā difference. For 12° difference, tithi extends to 60 daṇḍa approximately.

For 16 kalā the correction will be $\frac{60}{12} \times 16 =$ 80 pala approximately. By accurate calculation this comes to 78 pala for 78° sun position i.e. mithuna (18°).

Increase in value of sine is proprotional to its cosine which is maximum for 0° difference (from 78° position) and minimum for 90° difference. It is positive in Ist and last quadrants. Thus correction is positive for two signs more and 2 less than mithuna with decreasing values. In third rāśis from mithuna (168° or -12°) correction is zero and for others it is negative. These values are based on difference of sines of 18° and 25° for meṣa, 48° and 55° for vṛśa etc. For remaining days, correction is found by proportion.

Speed of tithi is speed of (moon-sun) which is 12/13 of moon speed. Speed of change of nakśatra is moon speed only. Since nakśatra changes 1/12 earlier, its time correction will be 1/12 less than that for tithis.

End of tithis increases progressively, hence its duration also will increase. For example, suppose 3rd tithi ends after 20 pala, 4th tithi after 25 pala more time. Then duration of 4th tithi will by 25-20 = 5 pala more. This is like correction of time for difference between true and mean sun (equation of time or velāntara) sanskāra. Roughly it is seen to be 2/5 of that value. Velāntara sanskāra becomes zero after every 3 months. If this interval is taken as 180° when sin function becomes zero at the ends, then 7 days are less than 1/6 of 3 months. Sin 180°/6 = 1/2. Hence, it is less than 1/2 of velāntara. These are approximate values based on observations and mathematical derivation will be very complicated, but still approximate.

Verses 18-19 : Half day for 7 days less than grāhya day will be the half day for iṣṭa day in present year. Hence, difference for 7 days in half

day will have to be added ot tithi, nakśatra and yoga for south krānti and deducted for north krānti. (18)

This will make the tithi etc sphuṭa according to old theories like sūrya siddhānta etc. From these rough tithis, we can find accurate tithis according to mandocca etc. (19)

Notes : We are taking the interval 7 days less than the solar year. Hence in south ayana, when day length is decreasing, this will be half day of 7 days earlier in previous year i.e. more than the value. Hence, sunrise will be earlier and the time of tithis etc, after sunrise will increase. This increase or decrease will be corresponding to difference in half day length for 7 days earlier in grāhya tithi.

Verses 20-24 : Correction of tithis

If grāhya tithis (of previous year) are accurate, then they should be made rough (because these calculation are based on rough method only). Tuṅgāntara and pākśika etc. corrections are calculated and applied to accurate tithis in reverse manner. (20)

In tithis from Ist to 7th we add 68, 125, 159, 166, 145, 98 and 35 kalās respectively and deduct from tithis from 8th to 14th in reverse order. (21)

In half pakśa, tuṅgāntara bhuja kalā is divided by 225. Result will be bhuja khaṇḍa of tuṅgāṇtara. (22)

According to these results, mandaphala of jupiter in kalā is multiplied by 5 and divided by 2. That will be parā etc for mid pakśa. These results

in parā for Ist to 7 are multiplied by 13, 25, 35, 45, 52, 57, 60. From 8th to 14th they are multiplied in reverse order - 60, 57, 52, 45, 35, 25, 13. In all results we divide by 66 and correct in reverse way as done for tungāntara. Then accurate tithi will become rough. (23-24)

Verses 25-27 : Correction of nakśatras

Similarly at the end of tithi also, the nakśatra to be completed, will be used after deducting 1/13 parts less than the correction of that tithi. If by end of tithi, nakśatra is not completed, then the time between tithi and nakśatra endings is multiplied by the difference of correction times of two tithis in which nakśatra falls. Product will be divided by 60 and added to the lapsed part of nakśatra or deducted from remaining part. Then the grahya (base years) nakśatra will become rough. (25-26)

We deduct from grāhya nakśatras (of base year), the following kalā (palas) in solar months of makara (26), kumbha (46), mīna, meṣa (53), meṣa (46) and vṛṣa (26). These are added to months beginning with karka. There is no change in dhanu and mithuna months. (27)

Notes : Correction for tithis are clear. Due to tuṅgāntara and pākśika correction, tithis had been revised. But the present calculations are for pre-revision tithis, hence reverse corrections are needed.

Nakśatra correction depends solely on moon motion. Same difference is caused in tithi which depends on sun motion also. Since moon is 13

times faster than sun, nakśatra change will be 13/12 times faster. Corection ratio will be 12/13 times the ratio for tithi.

Verses 28-29 : Yoga correction

For yoga starting with viṣkumbha etc, the correction is 1/7th less than that of nakśatras. Like lapsed and remaining parts of nakśatra, here also proportionate correction time will be found. By this method we get rough tithis from base year accurate values (28)

To make the rough tithi's etc. accurate, we make corrections for tungāntara and pākśika both results in order opposite to that explained above. For tithi, nakśatras, this correction will be positive for 5 months starting with solar karka month and negative for 5 months, starting with solar makara month. (29)

Notes : Speed of yoga will be still faster hence the difference from tithi correction will be double of nakśatra correction. Difference for nakśatra is 1/13.36, hence difference here will be 2/13.36 = 1/7 approximately.

Above were correction for making accurate values (for which tungāntara and pākśika corrections have been made to moon) to rough values. For changing rough values to accurate, we have to make opposite corrections.

Verse 30 : Adhimāsa

If there is an adhimāsa between grāhya (base year) month and iṣṭa (next year) month, then we take the month next to the grāhya month. Because there are 371 + 1/16 = 371/3/45 tithis in a solar year.

Notes : There are 365.256362 days in a sidereal solar year and 1 tithi = 0.98435 days (verse 13)

Hence no. of tithis in 1 year are

$$\frac{365.256362}{0.98435} = 371.0635$$

= 371+1/16 approx. (371/3/48)

Here it is given 371/3/45

Verses 31-35 - Nakśatra and rāśi of sun.

In one revolution of sun or a solar year, there are 365/15/32 sāvana (civil days) dina. Vāra of nakśatra is that vāra (week day) on which sun enters that nakśatra. (31)

Vāra of nakśatra in present year will be one more than vāra of nakśatra in previous year. In addition we have to add 15/32 daṇḍa also. Because solar year days etc divided by 7 leave a remainder of (1/15/32) days. If sum of base year daṇḍa of nakśatra and 15/32 added for present year is more than 60 daṇḍa, then two days are added to vāra for sun's entry in a nakśatra or rāśi (saṃkrānti). (32)

In base year sphuṭa sun is for some days after a particular śaṅkrānti. In present year, for same days interval after śaṅkrānti, we deduct 15/18 kalā to get sphuṭa sun. (33)

Saṅkrānti of a month will be 15/32 daṇḍa after previous years saṅkrānti. We take the previous week day in base year and from sphuṭa sun of that day we get the sphuṭa sun of this year (by substracting 15/18 kalās as before). (34)

But here it is asked to deduct 15/18 kalās. This can be made sphuṭa (more accurate) by

multiplying with sphuṭa gati of sun and dividing by 60. That sphuṭa kalā etc. should be substracted from previous year's sun of corresponding day (365 days before). (35)

Notes : Adding 1 week day and 15/32 daṇḍas have already been explained in the text. Civil days in a solar year divided by 7 give.

$$\frac{365/15/32}{7} = 52 + \text{remainder } 1/15/32$$

With average speed of 59/8 sun will move 15/32 X 59/8 = 15/18 kalā, hence 15/18 kalās are deducted, because sun's position will match after 15/32 daṇḍa. Its position of same time will be less by motion in 15/32 daṇḍa

More accurately, the movement is

$$\frac{15/32 \times \text{sphuṭa gati}}{60}$$

Verse 36 : Sun gati.

Sphuṭa gati of sun is obtained by substracting sun of grāhya dina (day of base year) from sun of next day.

If sañkrānti of sun falls after end of amāvasyā, then the month will be extra (adhimāsa).

Verses 37-38 : Moon gati

Lapsed portion of a nakśatra is found by multiplying period in daṇḍa etc by 800 and dividing it by total duration of nakśatra. Result will be in kalā etc.

The number of previous nakśatra will be multiplied by 800 and the kalās of present nakśatra are added to it. This gives kalā of sphuṭa moon.

By dividing it with 60 we get the degrees and degrees divided by 30 give rāśis. (37)

We divide (28,80,000) by total duration of nakśatra in palas. Result will be sphuṭa gati of moon. By this method, learned astronomers can know tithi and gati of moon for many years in advance. (38)

Notes : Motion of moon is considered constant within a small period of nakśatra of approximately one day.

$$\frac{\text{Kalā of past poriton}}{\text{period of past portion}} = \frac{\text{800 kalā of full nakśatra}}{\text{Period of full nakśatra}}$$

Each nakśatra is of 13°20′ = 800′. So we add the completed nakśatras and their value is found by multiplying with 800′ kalā. Adding the covered portion of present nakśatra will give positon of true moon.

Since moon moves in

d palas — 800 kalā of nakśatra

Hence in 3600 pala (= 60 X 60 pala of a day), it

moves $\frac{800 \times 3600}{d}$ kalā

Thus the speed is $\frac{28,80,000}{d}$ kalā

Remaining convensions are based on definitions

1 rāśi = 30°, 1° = 60 kalā

Verses 39-45 : Possibility of eclipse

To know the posibility of lunar or solar eclipse, sphuṭa position of moon and sun are to be calculated. For that, we find the ahargaṇa (count of days from a particular standard) for pūrṇimā (for solar eclipse) or for amāvasyā (in lunar eclipse).

It will be checked by comparing with vāra (week day). (39)

This ahargaṇa is multiplied by = 3/10/48 kalā daily speed of rāhu (candra pāta) to get the position of pāta. At pūrṇimā end, if distance between moon and its pāta is less then 13° then solar eclipse is probable. For lunar eclipse, distance between moon and pātā should be less than 9° (as stated earlier) when moon and shadow (i.e. sun + 6 rāśi) are in one position. (40)

We calculate the amānta period (when sun = moon) and it is corrected for lambana. For lambana corrected period, moon and its śara are calculated. Dṛkkśepa (south-north distance from zenith) is calculated from natāmśa of vitribha lagna etc. (41)

Śara and dṛkkśepa are added or difference is taken to find sphuṭa śara. If this sphuṭa śara is less than 32 kalā (sum of semi diameters of sun and moon) then solar eclipse is probable. Then method of finding grāsa, mokśa etc has already been stated. (42)

For iṣṭa (current) year, we find the interval of civil days from saṅkrānti of sun to amāvasyā or pūrṇimā day. In previous year, we calculate the gati of sun or moon for same days after that saṅkrānti. Motion for that period is added to the positions at previous year saṅkrānti to get the present position. Degrees and minutes are made equal by gati on current day to find true time of amāvasyā etc. (43)

In a lunar year, after one eclipse (of any type), next eclipse is possible after 15, 165, 180, 195, 345 or 360 tithis. (44)

After 10 years (lunar) again, the grahaṇa are probable at these intervals. (45)

Notes : We have already discussed the interval between eclipses while discussing maximum number of eclipses in a solar year. Repetition of this cycle is after 19 year's vedic yuga or saros cycle of Chaldea, because the revolution of rāhu is approximately in that period. In this period, solar year also matches lunar year with extra lunar months, as explained in calender (introduction to chapter 6)

19 solar years = 6939.6018 days (Tropical)

235 lunar months = 6939.688 days (synodic months)

235 lunar months = 19X12+7 = 19 years with 7 extra months.

Saros cycle is of 223 synodic months = 242 months of solar year relative ot rāhu (dragon year - dracontic months). Sun completes 1 revolution relative to rāhu in 346.62005 days. Thus

223 synodic months = 6585.321 days

242 draconitic months = 6585.357 days

This period of 242 draconitic months is equal to 18 years 11-1/3 days or (18 years 10-1/3 days if 5 leap years come).

Half of this period is 111 synodic months 15 tithis (3339 tithis - Viśvāmitra figure). This is mentioned as 10 lunar years approximately.

Verses 46-49 - Maṅgala position

Tārā graha like maṅgala do not make complete revolutions in an year. To calculate their positions,

we need calenders (pañcāṅga) of many years. From these ephemeries, many uses can be found. (46)

Position of maṅgala at present day and month of current solar year will be exactly same as position 32 years ago or 32 years after on same day and month of solar year. When it is 12 rāśis more than sun, 180 kalā is to be added. (47)

When maṅgala is 1, 2, 3 rāśis ahead of sun, we have to add 210, 270, 360, 480, 630 and 810 kalā to maṅgala. (48)

When maṅgala is in five rāśiṣ (ahead of sun) beginning with dhanu, we substract the kalā (last five figures above), increased by 1/15th value, 8th part, 5th part, 8th and 15th parts respectively. Then maṅgala will become sphuṭa after 32 years.

Notes : 32 solar years = 11,688.203 days

17 revolutions of maṅgala = 17 X 686.97982 = 11,678.656 days.

Thus after 17 revolutions of maṅgala, sun will be 9.547 days behind its complete 32 revolutions. Thus mars will be ahead by 9.547 X 0/31/25.52 kalā daily speed = about 300 kalā.

Thus at the time of 32 complete revolutions of sun, mars will move 300 kalā ahead. According to siddhānta figures, this difference will be about 180 kalā.(= 3°)

This difference arises when 32 years ago, mārs was in same position as sun. When mārs was 1 rāśi ahead of sun its true speed will be more than mean speed corresponding to śīghra phala for 1 rāśi.

Then correction will be 180 kalā + śīghra phala for 1 rāśi = 210 kalā. (actually it will be difference of śīghra phala for 30° and 33°). Similarly other corrections are made.

From dhanu, śīghra phala will be negative but śīghra paridhi will increase. Hence the corrections will be proportionately more, but negative.

Verses 50-51 : Budha positoin

Position of budha for a particular day of a solar month will be same as its position 13 years ago on same day of same solar month. There are many other calculations involved in it. Speed difference of spaṣṭa budha and mean sun is multiplied by 8 and divided by 3. (50)

If budha gāti is more than (59/8) mean sun gati, then this result will be added ot budha rāśi, otherwise substracted (if budha gati is less). If budha is vakrī, then sum of sphuṭa budha and mean sun gati is multiplied by 8 and divided by 3. Result will be substracted from vakrī budha. (51)

Notes : 13 solar years

= 4748.3326 days (Sidereal)

= 4748.3638 days (siddhānta)

54 revolutions of budha = 13 X 87.96926

= 4750.34 days

41 conjunctions of budha = 41 X 115.878

= 4750.998 days

Mean speed of budha is same as mean speed of sun as it moves around sun in a much smaller orbit.

41 conjunction time - 13 solar years = 4750.998 - 4748.333 = 2.665 = 8/3 days approx.

Hence, change from mean position of sun is the movement of budha relative to sun in 8/3 days.

Relative speed of budha = budha gati - sun gati (mean)

It is positive, if budha gati is more than mean sun gati (59/8) and negative if less.

If budha is vakrī , relative speed = - (budha-gati + sun gati)

Thus the corection to budha positon is 8/3 X relative speed.

Verses 52-53 - Guru position

Guru position will be almost same as its position 12 years ago. Difference of guru and sun in kalā is divided by 120 and result is added to 210 kalā. Sum is added to guru. (52)

Again when guru is ahead of sun by 5 rāśis starting with karka, then difference of sun and guru is divided by 33, 17, 11, 17, 33 Result is substracted from previous diffrence and remainder is added ot guru. When guru is in 5 rāśis starting with makara, this difference is added to 210 kalā and the sum is added to guru to find its sphuṭa (true) position. (53)

Notes : 12 solar years = 4383.0763 days

1 Guru year = 4332.5891 days

Thus at end of 12 solar years, guru will move ahead of complete revolution by 50.487 days. In this period guru will move ahead with mean speed by 50.487 X 4/59.13 kalā = 251.703 kalā.

Thus, corresponding to difference of 251.7 kalā śīghra kendra of guru, śīghra phala is to be substracted. Since guru is on average 5.2 times farther from sun compared to earth, it is about 6.2 times farther from earth at śīghra kendra zero. At 252 kalā it will be about 6 times farther. Then śīghra phala correction will be 252/6 = 42 kalā. Hence, change in guru for 0° śīghra kendra (at base year) will be 252-42 = 210 kalā.

Difference in śīghra phala correction decreases with increase in śīghra kendra. Hence reduction from 252 kalā mean difference will be less. Thus corresponding addition to 210 kalās is made according to śīghra kendra.

Verses 54-55 - Śukra position

On a particular day and month of solar year, position of śukra will be same, as it was on same month and day, exactly eight years ago. Difference of śukra sphuṭa gati and mean sun gati is multiplied by 5 and divided by 2. (54)

If sphuṭa śukra gati is more than mean sun gati (59/8), then this will be added to sphuṭa śukra of base year (8 years back). That will give sphuṭa śukra of current year. When śukra is vakrī , sum of gatis of sphuṭa śukra and mean sun is multiplied by 5 and divided by two. Result is deducted from vakrī śukra to get the sphuṭa position. (55)

Notes : 8 solar years = 2922.0508 days

13 revolutions of śukra = 13 X 224.7008 = 2921.1104 days

5 conjunctions of śukra = 5 X 583.921 = 2919.605 days.

Thus after 8 solar years, śukra will move ahead of its conjunction for

2922.051 - 2919.605 = 2.446 days

This is approximately 2.5 = 5/2 days.

In 5/2 days śukra will move ahead of sun, corresponding to its relative motion in that period.

As in case of budha,

relative speed of śukra = speed of sphuṭa śukra - mean sun speed

When sphuṭa śukra has more speed than mean sun speed (59/8) the correction is positive. Otherwise it is negative. For vakrī śukra, the relative speed of śukra is sum of the speeds.

Verses 56-57 : Śani position

Śani will be almost in same position on a particular solar month and day in which it was exactly 30 years ago on same solar month and day. Difference between mean sun and sphuṭa śani in kalā is divided by 120 and added to 375 kalā. Sum will be added to sphuṭa śani (of base year). (56)

When śani is in 5 rāśis beginning with meṣa from sun, the difference between sphuṭa śani and mean sun in kalā is divided by 20, 10, 7, 10, 20 respectively, and the result is added to corrected sphuṭa śani. Śani (its śīghra kendra) being in 5 rāśis beginning with tulā, these qualities will be deducted. (57)

Notes : 30 solar years = 10,957.69 days

1 revolution of śani = 10,759.2262 days

29 conjunctions of śani = 29 X 378.092 = 10964.668 days. Correction to śani can be calculated in two ways.

After 30 solar years, śani is before conjunction position by 6.978 days. In this period sun will be faster and it will cover more period, hence after 30 years sun will be behind śani. Sun will cover this difference in 6.978 days with relative speed of (59/8.19-2/0.45) = (57/7.74). Thus the difference will be 6.978X57/7.74 kalā = 398.646 kalā.

Another method will be to calculate the extra movement of śani in 198.464 days with mean motion.

This is correciton corresponding to mean motion. The correction for śīghra phala corresponding to 398.6 kalā is 398.6/10.55, as śani is on average 9.55 times earth distance away from sun. This will be about 36 kalā, which will be substracted for first quadrant.

Thus for zero śīghra kendra in base year, the correction will be 398.6 - 36 = 363 approx. Here it has been given as 375 kalā corresponding to siddhānta figures of solar year, and saturn revolution.

The further correction for other śīghra kendra is difference between śīghra phalas of 400 kalā and 30°+400 kalā, 60°+400 kalā etc. for meṣa, vṛṣa rāśis etc.

Verses 58-60 - Pāta and mandocca

Maṅgala, guru and śani will never be more than 6 rāśis ahead of sun. After 6 rāśis, same kalā difference will be added in next half of śīghra kendra. (58)

Ahargaṇa between grāhya and iṣṭa day is found. It is divided by 143 and result is added to

ahargaṇa. Sum is divided by 19. Quotient is substracted from candra pāta (rāhu) if iṣṭa day is after grāhya day. (59)

Ahargaṇa is added with its 1/440 part and divided by 9. Result is added to moon mandocca when iṣṭa day is after desired day. (60)

Notes : Calculation of other planets is already explained. Pāta and ucca of moon have uniform motion, hence they are directly calculated from no. of days lapsed since the base day. Pāta moves in reverse direction hence, its extra motion is deducted.

Pāta of moon (rāhu) completes 1 revolution in 6793.4598 days

Hence motion in A days (A = ahargaṇa)

$$= \frac{360^\circ \times A}{6793.4598} = \frac{A^\circ}{18.8707} = \frac{A}{19}\left(\frac{19}{18.8707}\right)$$

$$= \frac{A}{19}\left(1 + \frac{0.1293}{18707}\right) = \frac{A}{19}\left(1 + \frac{1}{145.9}\right)$$

Here, the formula is $\frac{A}{19}\left(1 + \frac{1}{143}\right)$

Similarly, ucca of moon moves 1 revolution in 3232.5885 days. Hence motion in A days is

$$\frac{360 \times A}{3232.5885} \text{ degrees} = \frac{A}{8.979413} \text{ degrees}$$

$$= \frac{A}{9}\left(1 + \frac{0.030587}{8.979413}\right) \text{ degrees} = \frac{A}{9}\left(1 + \frac{1}{293.57}\right)$$

Here the formula is $\frac{A}{9}\left(1+\frac{1}{440}\right)$ according to siddhānta values.

Verses 61-62 : Approximate complete revolutions of planets

Mean maṅgala increases by 64 kalā in 79 solar years. Budha decreases (101) kalā in 64 solar years. Guru decreases 74 kalāṣ in 83 solar years. (61)

Mean śukra increases 43 kalā in 243 solar years. Śani increases 57 kalā in 59 solar years. Rāhu decreases 20 kalā in 93 solar years. Sīghrocca of budha and śukra and mean values of other planets come almost correct by this calculation. (62)

Notes : Siddhānta figures are already given. Figures as per modern values are calculated here.

Maṅgala movement in 79 years

$$= \frac{79 \times 365.25636}{686.97982} \text{ revolutions}$$

= 42.003056 revolutions = 42 revs + 66.009 kalā (Here it is 64 kalā)

Budha movement in 64 solar years

$$= \frac{64 \times 365.25636}{87.96926} \text{ revs} = 265.73381 \text{ revs}$$

$= 266$ revs - $95°50'$

Here the difference is 101 kalā only instead of $95°50'$

Guru movement in 83 solar years in

$$= \frac{83 \times 365.25636}{4332.58912} \text{ revs} = 6.99727 \text{ revs}$$

= 7 - 0.00273 revs. = 7 revs - 59.06 kalā only

Here the difference is 74 kalā.

Sukra movement in 243 solar years is

$$= \frac{243 \times 365.25636}{224.7008} = 395.00213 \text{ revs.}$$

= 395 revs + 46.008 kalā

Here it is 43 kalā.

Śani movement in 59 solar years is

$$= \frac{59 \times 365.25636}{10759.2262} = 2.002944 \text{ revs}$$

= 2 revs. + 63.58 kalā

Here it is given 57 kalā.

Rāhu movement in 93 solar years is

$$\frac{93 \times 365.25636}{6793.4598} \text{ revolutions} = 5.0002269 \text{ revs}$$

= 5 revs. + 4.9 kalā

Here, it is given as + 30 kalā. Since rāhu moves backwards it is to be deducted.

Budha and śukra positions cannot be calculated from their revolutions, as it will indicate their distance from sun only. For earth, their difference from sun will be r sinθ / R, where r is radius of budha orbit, R is radius of earth orbit and θ is angle (budha-sun). R is more correctly R + r cosθ = manda karṇa approximately.

Budha figures here are not correct, hence, parocca correction was made by the author.

Verse 63 : Solar dates

For tithi and nakśatra of solar year, whatever time interval has been said for adding or deducting will be kept separately (it will be used for every tithi). Now grāhya tithi of (previous year) and corresponding tithi of present year will be 7 dyas before completion of solar year. We take the difference of sun between grāhya tithi and 8 tithis before it. Sun position at iṣṭa tithi of present year

will be less by that amount. For accuracy we take average of differences of 8th day before and 8th day after grāhya tithi. This figure and earlier figure kept separately, added to sun rāśi will give the solar date (equal to current degres in rāśi).

Verses 64-65 - Use of Kautuka pāñji

There may be error in degrees of tārā graha like maṅgala etc, in kautuka pañjikā of next year. There will be difference of minutes in moon and sun from their true positions. Correspondingly, errors in palas in tithi, yoga or nakśatra will be considered negligible by the learned. (64)

No body had constructed kautuka pañjikā earlier. People can know the auspicious moments of pilgrimage, bath etc much in advance. Hence, this kautuka pañjikā is very useful, even though it is rough. (65)

Verses 66-92 : Topics in various chapters.

Siddhānta darpaṇa has described many topics in detail with examples. Still the topics are enumerated chapter wise, so that contents of the book are easily known. (66)

Topics of first chapter are - time measures from truṭi to pralaya, criteria of siddhānta text, period of creation, parts of vedas, and its importance. (67)

Subjects in second chapter are - revolution numbers of graha, ucca, and their pāta etc, number of years, months and days in a kalpa, daily mean motions of graha, their ucca and pāta and methods to find them, times of complete revolution. (68)

Topics in third chapter are - method to find count of civil days, lords of day, month and year etc, mean planet, jovian year, dhruva (constants) of graha, ucca and pāta, hāra (divisor) for ucca and pāta. (69)

Topics in fourth chapter are - circumference of earth, longitude difference, difference in start of days, corrections for longitude, bhujāntara, udayāntara, cara kalā and dhruva padaka (position) at end of dvāpara. (70)

Thus madhyamādhikāra of planets is completed in first four chapters. In next two chapters, method to find true planets have been described. (71)

Fifth chapter describes - value of east and west motion of planets, difference is motion of planets due to ucca and pāta, sines and versines, gati-khaṇḍas for bhujaphala, paridhi and its manda and śīghra paridhis, elements of pañcānga like tithi and nakśatra etc. (72)

Sixth chapter describes accurate position of moon and elements of pañjikā, junction of tithi, nakśatra and rāśi, circle and eccentric circles, ayanāṃśa, śīghra phala, krānti jyā, dyujyā and bhūjyā, cara and lagna for any place. (73)

Seventh chapter gives - Cardinal directions, bhuja, koṭi and karṇa, lambajyā, akśajyā and agrā, sama maṇḍala and other circles, finding direction, place and time, sphuṭa sun, karaṇī and koṇa śañku etc. (74).

Eighth chapter describes diameters of sun, moon and shadow of earth, manda karṇa of sun and moon, position of śara, periods of sthiti and

marda in an eclipse, lunar and solar eclipse, valana (bending) due to āyana and ākśa, lunar day and extent of grāsa. (75)

Nirth chapter describes nati, śaṅku and vitribha lagna two types of laṁbana for sun and moon, difference in shadow length due to śaṅku, and solar eclipse. (76)

Tenth chapter describes variation in visible disc due to change in nata or unnata time, 3 circles according to śara and ḍiagram for eclipse. (77)

Eleventh chapter describes - finding equal position of sphuṭa graha, correction for āyana and ākśa dṛkkarma, conjunction of planets, types of discs of planet, types of conjunction, observation through a tube. (78)

Twelfth chapter descrbies conjunction of nakśatra and a graha, dhruva of nakśatra, south and north śara, shape of nakśatras, numbers of stars in them, size of stars, yogatārā, saptarṣi, stars like Agastya, lubdhaka etc. (79)

Thirteenth chapter describes rising and settings of planets in east and west, degrees of kśetra, kāla and māna, brightness of sun, moon and venus. (80)

Fourteenth chapter gives rising and setting of moon from difference of time degrees between sun and moon, diagram of moon phases, elevation of lunar horns and horns of budha and śukra. (81)

Fifteenth chapter describes two mahāpātas - vaidhṛti and vyatīpāta. Thus tripraśnādhikāra extends for nine chapters. Here ends the first half of the book. (82)

Sixteenth chapter describes various opinions, and raises questions about motion of earth and creation etc. (83)

Seventeenth chapter gives answers to those questions, refutation of Bauddha views, location of earth in space, rotation around it of moon and sun with other planets. (84)

Eighteenth chapter describes creation, nature of planets like sun, height of hills, dimensions of earth and its oceans, area of circle and earth surface etc. (85)

Ninteenth chapter gives dimensions of orbits of planets, stars and sky and its explaination, distance of planets and stars, order of lords of year, month, day and horā, limit of visible distance of śaṅku, extent of reach of sun rays. (86)

Twentieth chapter describes two types of gola yantra, time from instruments like hemisphere, cakra etc, finding planet positions by instruments, construction of automatic wheel. (87)

Twenty-first chapter explains duration of day, night of deva and asura, explaination of lagna, alternative revolution numbers of planets, seasons and method to find cube root. (88).

Thus golādhikāra extends upto 6 chapters. Now, last three chapters are described. Twenty second chapter tells about years and their meaning, months tithi, different periods of lunar, sidereal years etc. and solar saṅkrānti etc. (89)

Twenty third chapter describes Jagannātha temple of Purī, and prayer hymns of Lord Jagannātha which are also prayer of his forms like sky, oceans and other gods. (90)

Last, twenty fourth chapter describes kautuka pañjikā and list of topics in this book. Thus kālādhikāra ends in three chapters. Book is over in two halves gaṇita and gola. (91)

Some persons don't read whole text and go through the summary only. In conclusion, all topics are mentioned briefly so that rich and easy going can quickly glance through the topic of astronomy and its results which is like eyes of vedas. (92)

Verses 93-139 : Longitude and latitude of 109 places.

In seventh chapter, the topics of time, place and direciton have been described in detail. Still location of famous places of world is not known to the astrologers. Hence, they are unable to calculate eclipse etc at different places. (93)

Hencé, the longitude of places from assumed prime meridian (through Jūnāgarha of Gujarāta here) and distances from equator are given here in kalā (minutes of angle). With help of that, an astronomer can prepare ephemeries of any country by making slight changes in a standard ephemeries of a particular place.

Notes : At present, the distance of longitude is expressed from standard longitude O° of Greenwich, London. In India, since long, Ujjain has been taken as prime meridian, from which east and west longitudes were measured. Ujjain was central place of India from which all places of India and even outside in Asia, Africa were named according to direction from Ujjain. It is probable that the prime meridian in Asura kingdoms was between Cairo and Alexandria of Egypt, where

great pyramid was constructed to mark O° longitude. Reasons for giving this distance from Jūnāgarha of Gujrāt is not known. There is however, one significance of that meridian. Oṅ sea coast at that meridian Somanātha temple was constructed from where south meridian will not meet any island till south pole. However, the chart here appears to be copied from some Gujarāti text book for purpose of naval records. Source may be a nautical almanc or Jaina jyotiṣa book.

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
1.	Jina durga or Junāgarha	0	1294
2.	Morpur	0	1371
3.	Bholapur	0	1428
4.	Rāmagarh	0	1643
5.	Jalālābād	0	2067
6.	Gujarāt	5	1325
7.	Unāpura	7	1255
8.	Narmadā sāgara saṅgama	22	1300
9.	Mumbai (Bombay)	23	1137
10.	Udaipur	30	1470
11.	Lahore	36	1900
12.	Śrīnagara (Kashmīra)	43	2053

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
13.	Jammu	43	1952
14.	Jaipur	48	1620
15.	Ujjainī	50	1390
16.	Vindhyācala	52	1350
17.	Kurukśetra	56	1790
18.	Śrīraṅgapattana	62	742
19.	Karṇāṭaka	65	737
20.	Gaziabad	66	1717
21.	Kanyā-kumārī	68	428
22.	Bhopāl	69	1400
23.	Braja	70	1660
24.	Agrā	74	1631
25.	Kāñchī	83	660
26.	Śrī Veṇkaṭa (Tirupati)	80	780
27.	Rāmeśvaram	87	556
28.	Draviḍa middle	87	660
29.	Bundela khaṇḍa	87	1500
30.	Jabalapur	93	1395
31.	Chandrapura	97	784
32.	Krishnā mouth	102	950

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
33.	Raipur	109	1269
34.	Rāja-Mahendri	111	1022
35.	Prayāga	111	1520
36.	Ayodhyā	115	1605
37.	Ratnapur	115	1366
38.	Godāvarī mouth	117	1006
39.	Kāshī	124	1517
40.	Raigarh	127	1308
41.	Viśākhapattan	127	1062
42.	Sambalpur	130	1283
43.	Śrīkākulam	133	1094
44.	Nepal centre	134	1688
45.	Mahendra giri	139	1140
46.	Gayā	143	1488
47.	Brahmapur	143	1154
48.	Ganjām	145	1164
49.	Mādhava kśetra	146	1222
50.	Purī	152	1188
51.	Bhuvaneśvara	152	1214
52.	Cuttack	152/30	1228

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
53.	Mahānadī mouth	161	1213
54.	Baleśvara	161	1292
55.	Jaleśwara	165	1313
56.	Medinīpur	167	1350
57.	Vardhamāna	172	1390
58.	Gangā sāgara	175	1325
59.	Navadvīpa	177	1440
60.	Calcutta	177	1354
61.	Raṅgapur	186	1546
62.	Dhākā	199	1425
63.	Sylhet (Śrīhaṭṭa)	213	1495
64.	Lohit river	242	1654
65.	Rann of Cutch	2 west	1440
66.	Cutch	7 w	1388
67.	Sindhu	14 w	1560
68.	Dwārkā	15 w	1338
69.	Kabul	15 w	2067
70.	Afghanistan	19 w	1875
71.	Sindhu mouth	24 w	1417
72.	Dehayajna	37w	2088

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
73.	Brahmadesh (Myammar)	257	1103
74.	Irāvatī mouth	257	948
75.	Persia (Iran)	157 w	1920
76.	Arab Centre	237 w	1410
77.	Turky	337 w	2190
78.	Śrī Lankā	100	488
79.	Hindukush Centre	16	2125
80.	West Himālaya	38	2104
81.	East Himālaya	243	1687
82.	Central Himālaya	141	1733
83.	Gaurīśaṅkar (Everest)	150 Height	1700 19348 hand
84.	Mānasarovar	106 Height	1840 14667 hand
85.	Kailāśa	110	187
86.	Tibbet	193	187
87.	Central China	393	1860
88.	Capital Peking	393	2400
89.	Centre of Russia	493	3990
90.	Centre of Europe	507w	3180
91.	London	707w	3090

Sl. No.	Place	East of Junāgarh (kalā)	North of equator kalā
92.	Africa north coast	—	507
93.	Centre of north America	1707w	3006
94.	South America	1287w	1320 South
95.	Australia	613	1500 south
96.	Boudha	137	1250
97.	Manjūṣā	140	1135
98.	Aṅgula	144	1247
99.	Nayāgarh	145	1207
100.	Khaṇḍaparā	146	1216
101.	Talcher	146	1258
102.	Pārīkuda	147	1181
103.	Raṇapur	147	1205
104.	Barambā	147	1224
105.	Dhenkanal	150	1238
106.	Konark	155	1193
107.	Jājpur	158	1251
108.	Mayūrbhanj	158	1312
109.	Nīlagiri	160	1287

Verses 140-141 : Distance between two places.

Difference between akśakalā of two places is multiplied by (5026) and divided by (21,600). Result will be squared. (140)

East west longitude difference in minutes is multiplied by 5026 and divided by 360. Result will be multiplied by half the sum of sine of colatitude of two places and divided by trijyā (3438). Result will be squared and added to the previous square. Square root of the sum will be straight distance between two palces.

Notes : North south distance is proportional to difference between latitudes. For difference of 21,600 kalā, we get the circumference 5026 yojana.

Hence for given difference, north south distance is

$$\frac{\text{Akśa kalā} \times 5026}{21{,}600} \text{ Yojāna}$$

East west difference depends on sphuṭa paridhi which changes with latitude and is proportional to sine of colatitude. Hence we have taken average of sines of colatitudes of two places. This multiplied by 5026 and divided by trijyā gives sphuṭa paridhi.

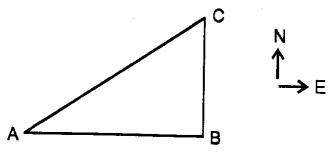



Figure 1 - Distance between places

East west difference in degrees (not mentioned in text) is multiplied by sphuṭa paridhi and divided by 360 to give result in yojanas.

From A to C, AB is east west distance and BC is north west distance as shown in figure 1. Then ABC is a right angled triangle. Hence AC = $\sqrt{AB^2 + BC^2}$

Verses 142-143 - I have not accepted without evidence anything written by earlier astronomers about maximum latitude of planets, oscillation of ayana, valana etc. I have verified everything with observations. (142)

In dark room we make a small hole at the roof. At mid day, we keep a water pot at place where sun light falls on ground. The incident rays of sun will be reflected at same angle to the vertical. This angle will be spaṣṭa (apparent) declination of sun at that place. By adding or substracting north or south krānti, we get latitude of the place. (143)

Verse 144 : Reasons for writing the book

In many ways learned men have inspired me to write this book. Old calculation methods do not give correct positions of planets as observed, so that, religion can be protected. This will refute the arguments of those who accept the rotation of earth. My stray talks were laughed at in gathering of the learned. For all these inspirations, I pray to these learned men. (144)

Verses 145-147 : Family of author

In Khaṇḍaparā of Purī district (Orissa, India), a king named Vairāgī was born, who contained the elements of eight lokapālas (protector of eight

directions and angles) - Varuṇa, Indra, Rudra, Yama, Kubera, Nairṛti, Agni and Pavana. For his achivements, king Vairāgī, had been given titles of Marddarāja and 'Bhramara vara' by king of Purī himself. He was like moon of Khaṇḍaparā, the brightness of whose knowledge and greatness was spread all around. (145)

Due to grace of Lord Kṛṣṇa, his devotee king Vairāgī got a son named Śrī Nīlādri Singh, who defeated his enemies with his fierce prowess like Bhīṣma, son of Gaṅga. Son of Śrī Nīlādri was Śrī Nṛsiṃha, who excelled sun and moon with his fame and influence. He got his family titles (mardda rāja and bhramaravara) and, by meditating upon the feet of the undefeated (Lord Viṣṇu), he was freed of three troubles (of body, mind and world). (146)

Family of Baghela Kśatriyas is as spotless as ocean of milk where goddess of wealth (Lakśmī) resides. In this family itself, Srī Nṛsimha had risen like a full moon. His own son Śrī Śyāma-Bandhu was very learned, and destroyed his ignorance and darkness through feet nails of 'Dīnabandhu' (Lord Viṣṇu, friend of poor). He belonged to Orissa, was called a Siṃha (Lion) and was like a sun for Bharadvāja family (gotra) as a lotus. (147)

Verses 148-149 : Author

I am son of the same Śrī Śyāma Bandhu Siṃha. I have been blessed by my guru Śrī Madhusūdana Mahāpātra. Kadgarāya (wielder of sword - a title) Śrī Ānanda Miśra taught me (astronomy). My only shelter is feet of Lord in three forms - Brahmā, Viṣṇu and Maheśa. I have

always erred in following my duties according to demand of the times,' Still I could complete this siddhānta by grace of god. It is offered in the feet of the same lord. (148)

I was born in Kali year 4936. It took me 34 years to complete this text. No man has the complete knowledge. Hence, the learned are requested, only to accept the truth in this text and ignore the errors. (149)

Verses 150-166 : Conclusion of chapter

The learned are requested not to criticise the innovations in this book only because they are new. For tallying the calculations with observations Lalla, Bhāskara, Śatānanda and Āryabhaṭa etc also have written many new things in their texts, and people accept them. What is the fault, if I too have done the same? (150)

Even before a formal education of siddhānta texts, I developed intense curiosity about limit of sky, motion of graha and nakśatras. Then, I was engaged in their research. With completion of this second half, famous as gola gaṇita consisting of two out of nine adhikāra (i.e. golādhikāra and kālādhikāra), siddhānta darpaṇa is complete. (151)

Siddhānta darpaṇa is specially famous and respected for its true calculation of planets as they are observed. This contains 24 chapters in 5 adhikāras. This text is over. I wish that this should remain useful for long. (152)

I pray to Lord Kṛṣṇa, who controls time divisions starting with truṭi to the end of pralaya, who subdues the king of dead, Yama, being the blackest, always blissful, who is crest jewel of Nīlācala hill and death of death itself. (153)

Lord of my heart may always reside in Nīlācala (i) who makes the planets move as per astronomical calculations so that people timely obey the duties described in vedas and smṛtis written by direction of god, (ii) who takes care of the whole moving and non-moving world (iii) who is ever ready with his raised hands to protect fallen like me, (iv) and whose blue colour is bright as Indranīla (blue emerald). (154)

Thus ends the concluding 24th chapter of siddhānta darpaṭa written as a text bok for true calculation of plaents by śrī Candraśekhara born in famous royal family of Orissa. (155)

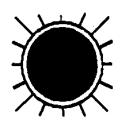
This book contains a total of 2,500 verses out of which 2,284 are written by me and the rest 216 are quoted from others. May these beautiful verses give good results. (156)

Sometimes the great warrior Bhīma also was defeated in war. What can be spoken about a paṇḍita will little knowledge like me ? Hence, the learned are requested to correct the errors and repeatitions, wherever they occur. (157)

Siddhānta darpaṇa is essence of astronomy, which is most important of the three parts of jyotiṣa, and is difficult to understand. This text has come out like a lion from cave mouth of the family of Nīlādri Siṁha (Alternate meaning - This is pronouncement of lord Kṛṣṇa himself who is lion of Nīlācala Purī). Like a lion, this may vanquish the power of England scholars like elephant tusks, who reject the theory that earth is fixed in the infinite sky and think it as moving. (158)

Nīlācala is on coast of ocean which is king of holy places. This is like a pot of verses made by Indra and other gods through which Lord Jaganātha is worshipped. My body may fall in the same Puruśottama (The great being) region. (159)

Note : Śrī Chandraśekhara had really expired while praying to Lord Jagannātha in Purī temple.

In 914th year of Mukundadeva or śaka year 1814, mārgśirṣa kṛṣṇa 9th, saturday, compilation of the book was over. (160)

Appendix

Sanskrit terms according to context

1. Names of planets

Sun - Ravi, sūrya, arka

Moon - Candra

Mercury - Budha

Venus - śukra

Mars - Maṅgala

Jupiter - Guru, Bṛhaspati

Saturn - Śani

Adjectives of planets -

Saura - of sun, solar

Cāndra - of moon, lunar

Bārhaspatya - of juipter, jovian

2. Terms of ecliptic etc

Ecliptic - Krānti vṛtta,

Equator - Viṣvra vṛtta, Nāḍī vṛtta

Vṛtta = Circle

Nakśatra = stars, constallations or groups of stars marking position of ecliptic.

Kśitija - horizon

Svastika - Zenith or nadir point

Kha svastika - Zenith

Bha = nakśatras (27 in number)

Bhagaṇa = Revolution (round nakśatra circle)

Śara or vikśepa - Distance from ecliptic along perpendicular to it.

Krānti - Distance from equator along perpendicular to equator.

Rāśi - 12 divisions of zodiac of 30° each,

- position of a planet in ecliptic expressed as rāśi and its subdivisions

- any quantity or heap (literal meaning)

Dhruva - Pole of equator, north or south

Kadaṁba = Pole of ecliptic (north)

Kalaṁba = south pole of ecliptic

Dhruva or dhruvāmśa = Distance on ecliptic measured along the line from pole of equator used for a star.

Digamśa = Distance in hoirzon circle measured south ward from east point

Unnata amśa = Degree of elevation

Nata aṁśa = Depression angle from zenith

3. Terms of time, angle etc

Bhagaṇa = 1 complete revolution of 360°

Cakra = 1 complete revolution, posiiton at 0° from ecliptic beginning or centre of its slow or fast motion.

Rāśi = A division of 30° length

- position of a planet on ecliptic from its 0° or from centre of fast or slow orbit with its divisions

Amśa = Part, degree of angle

Kalā or liptā = minute of angle

vikalā or viliptā = seconds of angle

Parā = 1/60 of seconds of angle

Viparā = 1/60X60 of seconds of angle

Kalpa = 1 day of Brahmā of 4.32 bilion years

- The period in which all planets, their apogee and pāta make a complete revolution

- Equal to 1000 yuga or mahāyuga

Pralaya = Dissolution of world. Time till pralaya is same as kalpa. Bigger pralaya time is of 100 years of Brahmā = 72,000 kalpa

Yuga or Mahā yuga = A period of 43,20,000 years.

Pāda yuga - 4 parts of yuga or mahā yuga, named as satya or kṛta, tretā, dvāpara and kali. According to Āryabhaṭa each is 1/4th of yuga. According to others, the above are in ratio of 4 : 3 : 2 : 1. Thus Kali is 1/10 of a yuga equal to 4,32,000 years.

Vatsara = Tropical year, any year

Saṁvatsar = Any type of year, a civil year of 360 days i.e. 12 mouths of 30 days each used in context of vedāṅga jyotiṣa of yajurveda.

Vatsara types - Saṁvatsara = normal year (solar) starting between śukla Ist to 6th tithi

Anuvatsara - year lagging behind, the solar year which starts between śukla 7th to 12th tithi

Parivatsara - year at opposite end, solar year starting between lunar dates of śukla 13th to kṛṣṇa 4th.

Idvatsara = Advanced year, solar year starting between lunar dates kṛṣṇa 4th to 9th.

Idāvatsara = more advanced year, solar year starting with lunar dates 10th to 15th of dark half.

Guru or Bārhaspatya varṣa = Jovian year in which jupiter covers 1 rāśi 30° with mean motion.

Varṣa = year

Varṣa of different duration = Nākśatra of 360 sidereal days

Cāndra = 12 synodic lunar months

Guru = 30° movement of mean guru in 1 rāśi

Divya = 360 years

Pitṛ = 360 lunar months

Saura - Solar year (sidereal)

Māsā - Measured time, month

Lunar month - period between successive full moon or new moon

Saura māsa - Movement of sun in 1 rāśi

Sāvana māsa = 30 sāvana (civil days) which is interval between sunrise to next sunrise.

Nākśatra māsa = 30 sidereal days

Dina = Day (day + night both)

- Only bright part with day light

Types are - Nākśatra dina - period between two successive rising of a star.

Sāvana dīna - period between successive rising of Sāvana dina of other planets is period between two risings of the planet.

Cāndra dina = sāvana dina of moon

- tithi = when moon advances by intervals of 12° over sun

Nakśatra - star or constellation

- As a time measure it is duration for which moon remains in a particular star.

Daṇḍa or nāḍī = 1/60 of a nākśatra dina, period of 1/60 revolution of earth.

Pala or vināḍī = 1/60 daṇḍa

Ghaṭī = same as daṇḍa

Vipala = 1/60 pala = 1/60 X 1/60 daṇḍa

Asu, or prāṇa - 10 vipala = 1/6 pala (about 4 second) Defined as average breathing cycle of man or period for 1 kalā rotation of earth. Thus 1 kalā at equator is same as 1 asu time. Since asu is felt by breathing, the time units of asu and bigger are tangible. Others are intangible, so small that they cannot be felt.

4. Zodiac Divsions

1. Meṣa - Aries	2. Vṛṣa - Taurus
3. Mithuna - Gemini	4. Karka - Cancer
5. Simha - Leo	6. Kanyā - Virgo
7. Tulā - LIbra	8. Vṛścika - Scorpio
9. Dhanu - Sagittarius	10. Makara - Capricorn
11. Kumbha - Aquarius	12. Mīna - Pieces

In western astronomy, these rāśis of 30° intervals are measured from point of ecliptic which meets equator (equinox) after which it rises north of equator. This point moves west wards relative to fixed stars called 'ayana gati' at speed of 50.7" seconds per year. In India, this rāśi measurement from equinox point is called sāyana, because the movement of equinox is west wards, and its negative motion is added to position with respect to fixed stars.

There are two other systems of rāśi measurement Rāśi without any prefix means position from

fixed stars (citrā 179.9° or Revatī 0.0°). This is also called nirayana. The distance of planets from their mandocca or śīghrocca also is expressed by names of these rāśis. This distance is called manda or śīghra kendra.

27 constellations of 13°20′ intervals

1. Aśvinī
2. Bharaṇī
3. Kṛttikā
4. Rohinī
5. Mṛgasirā
6. Ārdrā
7. Punarvasu
8. Puṣya
9. Aśleṣā
10. Maghā
11. Purvā Phālgunī
12. Uttarā Phālgunī
13. Hasta
14. Citrā
15. Svātī
16. Viśākhā
17. Anurādhā
18. Jyeṣṭhā
19. Mūla
20. Pūrva āṣḍha
21. Uttara Āśāḍha
22. Śravaṇa
23. Dhaniṣṭhā
24. Śatabhisaj
25. Pūrva bhādarpada
26. Uttara bhādrapada
27. Revatī

There is system of unequal divisions also in which 28th nakśatra is inserted between uttara āṣāḍha and śravaṇa. In both systems aśvinī starts with start of meṣa and revatī ends with end of mīna rāśi. In unequal divisions, extent of nakśatras are generally the distance covered by mean moon in one day i.e. (790′35″), where as, in equal division all are of 13°20′ = 800′.

In unequal divison extra duration of moon's revolution period (254′18″35‴) is allotted to abhijit. Three uttarā nakśatra (12, 21 and 26), Rohinī (4),

punarvasu (7) and anurādhā (17) have double the length (1185′52″18‴) To compensate, six nakśatras have half duration, (2) bharaṇī, (6) ārdrā (9) aśleṣā (15) svāti (18) jyeṣṭhā (24) śatabhiṣaj

Length units

1 yojana = Distance travelled by light in 1 truṭi i.e. 1/1,12,500 sec.

- 1/1600 of earth's diameter according to sūrya siddhānta = 5 miles or 7.9 Kms approximately.

- or 1/3200 of earth's circumference according to Varāhamihira and Āryabhaṭa = 7.52 miles.

- yojana or mahā yojana for stellar measurements is 5 terrestrial yojana = 40 miles (8 yojanas of sūrya siddhānta

- 16000 hands (1 hand = 1-1/2 ft.) according ot sūrya siddhānta or 32000 hands according to Jaina measures.

Hand or hasta = about 1-1/2 feet.

1 aṅgula = 1/24 hand = 3/4 inch approx.

- any measure of length, which is subdivided into 60 units. All measurements will be same aṅgula measures. This indicates dimension of length.

5. Terms used in planetary motion

Kakśā or kakśā vṛtta - Approximate circular orbit of a planet's motion.

Mandaparidhi - Small circular orbit around mean planet in which true planet moves with same angular speed in opposite direction. With varying radius this becomes equivalent to an elliptical orbit.

Prati vṛtta or Prāti maṇḍala - Eccentric circle of same size as kakśā paridhi, but centre removed towards mandocca or śīghrocca by distance equal to maximum value of correction to mean planet.

Śīghra paridhi - To convert the heliocentric position of a planet to geocentric position, planet corrected for elliptical orbit has another small orbit around it on which true planet moves. This corresponds to smaller orbit between planetary orbit or earth's orbit.

Mandocca - Apogee or farthest point of elliptical orbit where plaent is farthest (ucca) and hence slowest (manda)

Śīghrocca - Position of planet moving in slower orbit around earth. For outer planets it is position of sun itself. For inner planets, it is that planet, i.e. budha śīghrocca is budha itself.

Kendra - Centre of a circle

- Distance of planet from apogee or śīghrocca. They are called manda and śīghra kendra. Manda kendra is like anomaly, which is distance from perigee (180° from apogee). Thus manda kendra = anomaly + 180°.

Karṇa - Hypotenus of a right angled triangle

- Radius of a circle

- Distance of a planet from centre of orbit

Manda and śīghra karṇa - True distance of a planet after calculating manda (elliptical) correction is manda karṇa. Geocentric position is śīghra karṇa.

Cāpa - Arc, part of a circumference measuring angle. Its length is measured in kalā units where

whole circumference is 21,600 kalā, or degree when circle is 360°

Jyā - Chord of arc, straight line joining end points of a cāpa. This is also measured in kalā.

Cakra - Circle

- 1 revolution of 360° or 21,600 kalā (liptā)

Jyā - Jyā is short form of Jyārdha (half chord). It is half the chord of double the arc. In radius of unit circle it is equal to sine ratio of trigonomentry. This is measured in kalā, for circumference of 21600 kalā or radius 3438 kalā. Thus jyā in kalā = 3438 sine i.e. radius X sine.

Trijyā = Jyā of 3 rāśi. Since sine of 3 rāśi is 1, trijyā is equal to radius 3438 kalā.

Vyāsa - Diameter

Vyāsārdha - half of diameter. This is more used for radius of any other circle. In main orbit under consideration trijyā = 3438 only.

Koṭijyā - Distance of Jyā from centre or equivalent to Jyā of the koṭi angle i.e. difference of angle and 3 rāśis. This is trijyā X cosine

Utkrama jyā - Distance of Jyā from circumference along perpendicualr radius. This ratio is not used in modern trigonometry. This may be called verse-sine or versine. Versine of angleθ is versθ = R (1-cosθ) where R is radius

Bhuja - Base of a right angled triangle

- Angle from starting point in first and third quadrant and the complement in that quadrant in 2nd and fourth quadrants.

Bhuja jyā - Jyā of bhuja of that angle. Bhuja of angle θ in differnt quadrants is $\theta°$, $180° - \theta$, $\theta - 180°$ and $360° - \theta$ in Ist to 4th quadrants.

Koṭi - Perpendicular of a right angled triangle

- Complement of bhuja angle

Bhujaphala - Correction to mean planet for manda or śīghra motions which is proportional to Jyā of main of kendra (manda or śīghra) measured in main orbit.

Dohphala - Bhujajyā of radius of manda or śīghra circle. This distance measured at circumference of main circle is bhujaphala.

Koṭiphala - Koṭijyā for radius of manda or śīghra circle. This is component of change in distance of planet in direction of mean planet.

Mandaphala - bhujaphala of mandaparidhi

Manda koṭiphala - Koṭiphala in mandaparidhi

Śīghra phala - Bhujaphala in śīghra paridhi

Śīghra koti phala - Koṭi phala in śīghra paridhi

Khaṇḍa - Parts of angle at which calculation of variables is done. For jyā, khaṇḍa is of length $3°45' = 225'$ i.e. 1/24 of a right angle. Rising times are calculated at 1 rāśi intervals and krānti at interval a half rāśi. Within the interval, variation is considered proprotional.

Mārgī - Planet moving forward i.e. east ward

Vakrī - Planet moving west ward, i.e. retrograde motion

Calakarṇa = same as śīghra karṇa

Pākśika phala - Correction in moon's position which varies fortnightly (pākśika). It is called variation.

Tuṅgāntara correction - Correction in moon's motion due to attraction of mandocca (Tuṅga = top, antara = difference) by sun. It is called evection. The equation combines second order correction of elliptic orbit also.

Digamśa correction - This is annual variation in correction ot moon due to vaying distance from sun.It equals 1/10 of sun's mandaphala, hence is called diagamśa (dig = directions i.e. 10, amśa = part).

6. Calender Elements

Pañcāṅga = A calendar with 5 limbs (aṅga) - Vāra, tithi, nakśatra, karaṇa and yoga

Vāra = Weekdays

Tithi - Lunar day depending on its phase. These are 30 tithi in a synodic revolution of moon i.e. 1 revolution ahead of moon. Hence tithi

$$= \frac{\text{moon} - \text{sun}}{12^\circ}$$

Karaṇa - Half part of a tithi is called karaṇa

Thus karaṇa = $\frac{moon - sun}{6^\circ}$. From 14th tithi 2nd half in kṛṣṇa pakśa to first half of śula pakśa Ist tithi, 4 kararas are fixed - śakuni (hawk), catuṣpada (quadruped), nāga (serpent), kinstughna is an animal. In remaining 56 half tithis 7 movable kararas rotate 8 times 1. Bava (= lion, Bābara in Hindi), 2. Bālava (=tiger, powerful), 3. Kaulava (kola = boar), 4 Taitila (-donkey), 5 Gara or gaja

(elephant), 6. Vaṇija (=Trader) and 7. Viṣṭi or Bhadrā (=cow).

Yoga = Sum of longitudes of moon and sun. At 13°20′ interval equal to a nakśatra divison, 1 yoga changes. Thus there is a cycle of 27 yogas in about 25.42 days

Nakśatra - The nakśatra division in which moon is located.

Pakśa = Literally means wings. This is two halves of a lunar month. Fortnight

Śukla pakśa - Bright half of lunar month or bright fortnight

Kṛṣṇa pakśa - Dark half of a lunar month or dark fortnight.

Amāvasyā - Last day (tithi of dark half).

Amānta - Ending point of amāvasya or beginnig of bright half, new moon (when moon = sun)

Pūrṇimā = Full moon or last tithi of bright half.

Pūrṇimā - Ending point of pūrṇimā when moon-sun = 180°.

Names of lunar months

Lunar months are named according to nakśatra near which moon goes on pūrṇimā day. Amānta months ends with amavasyā and purnimā is in middle. This is generally followed. Pūrṇānta month ends with pūrnimā. Nakśatra numbers are given in bracket after the month name.

1. Caitra (14)
2. Vaisakha (16)
3. Jyeṣṭha (18)
4. Āṣāḍha (20, 21)

5. Śrāvaṇa (22)
6. Bhādrapada (25, 26)
7. Āśvina (1)
8. Kārttika (3)
9. Mārgaśīrśa (5)
10. Pauṣa (18)
11. Māgha (10)
12. Phālguna (11, 12)

At present Caitra month starts in which sun, enters meṣa rāśi with mean motion. Solar months also are given the same names.

Ayanas

Ayana is movement of sun north or south of equator. Krānti is position north of south of equator. When sun krānti is southern most, it starts its northward motion called uttarāyaṇa.

When sun is in northern most point it starts southward motion called dakśiṇāyana.

Two ayana make one 'hāyana' (full year). Earlier solar year started with Mārgśīrṣa month (9th) hence it was called agra-hāyaṇa (first of year). Since it was having longest nights it was called Kṛṣṇamāsa (Ref. Gītā), which has become 'Christmas'.

As it was start of 'divya dina' of 1 solar year it is called 'baḍā dina' (great day).

Seasons

Two months make a season. Thus there are six seasons which start with year, covering two month each -

1 Vasanta (spring)
2 Grīṣma (summer)
3 Varṣā (rains)
4 Śarat (autumn)
5 Hemanta (cold)
6 Śiśira (very cold).

Tithi names

Tithi names are same and given same numbers also in each fortnight. Only last tithis are named and numbered differently. Pūrnimā is 15th and amāvāsyā is 30th as it is last tithi of month.

1 Pratipadā	2 Dvitīyā
3 Tṛtīyā	4 Caturthī
5 Pañcamī	6 Ṣaṣṭhī
7 Saptamī	8 Aṣṭamī
9 Navamī	10 Daśamī
11 Ekādaśī	12 Dvādaśī
13 Trayodaśī	14 Caturdaśī

15 or 30 Pūrṇimā or Amāvāsyā

7. Terms used in Eclipse

Biṃba - Diameter of a planet or star, angular diameter

Biṃba yojāna - Diameter in yojana

Sparśa (kāla) - Time of first contact

Nimīlana - Second contact, when eclipse is just complete or maximum

Unmīlana - Third contact, last point of complete eclipse

Mokśa - Fourth contact, when eclipse is to end.

Sthithi kāla - Complete duration of eclipse from sparśa to mokśa

Sthiti ardha - Time between mid point and sparśa or mokśa time

Marda (or vimarda) kāla - Duration of total eclipse between nimīlana and unmīlana

Marda (or vimarda) ardha - Time between mid eclipse and nimīlana (or unmīlana)

Grāsa - Length of diameter (angle or proportionate) eclipsed at a particular time.

Chādaka - Covering planet or shadow of earth

Chādya - Covered or eclipsed planet

Chāyā - Shadow (of earth)

Bhū - earth

Laṁbana - Parallax due to seeing a planet from surface of earth in stead of calculated position from centre of earth, in east west direction

Nati - parallax in north south direction

Valana - Change in direction of peripheral points of disc due to latitude and declination at a place

Ākśa valana - valana due to latitude

Āyana valana - Valana due to krāntī

8. Miscellaneous terms

Gola - Sphere, spherical trigonometry

Akśāmśa = Latitude in degrees

Deśāntara - Longitude difference in degrees, or yojanas

Śaṅku - Gnomon - a vertical rod whose height is 12 units called aṅgulas

- Height of a heavenly body on celestial sphere above horizon = R cos Z where R = 3438 kalā and Z = angular distance from zenith

Yāmyottara - meridian circle, passing through north and south poles and zenith

Samamaṇḍala = Great circle passing through east west points and zenith

Unmaṇḍala - Great circle through east-west points and dhruva. It is horizon at equator for same longitude.

Dṛkmaṇḍala - Great circle passing through zenith and a planet.

Lagna = Point of krānti vṛtta (ecliptic) rising on east horizon for a place at a time.

Madhya lagna - Point of krānti vṛtta on yāmyottara

Asta lagna - Point of kranti vṛtta on west horizon

Vītribha lagna - 3 rāśi's less than lagna. It is nearest point to svastika (zenith) and close to madhya lagna.

Koṇa - Angle directions

Koṇa Śaṅku - Śaṅku of sun when it comes on circle through zenith and angle direction.

Cara (Cara jyā) - Difference of half day length compared to equator (15 daṇḍa)

Kujyā - Extra drameter of diurnal circle for increased half day length

Dyujyā = Diameter of diurnal circle

Ahorātra = Day and night

Yoga - (Sun + moon) longitude

- Conjunction betwen planets and stars

- Conjunction of nakśatra, tithi, vāra etc. which is auspicous or bad.

Chāyā - Shadow length

Chāyā karṇa = Distance from śaṅku end to shadow end

Palabhā - Shadow of a 12 aṅgula śaṅku on equinox mid day.

Pala karṇa - Chāyā karṇa of 12 aṅgula sadow on equinox mid day.

Agrā - Azimuth or digamśa from east horizon point at the time of rising

Karṇa vṛttāgrā - North south distance between shadow end of śaṅku and palabhā.

Pāta or mahāpāta - When (sun + moon) = 180° (sāyana values) or 360°, their krāntis are same. These are called pāta or mahāpāta. When it is 180° pāta is called vyatī pāta, when it is 360° it is called vaidhṛti.

Saṅkrānti - Crossing of sun from one rāśi to another. Sankrānti period is that period when some point of sun disc is in both the rāśis.

Dṛkkarma - Finding corrections due to valana

Sīta - Bright part of moon

Asita - Dark part of moon

Yaṣṭi - Any stick

- A measured stick with plumb line to measure elevation of a heavenly body, or hill etc.

- Height of sun in vertical direction from horizon of equator

Drgjyā - Jyā of dṛggati

Dṛg gati - Arc of ecliptic between sun or moon and central ecliptic point (vitribha)

Dṛkkśepa - Ecliptic zenith distance (zenith distance of vitribha) or its jyā.

Name and subject index

Names are arranged in order of English alphabet. Numbers are in two components separated by dash. First number indicates chapter number. O indicates general introduction in beginning. Suffix A indicates introduction before chapter. Suffix B is appendix after chapter. Second number indicates paragraph no. of introduction or appendix. It is verse number of that chapter at end of that topic. Suffix n indicates notes after that verse. This follows by paragraph number of the note.

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